Analysing dynamics of risk aversion in FX market using the Heston volatility model

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Abstract

In this paper we investigate how dynamics of FX implied volatility for EUR/RON rate evolved against an important event that was considered the moment of Lehman Brothers default. The main objective of this study is in fact twofold: we intend to analyze the way in which the expectations of professional investors are changing in the face of turbulences and corroborating with the FX evolution after that trigger point to look also if there it could be the case of an epidemiological relation between professional forecasters views and short-run movements of EUR/RON rate. In a second timeframe, our results could be interesting to look at how quickly these views are reflected in the future FX levels and movements, thus making a connection with the theory related to informational efficiency. Not lastly, we intended to extrapolate the use of a micro model of pricing to obtain information at a macro level about the local FX features. Calibrating the Heston model to market data surprisingly we found that despite the high volatility and jumps in EUR/RON evolution, the model performs very well for short-, mid- and long-run. The most important conclusion underlined by our results is that on the first ground, investors react quickly to news in international markets and their views are very well and in a short time incorporated in future FX market evolution.

Keywords: risk-aversion, Fx market, implied volatility, Heston, smile, stochastic volatility.

1. Introduction

The use of micro models of option pricing to extract information on the expectations in regard with future developments in FX, bond, equity or short term markets was pioneered within the Bank of England and

Federal Reserve research departments. In place of their traditional purposes related to the pricing or hedging activities in derivatives market, these continuous stochastic models could be used for policy purposes as to look at the expectations on future inflation or financial stability. Taking into account the considerations made before, we investigate the evolution of market expectations from Romanian FX market using the volatility smile. For this purpose we used market quotes of risk reversal and butterflies strategies to calculate the implied volatilities. In order to analyze the movements in investor's perceptions we have split the data before and after the starting of actual financial crisis. We set the day of Leman Brothers' collapse, 15 September 2008, as the beginning of adverse spill-over scattering worldwide and implicitly in Romania. Based on market data we calibrate the Heston model. In this way we obtained the parameters and the volatility smile in comparison before and after the threshold. Our view is that expectations of professional investors have an important impact on the short movements of local exchange rate in an epidemiological way. In this paper we investigate in fact the case of existing "epidemiological" relationship between forecasts of professional investors and short-term developments in the EUR / RON exchange rate. Even that we don't call a typical epidemiological model as those ones used in biology fields of research, we investigated the hypothesis according to which after the Lehman Brothers crash and implicit the generation of the current financial crisis, the forecasts of professional investors pose a significant explanatory power on the futures short-run movements of EUR / RON. How it works this mechanism? Firstly, the professional forecasters account for the current macro, financial and political states, then they elaborate forecasts. Secondly, based on that forecasts they get positions in the Romanian exchange market for hedging and/or speculation purposes. But their positions incorporate in addition different degrees of uncertainty. In parallel, a part of their anticipations are disseminated to the public via media channels. Since some important movements are viewed within macro, financial or political fields, the positions of professional investors from FX derivative market are activated. The current study represents a first step in that direction of analysis for Romanian case.Even that our analysis don't use specific epidemiological models, the results are very informative in regard with the previous mention.

Merton in 1973 suggested that the volatility can be modelled as a deterministic function of time. Although this might explain the volatility levels at different maturities but still not explain the smile shape for different strikes. Dupire (1994), Derman and Kani (1994), and Rubinstein (1994) came up with the idea of allowing not only time, but also state dependence of the volatility coefficient. The local volatility approach leads to a complete market model but anyhow cannot explain the persistent smile that doesn't disappear as time passes. The further approach was the stochastic volatility concept and the initiators have been Hull and White (1987), Stein and Stein (1991), and Heston (1993. These are multi-factor models with one of the factors being used for the dynamics of the volatility coefficient. Different driving mechanisms for the volatility process have been proposed, including GBM and Ornstein-Uhlenbeck type processes.

2. Theoretical Model

Considering a set of European options of different strikes on the same underlying, two types of volatility appears: the implied volatility surface and local volatility surface. In case the market prices of options are used to obtain these areas, they are market implied volatility.

