

MANAGEMENTUL RISULUI DE CREDIT

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Elemente de inferenta bayesiana

➤ Regula lui Bayes:

$$p(\theta | y) = \frac{l(y | \theta) \cdot p(\theta)}{\int l(y | \theta) \cdot p(\theta) \cdot d\theta}$$

unde:

- y - vectorul observatiilor;
- θ - vectorul parametrilor (inclusiv valori ce lipsesc din vectorul y)
- $l(y | \theta)$ – functia de verosimilitate;
- $p(\theta)$ – distributia a priori
- $p(\theta | y)$ – distributia a posteriori

➤ Distributii conjugate (exemple):

$p(\theta)$	$l(y \theta)$	$p(q \theta)$
Gamma(a,b)	Poisson(θ)	Gamma(x,y)
Gamma(a,b)	Exponential(θ)	Gamma(x,y)
Beta(a,b)	Binomial(θ, N)	Beta(x,y)

Elemente de inferenta bayesiana

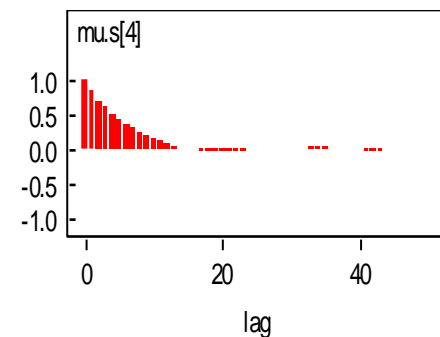
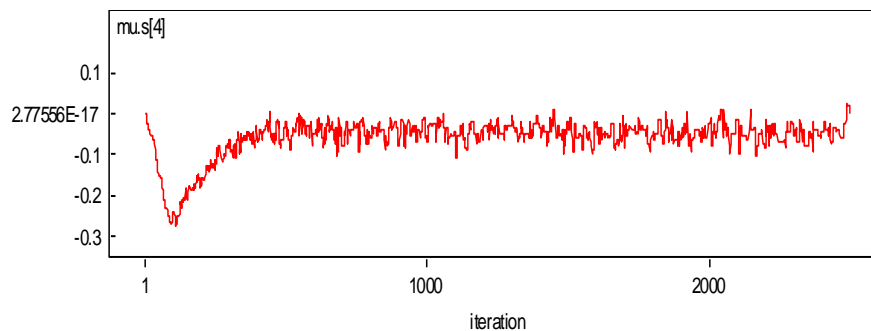
Metode bazate pe simulari

- Pana la inceputul anilor '90, metodele bazate pe simulari erau folosite doar in fizica.
- Incepand cu Gelfand, Hills, Racine-Poon si Smith (1990) acestea au fost introduse si in statistica bayesiana (Gilks, Richardson si Spiegelhalter (1996), Gelman, Carlin, Stern, Rubin (2004), Ntzoufras (2009)).
- Metodele bazate pe simulari pot fi grupate in:
 - Metode non-iterative:
 - Algoritmul “accept/reject”;
 - Algoritmul “Sampling-Importance Resampling” (SIR) – Smith si Gelfand (1992);
 - Metode iterative (algoritmi MCMC):
 - Algoritmul Metropolis – Hastings (Hastings, 1970);
 - Algoritmul Gibbs (Geman si Geman, 1984)
- Programe:
 - WinBUGS: Lunn, D.J., Thomas, A., Best, N., and Spiegelhalter, D. (2002) - MRC Biostatistics Unit si Imperial College School of Medicine din Londra;
 - pachete de functii ce pot fi rulate prin R (codurile R au fost scrise initial de Ross Ihaka si Robert Gentleman de la Universitatea Aukland, dep. Statistica)