The Black-Scholes-Merton (BSM) formula lead to the price of the option as a function depending on the following factors:

a. The sport price of the underlying asset (S_t)
b. The exercise price of the option –strike (X)

c. The risk-free interest rate (r)
d. The option's maturity (T-t)
e. The volatility of the underlying asset's returns (σ)

The volatility that input in the Black-Scholes-Merton (BSM) formula yields the price represents the implied volatility, also known as implicit volatility. The model assumes that the volatility is constant, given that all the options should have the same market implied volatility. Anyhow, the expectations in the market are different since the market prices of options ordered by strike and maturity leads to a surface that is not flat-shaped.

When considering that the underlying asset is a currency, the analytical formula Black-Scholes/Garman-Kohlhagen takes the following form:

$$c = e^{-r^*(T-t)} S_t \Phi\left[\frac{\ln(S_t/X) + \left(r - r^* + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{(T-t)}}\right] e^{-r(T-t)} X \Phi\left[\frac{\ln(S_t/X) + \left(r - r^* - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{(T-t)}}\right], \quad [1]$$

where r^* represents the foreign risk free interest rate and $\Phi(.)$ is the standard normal cumulative distribution function.

Castagna (2011) gives the following definition for the volatility smile: for an expiry T, the volatility smile maps the implied volatilities to a given function $\zeta = f(K)$ of the strike K. Hence the smile indicates the volatility parameter to plug into the BS formula struck at level $K = f^{-1}(\zeta)$. Currency options are in general used for strategies to hedge against the foreign exchange rate movements. One type of strategy is risk reversal, which implies a long position on call and short position on put, the call having a higher strike than put. That strategy represents a measure of the relative value of options with strikes above or below the current at the money forward rate, actually expressing the skew that might exist in the volatility smile. Forward looking financial indicators represents informative predictors of the behaviour of the asset price, therefore those options gained attention in recent years. Neftci (2008) explained how risk reversal represents a measure of the bias in a volatility smile, since a symmetric smile implies that zero cost risk reversal could be achieved.The measure of an option's moneyness is called delta, such as at the money (ATM) options have a delta around 50 and the following relationship holds:

$$\sigma(25 - \text{deltaRR}) = \sigma(25 - \text{deltaput}) - \sigma(25 - \text{deltacall}) \quad [2]$$

where $\sigma(25 - \text{deltaRR}), \sigma(25 - \text{deltacall}), \sigma(25 - \text{deltaput})$represents the implied volatilities of a risk reversal, namely the 24 delta call and 25 delta put. The curvature of the smile can be measured using butterfly strategy. One way to construct a butterfly is a long straddle which forms a V-shaped head of the butterfly and short a strangle that creates the flattened wings.

$$\sigma(25 - \text{deltacall}) = \sigma(25 - \text{delta ATM}) + \sigma(25 - \text{deltaBFY}) + 0.5 \cdot \sigma(25 - \text{deltaRR}) \quad [3]$$

$$\sigma(25 - \text{delta put}) = \sigma(25 - \text{delta ATM}) + \sigma(25 - \text{deltaBFY}) - 0.5 \cdot \sigma(25 - \text{deltaRR}) \quad [4]$$

Nakisa(2010) plot the implied volatility for ATM options on the S&P 500 index against expiry term and determined two term structure curves: the first one for January 24th 2007 which had a low level of volatility, respectively the second one for November 20th 2008 which was just after the collapse of Lehman Brothers when volatility was across the entire term structure.

In order to approach the assumption of constant volatility, Heston (1993) proposed the following model where volatility is a stochastic process. The model is an extended version of the Black Scholes SDE with a volatility that follows a so called CIR-process:

$$dS_t = \mu S_t dt + \sqrt{V_t} S_t dW_t^1 \quad [5]$$

$$dV_t = k(\theta - V_t)dt + \sigma\sqrt{V_t} dW_t^2 \quad [6]$$

$$dW_t^1 dW_t^2 = \rho dt \quad [7]$$

where $\{S_t\}_{t\geq 0}$ and $\{V_t\}_{t\geq 0}$ represents the price and volatility processes and ρ is the correlation parameter between the Brownian motion processes, $\{W_t^1\}_{t\geq 0}$ and $\{W_t^2\}_{t\geq 0}$. $\{V_t\}_{t\geq 0}$represents a square root mean reverting process, first used by (Cox, Ingersoll & Ross 1985), The parameters μ, θ, k and σ could be interpreted as the drift, the long–run variance, the mean reversion rate to the long-run variance and the volatility of the variance[1]. The correlation parameter could be seen as correlation between the log-returns and volatility of the asset, affecting the heaviness of the tails. When this parameter is positive the volatility will increase as the price of the asset increases and therefore affects the skewness of the distribution since it shifts to the right. The parameter σ impacts the kurtosis of the distribution, a null value will lead to a normally distribution of log-returns. In this way higher values of σ leads to a more prominent smile since higher kurtosis means that the market has a greater chance of extreme events and the volatility is fluctuates more. The mean reversion κ parameter can be seen as the degree of "volatility clustering".

By applying the Ito formula the Garman's partial differential equation is:

$$\frac{\partial C}{\partial t} + \frac{S^2}{2}\frac{\partial C^2}{\partial S^2} + rS\frac{\partial C}{\partial S} - rC + [k - (\theta - V) - \lambda V]\frac{\partial C}{\partial V} + \frac{\sigma^2 V}{2}\frac{\partial C}{\partial V^2} + \rho\sigma SV\frac{\partial C}{\partial S\,\partial V} \quad [8]$$

where the market price of the volatility risk is λ., which means the Heston approximation is built on eq. [5].

$$C(S_t, V_t, t, T) = S_t P_1 - Ke^{-r(T-t)}P_{2.} \quad [9]$$

$P_1$and $P_2$are defined by the inverse Fourier transformation:

$$P_j(x, V_t, T, K) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \left(\frac{e^{-i\phi \ln(K)} f_j(x, V_t, T, \phi)}{i\phi}\right) d\phi, \quad j = 1,2 \quad [10]$$

where the asset price is

$$x = \ln(S_t) \quad [11]$$

[1]Also called the volatility of the volatility

and the characteristic functions has the form [12]

$$f_j(x, V_t, T, \phi) = e^{C(T-t,\phi)+D(T-t,\phi)V_t+i\phi x}$$

Introducing the characteristic functions in Garman equation [8] and solving the system knowing the zero initial conditions $C(0,\phi) = D(0,\phi) = 0$, we obtain the following:

$$C(T-t,\phi) = r\phi ir + \frac{a}{b^2}\left[\left(b_j - \rho\sigma\phi i + d\right) - 2\ln\left(\frac{1-ge^{dr}}{1-g}\right)\right] \quad [13]$$

$$D(T-t,\phi) = \frac{b_j - \rho\sigma\phi i + d}{\sigma^2}\left(\frac{1-ge^{dr}}{1-g}\right) \quad [14]$$

$$g = \frac{b_j - \rho\sigma\phi i + d}{b_j - \rho\sigma\phi i - d} \quad [15]$$

$$d = \sqrt{\left(\rho\sigma\phi i - b_j\right)^2 - \sigma^2\left(2\mu_j\phi i - \phi^2\right)} \quad [16]$$

and

$$\mu_1 = \frac{1}{2}, \qquad \mu_2 = -\frac{1}{2}, \qquad a = \kappa\theta, \qquad b_1 = \kappa + \lambda - \rho\sigma, \qquad b_2 = \kappa + \lambda \quad [17]$$

An important property of being non negative is satisfied by the process of the variance [] [2] and in order to have a strictly positive condition the CIR process remains like this when:

$$.\ \alpha := \frac{4\kappa\theta}{\sigma^2} \geq 2 \quad [18]$$

which is also known as the Feller condition. The dimensionality of the corresponding Bessel process is α. If the condition is not satisfied, i.e. for $0 < \alpha < 2$, the CIR process will visit 0 recurrently but will not stay at zero.

3. Calibration approach

The estimation becomes crucial as the model itself and since Heston model has six parameters leads to a more complex process. Bakshi, Cao and Chen (1997) showed that implied parameters and their time series estimate counterparts are different. The most used approach is to solve the inverse problem by minimizing the error or discrepancy between model prices and market prices, which turns out to be a non-linear least square optimization problem. Since one issue when considering historical data for estimation is the market price of volatility risk, we calibrate the model to the volatility smile instead of using spot data. First we fixed two parameters, initial variance and mean reversion k and fit the remaining three: volatility of the variance, long-run variance and the correlation parameter. After fitting those parameters we defined the objective function, which is Sum of Squared Errors (SSE):

$$SSE(\kappa,\theta,\sigma,\rho,v_0) = \sum_{i=1}^{n}\{\hat{\sigma}_i - \sigma_i(\kappa,\theta,\sigma,\rho,v_0)\}^2 \quad [19]$$

[2]For instance, is not satisfied by the Ornstein-Uhlenbeck process.