Elemente de inferenta bayesiana

Criterii de monitorizare a convergentei

- Vizualizarea lantului simulat
- Monitorizarea mediilor recurente
- Aplicarea de teste statistice asupra seriilor generate (lanturi):
 - Testul Geweke (1992);
 - Testul Gelman Rubin (1992);
 - Raftery, Lewis (1992, 1996)
- Programe de monitorizare a convergentei
 - CODA (Convergence Diagnostic and Output Analysis)
 - BACC (Bayesian Analysis, Computation and Communication)



Elemente de inferenta bayesiana

Estimare bayesiana recursiva

- Model dinamic de tip “state - space”:

$$\begin{cases} y_t = h(\alpha_t, v_t) \\ \alpha_t = f(\alpha_{t-1}, w_t) \end{cases}$$

unde:

- y - variabila observabila la momentul t ;
- α - variabila de stare (neobservabila) la momentul t ;
- v - eroarea de evaluare/ masurare a variabilei observabile;
- w - eroarea ecuatiei de tranzitie a variabilei de stare.

- Estimarea bayesiana recursiva presupune doua etape

1. Etapa de predictie

$$p(\alpha_t | \Psi_{t-1}) = \int p(\alpha_t | \alpha_{t-1}) \cdot p(\alpha_{t-1} | \Psi_{t-1}) \cdot d\alpha_{t-1}$$

2. Etapa de actualizare

$$p(\alpha_t | \Psi_t) = \frac{p(y_t | \alpha_t) \cdot p(\alpha_t | \Psi_{t-1})}{\int p(y_t | \alpha_t) \cdot p(\alpha_t | \Psi_{t-1}) \cdot d\alpha_t} \propto p(y_t | \alpha_t) \cdot p(\alpha_t | \Psi_{t-1})$$

- Pentru cazul general non-liniar si/sau non-Gaussian s-au propus o serie de metode precum:

- Filtrul Kalman Extins;
- Filtrul UKF – “Unscented Kalman Filter” (Wan si Van der Merwe, 2000)
- Metoda verosimilitatii maxime simulate sau “Importance Sampling” (Durbin si Koopman, 1997)
- Filtre cu Particule:
 - Filtrul SIR (Gordon, Salmond si Smith, 1993);
 - Filtrul cu Particule Auxiliare (Pitt si Shephard, 1999);

Modele structurale de risc de credit

- Primul model structural: Merton (1974)
- Extensii ale modelului Merton:
 - Definirea evenimentului de nerambursare:
 - Black si Cox (1976) – firma poate intra in incapacitate de plata a datoriilor in orice moment pana la scadenta datoriei (“first passage models”);
 - Pragul de nerambursare:
 - Exogen si determinist (Kim, Ramaswamy si Sundaresan (1993), Longstaff si Schwartz (1995), Black si Cox (1976));
 - Exogen si stohastic (Briys si Varenne (1997));
 - Endogen si determinist (Leland (1994), Leland si Toft (1996));
 - Rata dobanzii fara risc stohastica:
 - Shimko, Tejima si van Deventer (1993);
 - Kim, Ramswamy si Sundaresan (1993),
 - Nielsen, Saa-Requejo si Santa-Clara (1993),
 - Longstaff si Schwartz (1995),
 - Briys si Varenne (1997), etc.

Modele structurale de risc de credit

Modele cu rata dobanzii fara risc constanta

Modelul Merton (1974):

- valoarea de piata a activelor firmei urmeaza un proces stohastic Brownian geometric:

$$dV = \mu \cdot V \cdot dt + \sigma \cdot V \cdot dW$$

- datoria sub forma obligatiunilor zero cupon;
- firma poate intra in incapacitate de plata doar la momentul scadentei datoriei;
- pragul de nerambursare = valoarea nominala a obligatiunii;

$$E(V_t, t) = V_t \cdot \Phi(d_1(t)) - D_T \cdot e^{-r \cdot \Delta t} \cdot \Phi(d_2(t))$$