By minimizing the objective function we find the optimal set of parameters.

4. Data and empirical results

In order to evaluate the volatility smile before and after the Lehman Brothers collapse, we took the quotations of risk reversal and butterflies strategies for call and put. Next, basing on equations (3) and (4), we calculated the implied volatilities. Then using those implied volatilities we calibrate the market data in order to obtain the implied volatilities from Heston Model for both periods, namely before and after the Lehman Brothers collapse.

Before calibrating the model to market data we will analyse the input parameter and how the shape of the smile curve will be affected. The first parameter which we analyze *a priori* is the initial variance v_0, which allows the modification on the heights of the smile curve. The volatility of the volatility parameter σ has a different impact on the smile, for instance when setting to zero the smile doesn't exist and an increase in the parameter will increases the convexity of the fitting function. The effects of the changing the parameter, long-run variance θ, are similar to those from initial variance parameter. The mean reversion level k, has been set up to 1.5 and then the other three parameters have been subject to modifications. The effects in modifying the values for k are that for low and high deltas the implied volatility remains almost unchanged. In the figure 1 from Annexes is presented the behaviour of implied volatility on different values for input parameters, before and after the crisis. What is important to note here is that both long run variance and the variance of volatility parameters have recorded higher values after crisis comparing with the changes in implied volatility occurred before crisis.Based on the hypothesis mentioned in the calibration scheme part, the parameters are detailed bellow in table 1. Using those parameters we calibrate the Heston model to market data. The data used is based on the volatility surface EUR/RON 3m before and after the threshold point[3].

Before	v_0	σ	κ	θ	ρ	**Feller**	**SSE**
1M	0.01080	0.7105	1.5000	0.0539	-0.4411	-0.3431	0.00000
3M	0.01200	0.4877	1.5000	0.0316	-0.5718	-0.1430	0.00000
6M	0.01220	0.4135	1.5000	0.0250	-0.6317	-0.0959	0.00000
12M	0.01460	0.3958	1.5000	0.0246	-0.6386	-0.0829	0.00000

After	v_0	σ	κ	θ	ρ	**Feller**	**SSE**
1M	0.01963	0.784056	1.5000	0.0763	-0.3918	-0.4031	0.00000
3M	0.01874	0.508258	1.5000	0.0418	-0.4834	-0.1318	0.00000
6M	0.01320	0.410669	1.5000	0.0259	-0.5816	-0.0905	0.00000
12M	0.01481	0.387854	1.5000	0.0244	-0.6073	-0.0873	0.00000

Table 1. Resulted parameters from calibrating Heston model to market data for the period before, respectivelly after the Lehman Brothers crash..

The results indicate the fact that for 1M maturity for instance, the long run variance is higher after the crisis[4] and also the variance of the volatility showed the same pattern due to levels recorded by the initial variance

[3] The Lehman Brothers collapse, 15.09.2008

[4] This could be indicative for the use of a more complex model that allows for a stochastic occurring of jumps.

that is higher after the crisis. The ρ parameter indicates the asymmetry of the smile in particular, positive correlation makes calls more expensive, negative correlation makes puts more expensive. The negative values recorded indicate a skew in the volatility smile for FX EUR RON

But on the other side, another relevant tool that could be used to draw some information on the investor expectations in regard with future evolution of exchange rates is the term structure of implied volatility. The volatility term structure it is also use to pricing FX options and provides thus insights on the relation between uncertainty and horizons of time. It is important to note that between volatility and maturity is an increasing function if the short-dated volatility is historically low and vice-versa. Putting together, we constructed a volatility surface in order to achieve a general overview on the dynamics of expectations from FX market. The figure 5 shows the two surfaces[5] plotted against the moment of Lehman Brothers default. Because we have only a few discrete points as times to maturity, we interpolated between two points in order to achieve a smother view related to the function with implied volatilise for the five deltas. Even that before the Lehman Brothers default we can observe that between the implied volatilities and the option's life time was an increasing function, this relation was reversed after the Lehman moment. In fact this is the most important evidence of our analysis: the increase of risk aversion[6] in international markets as result of Lehman Brothers default determined an important shift in the level and the curvature of EUR/RON smile for short-times to maturity, even that investor's expectation for longer horizon of time remained almost unchanged. Thus coming back to our ex-ante assumption on the epidemiological structure between expectation of professional investors and short-run movements of EUR/RON exchange rates, we observed that corroborated with an increase of volatility smile for short-times to maturity, also the historical volatility posted significant increases in the next few moths after Lehman crash. Maybe more important is the fact that even we do not used specific instruments as the risk neutral densities[7], from figure 5 we can observe that after Lehman moment, the most important shifts in levels of implied volatilities were for times to maturity up to 3 months. If we look at the evolution of EUR/RON rate from that period, we can observe that through the end of October, the level of FX decreased as the NBR's interventions were substantial, but in the next three months the EUR/RON jumped irreversibly to a range between 4.2-4.3 and continued to situate around this range up to the end of 2009.

5. Conclusions

In this paper we investigate the evaluation of market expectations using volatility smile. The implied volatility has been calculated from quotes of risk reversal and butterflies strategies. In order to analyze the movements in investor's risk aversion we have split the data before and after crisis. We set the day of Leman Brothers' collapse, 15 September 2008, as the beginning of crisis in Romania. Based on market data we calibrate the Heston model and we've obtained the parameters and the volatility smile in comparison before and after the threshold. Here we also intend to investigate the case of existing "epidemiological" relationship between forecasts of professional investors and short-term developments in the EUR/RON exchange rate. Even that we don't call a typical epidemiological model as those ones used in biology fields of research, we investigated the hypothesis according to which after the Lehman Brothers crash and implicit the generation of the current

[5] Because the calibration errors were very low for short-, mid- and long-runs, in figure 4 we reported only the values resulted from Heston models.

[6] That caused a series of adverse (specific) spill-overs for emerging countries.

[7] And other tools related to that one.

financial crisis, the forecasts of professional investors pose a significant explanatory power on the futures short-run movements of EUR/RON. How it works this mechanism? Firstly, the professional forecasters account for the current macro, financial and political states, then they elaborate forecasts. Secondly, based on that forecasts they get positions in the Romanian exchange market for hedging and/or speculation purposes. But their positions incorporate in addition different degrees of uncertainty. In parallel, a part of their anticipations are disseminated to the public via media channels. Since some important movements are viewed within macro, financial or political fields, the positions of professional investors from FX derivative market are activated. The current study represents a first step in that direction of analysis for Romanian case.

The main conclusion is that coefficient of the volatility of the variance on short term is higher after crisis comparing with the one before, underlying the fact that the smile captured the risk aversion. On long term on the other hand, the long run variance is stabilizing on maturities higher than six months.

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Figure 1. Calibration of Heston model parameters on market data (before threshold)