$$D(V_t, t) = V_t - E(V_t, t) = V_t \cdot \Phi(-d_1(t)) + D_T \cdot e^{-r \cdot \Delta t} \cdot \Phi(d_2(t))$$

$$d_1(t) = \frac{\ln\left(\frac{V_t}{D_T}\right) + \left(r + \frac{\sigma^2}{2}\right) \cdot \Delta t}{\sigma \cdot \sqrt{\Delta t}} \quad d_2(t) = d_1(t) - \sigma \cdot \sqrt{\Delta t} = \frac{\ln\left(\frac{V_t}{D_T}\right) + \left(r - \frac{\sigma^2}{2}\right) \cdot \Delta t}{\sigma \cdot \sqrt{\Delta t}}$$

Modele structurale de risc de credit

Modele cu rata dobanzii fara risc constanta

Modelul Leland (1994)

- valoarea de piata a activelor firmei urmeaza un proces stohastic Brownian geometric: $dV = \mu \cdot V \cdot dt + \sigma \cdot V \cdot dW$
- datoria este sub forma unei obligatiuni perpetue ce plateste un cupon constant (C)
- pragul de nerambursare (V_B) este endogen;
- firma poate intra in incapacitate de plata cand valoarea activelor firmei atinge pragul de nerambursare;
- introduce impozitul pe profit (τ) si costurile de recuperare (γ) a datoriei in caz de nerambursare.

$$E(V) = V - \frac{(1-\tau) \cdot C}{r} + \left[\frac{(1-\tau) \cdot C}{r} - V_B \right] \cdot \exp(-\tilde{\alpha} \cdot x)$$

$$D(V) = \frac{C}{r} + \left[(1-\gamma) \cdot V_B - \frac{C}{r} \right] \cdot \exp(-\tilde{\alpha} \cdot x)$$

$$\tilde{\alpha} = \ln\left(\frac{V_T}{V_B}\right) \quad V_B = \frac{(1-\tau) \cdot C}{r + \frac{\sigma^2}{2}} \quad x = 2 \cdot r / \sigma^2$$

Modele structurale de risc de credit

Modele cu rata dobanzii fara risc constanta

Modelul Leland & Toft (1996)

- valoarea de piata a activelor firmei urmeaza un proces stohastic Brownian geometric de forma:

$$dV = (\mu - \delta) \cdot V \cdot dt + \sigma \cdot V \cdot dW$$

- firma emite in mod continuu obligatiuni cu o scadenta constanta (T) si un cupon constant;
- datoria ajunsa la maturitate este inlocuita cu o noua emisiune de obligatiuni de valoare egala cu suma rascumparata si cu acelasi cupon de dobanda, astfel incat principalul datoriei totale (P) si platile totale anuale cu dobanzile (C) sunt constante.
- pragul de nerambursare este endogen;
- pastreaza din modelul Leland (1994) impozitul pe profit (τ) si costurile de recuperare (γ) a datoriei in caz de nerambursare.

Modele structurale de risc de credit

Modele cu rata dobanzii fara risc constanta

Modelul Leland & Toft (1996) - continuare

$$E(V; V_B, T) = v(V; V_B) - D(V; V_B, T)$$

$$D(V; V_B, t) = \frac{C}{r} + \left(P - \frac{C}{r}\right) \cdot \left(\frac{1 - e^{-rT}}{r \cdot T} - I(T)\right) + \left((1 - \gamma) \cdot V_B - \frac{C}{r}\right) \cdot J(T) \quad v(V; V_B) = V + \frac{\tau \cdot C}{r} \cdot [1 - \exp(-\tilde{\alpha} \cdot x)] - \gamma \cdot V_B \cdot \exp(-\tilde{\alpha} \cdot x)$$