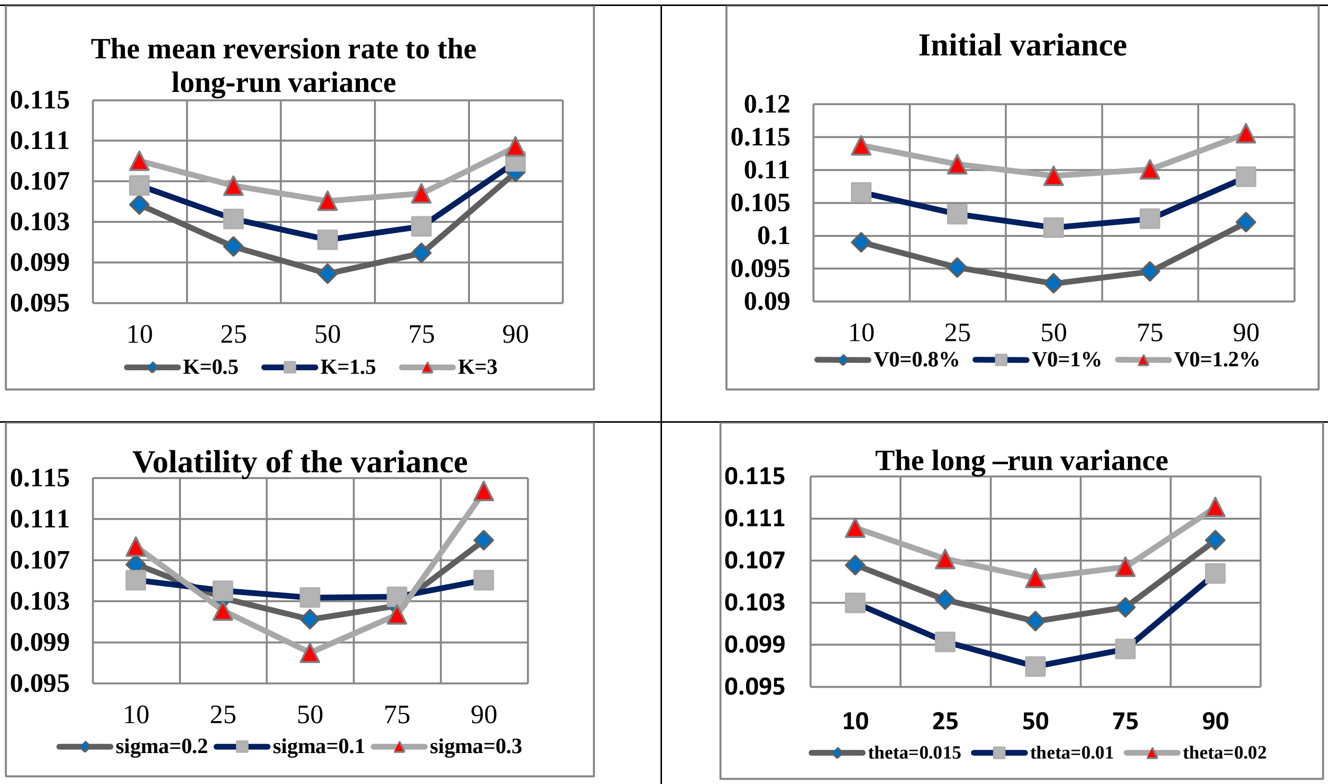
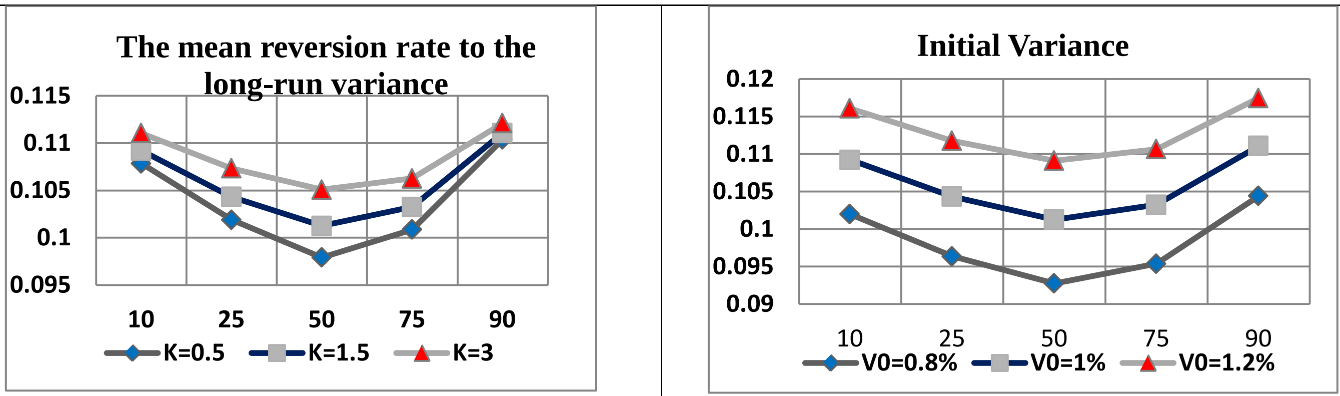
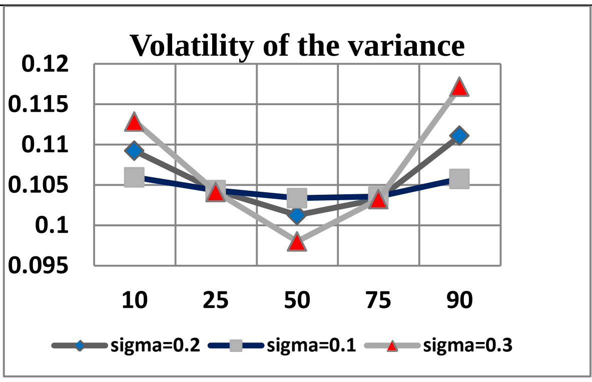


Figure 2. Calibration of Heston model parameters on market data (after threshold)