$$x = \frac{\beta_r + z}{\sigma^2} \quad \beta_r = r - \delta - \frac{\sigma^2}{2} \quad V_B = \frac{\frac{C}{r} \cdot \left(\frac{K}{r \cdot T} - L\right) - \frac{K \cdot P}{r \cdot T} - \frac{\tau \cdot C \cdot x}{r}}{1 + \gamma \cdot x - (1 - \gamma) \cdot L} \quad I(T) = \frac{1}{r \cdot T} \cdot (G(T) - e^{-rT} \cdot \Pi(T))$$

$$J(T) = \frac{\sigma}{z \cdot \sqrt{T}} \cdot \left[-\exp\{\tilde{\alpha} \cdot (z - \beta_r) / \sigma^2\} \cdot q_1(T) \cdot \Phi(q_1(T)) + \exp\{-\tilde{\alpha} \cdot (z + \beta_r) / \sigma^2\} \cdot q_2(T) \cdot \Phi(q_2(T)) \right]$$

$$K = 2 \cdot \frac{\beta_r}{\sigma^2} \cdot e^{-rT} \cdot \Phi(\beta_r \cdot \sqrt{T} / \sigma) - 2 \cdot \frac{z}{\sigma^2} \cdot \Phi(z \cdot \sqrt{T} / \sigma) - \frac{2}{\sigma \cdot \sqrt{T}} \cdot \phi(z \cdot \sqrt{T} / \sigma) + \frac{2}{\sigma \cdot \sqrt{T}} \cdot e^{-rT} \cdot \phi(\beta_r \cdot \sqrt{T} / \sigma) + \frac{z - \beta_r}{\sigma^2}$$

$$L = -2 \cdot \left(\frac{z}{\sigma^2} + \frac{1}{z \cdot T}\right) \cdot \Phi(z \cdot \sqrt{T} / \sigma) - \frac{2}{\sigma \cdot \sqrt{T}} \cdot \phi(z \cdot \sqrt{T} / \sigma) + \frac{z - \beta_r}{\sigma^2} + \frac{1}{z \cdot T}$$

$$G(t) = \exp\{\alpha \cdot (z - \beta_r) / \sigma^2\} \cdot \Phi(q_1(t)) + \exp\{-\alpha \cdot (z + \beta_r) / \sigma^2\} \cdot \Phi(q_2(t)) \quad \Pi(t) = \Phi(h_1(t)) + \exp(-2 \cdot \beta_r \cdot \tilde{\alpha} / \sigma^2) \cdot \Phi(h_2(t))$$

Modele structurale de risc de credit

Modele cu rata dobanzii fara risc stohastica

Modelul Shimko, Tejima & van Deventer (1993)

- datoria sub forma obligatiunilor zero cupon;
- firma poate intra in incapacitate de plata doar la momentul scadentei datoriei;
- pragul de nerambursare = valoarea nominala a obligatiunii;
- rata de dobanda fara risc stohastica, urmeaza un proces Ornstein-Uhlenbeck
$$dr_t = k \cdot (m - r_t) \cdot dt + \sigma_r \cdot dW_r$$
- structura la termen a ratelor de dobanda fara risc se determina conform modelului Vasicek (1977)

$$E(V_t, t) = V_t \cdot \Phi(d_1(t)) - D_T \cdot P(t, T) \cdot \Phi(d_2(t))$$

$$d_1(t) = \frac{\ln(V_t) - \ln(D_T \cdot P(t, T)) + 0.5 \cdot \Sigma(t)}{\sqrt{\Sigma(t)}} \quad d_2(t) = d_1(t) - \sqrt{\Sigma(t)}$$

$$\Sigma(t) = \Delta t \cdot \left(\sigma^2 + \frac{\sigma_r^2}{k^2} + \frac{2 \cdot \rho \cdot \sigma \cdot \sigma_r}{k} \right) + 2 \cdot (e^{-k \cdot \Delta t} - 1) \cdot \left(\frac{\sigma_r^2}{k^3} + \frac{\rho \cdot \sigma_r \cdot \sigma}{k^2} \right) - \frac{\sigma_r^2}{2 \cdot k^3} \cdot (e^{-2 \cdot k \cdot \Delta t} - 1)$$