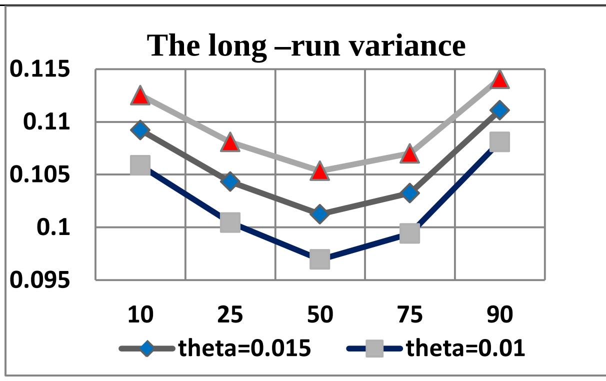


Figure 3. Implied Volatilities for the period before Lehman Brothers crash.

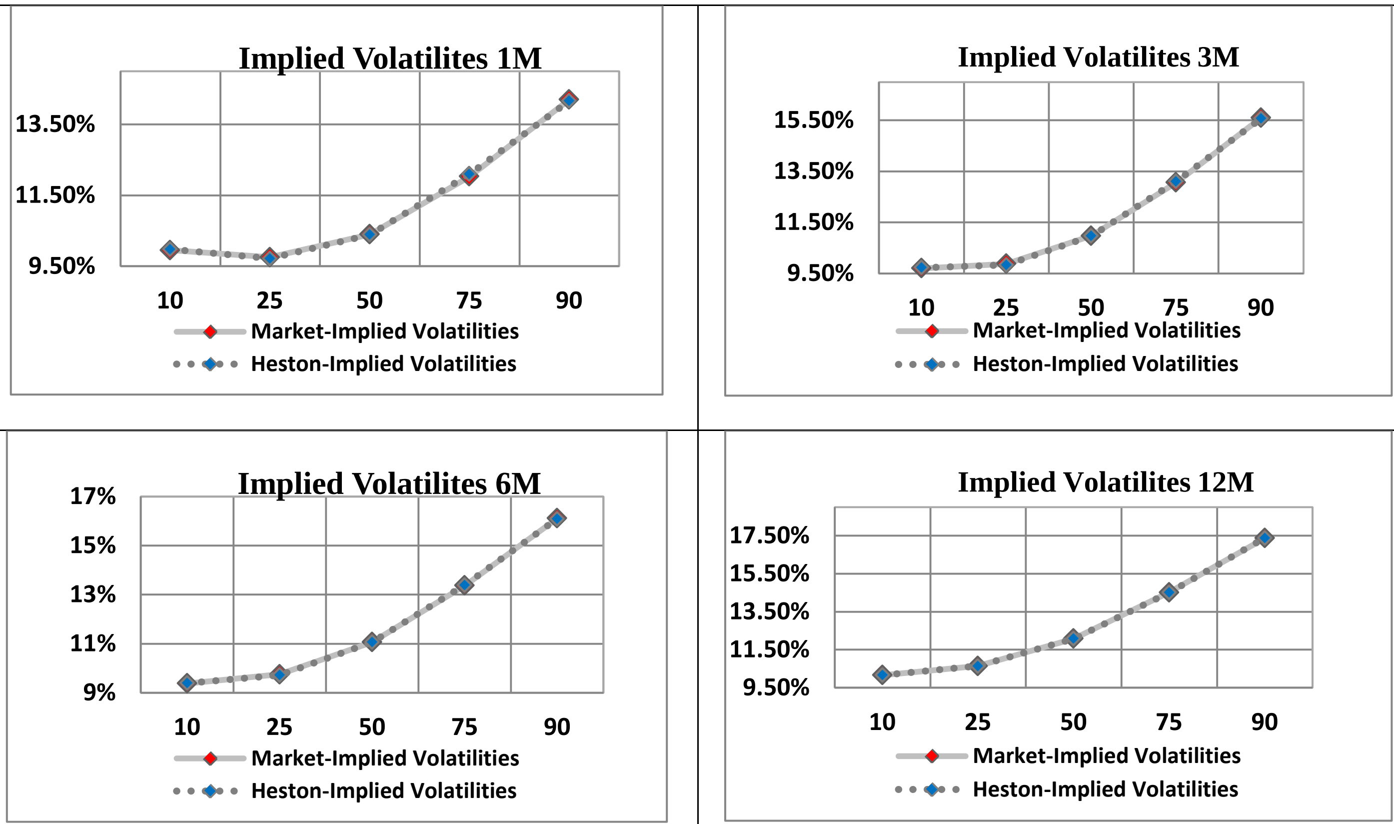
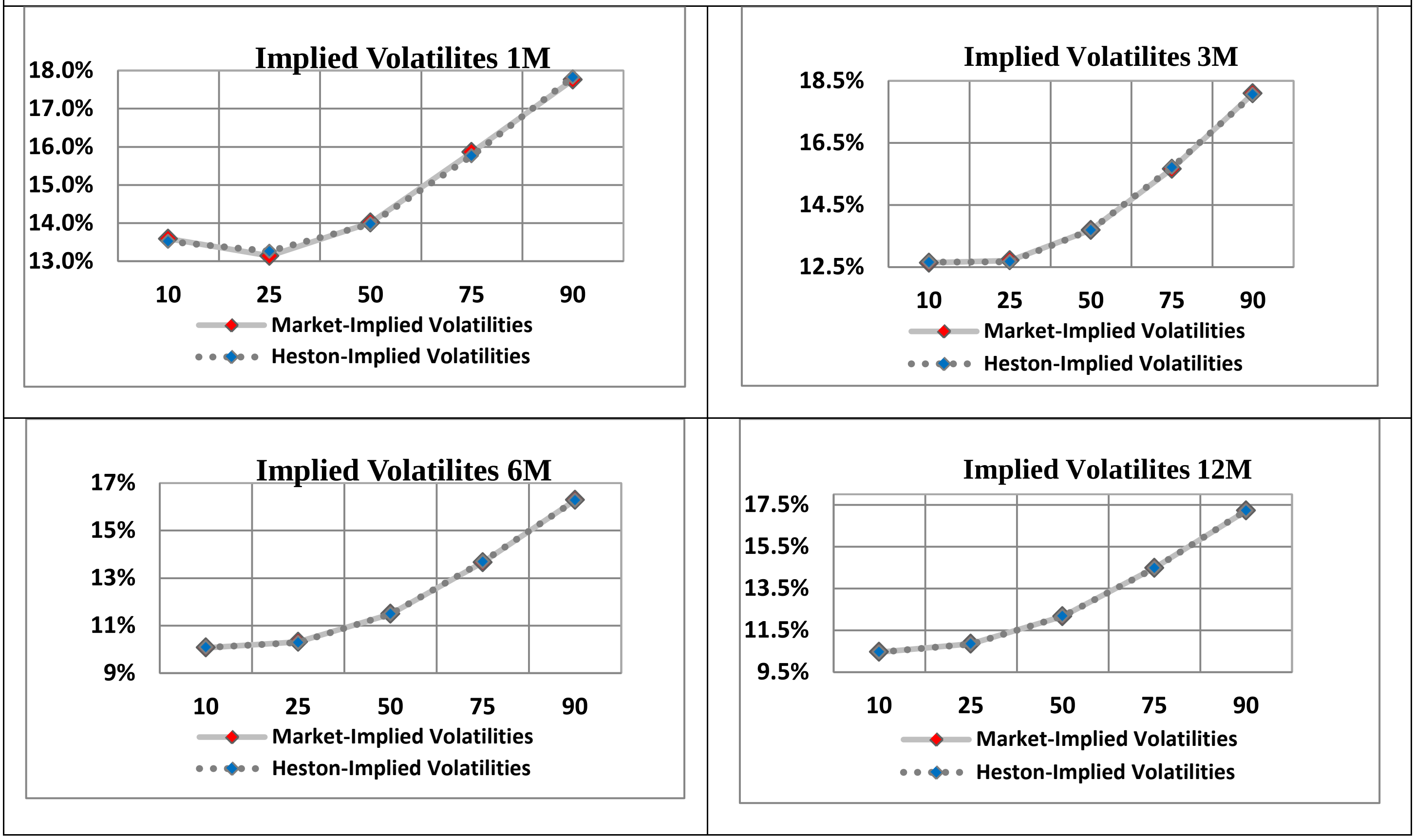


Figure 4. Implied Volatilities for the period after Lehman Brothers crash.



5. Heston smile surface for EUR/RON rate calibrated on one-month data before, respectively after the Lehman Brother default.