$$P(t, T) = A(t, T) \cdot \exp\{-B(t, T) \cdot r_t\}$$

$$A(t, T) = \exp\left\{ \gamma \cdot (B(t, T) - \Delta t) - \frac{\sigma_r^2 \cdot B^2(t, T)}{4 \cdot k} \right\} \quad B(t, T) = \frac{1}{k} \cdot (1 - e^{-k \cdot \Delta t}) \quad \gamma = m + \frac{\lambda \cdot \sigma_r}{k} - \frac{\sigma_r^2}{2 \cdot k^2}$$

Modele structurale de risc de credit

Modele cu rata dobanzii fara risc stohastica

Modelul Bryis & Varenne (1997)

- datoria firmei sub forma obligatiunilor zero cupon;
- debitorii dispun de o clauza de protectie prin care pot cere rascumpararea datoriei inainte de scadenta daca valoarea activelor firmei scade sub un anumit prag;
- pragul este exogen si stohastic:

$$V_B(t) = (1 - \gamma) \cdot D_T \cdot P(t, T)$$

- in caz de faliment, regula de prioritate poate fi incalcata in practica in functie de puterea de negociere a actionarilor. In acest sens, Briys si Varenne introduc in model doi factori (f_1, f_2) care indica ponderea recuperata din valoarea activelor ca urmare a nerespectarii prioritatii debitorilor
- rata dobanzii fara risc urmeaza un proces stohastic de forma:

$$dr_t = k(t) \cdot (m(t) - r_t) \cdot dt + \sigma_r(t) \cdot dW_r$$

Modele structurale de risc de credit

Modele cu rata dobanzii fara risc stohastica

Modelul Bryis & Varenne (1997) - continuare

$$D(t,T) = D_T \cdot P(t,T) \cdot \left[1 - P_E(l_t, 1) + P_E\left(q_t, \frac{l_t}{q_t}\right) - (1-f_1) \cdot l_t \cdot \left(N(-d_3) + \frac{N(-d_4)}{q_t} \right) - (1-f_2) \cdot l_t \cdot \left(N(d_3) - N(d_1) + \frac{N(d_4) - N(d_6)}{q_t} \right) \right]$$

$$l_t = \frac{V_t}{D_T \cdot P(t,T)} \quad q_t = \frac{V_t}{(1-\gamma) \cdot D_T \cdot P(t,T)}$$

$$d_1 = \frac{\ln(l_t) + 0.5 \cdot \Sigma(t,T)}{\sqrt{\Sigma(t,T)}} \quad d_2 = d_1 - \sqrt{\Sigma(t,T)} \quad d_3 = \frac{\ln(q_t) + 0.5 \cdot \Sigma(t,T)}{\sqrt{\Sigma(t,T)}}$$

$$d_4 = d_3 - \sqrt{\Sigma(t,T)} \quad d_5 = \frac{\ln\left(\frac{q_t^2}{l_t}\right) + 0.5 \cdot \Sigma(t,T)}{\sqrt{\Sigma(t,T)}} \quad d_6 = d_5 - \sqrt{\Sigma(t,T)}$$

$$P_E(l_t, 1) = -l_t \cdot N(-d_1) + N(-d_2) \quad P_E\left(q_t, \frac{l_t}{q_t}\right) = -q_t \cdot N(-d_5) + \frac{l_t}{q_t} \cdot N(-d_6)$$

$$\Sigma(t,T) = \int_t^T \left[(\rho \cdot \sigma + \sigma_p(s,T))^2 + (1-\rho^2) \cdot \sigma^2 \right] \cdot ds$$

Modele structurale de risc de credit

Metode de estimare a modelelor structurale

- Metoda volatilitatii constante (Jones, Mason si Rosenfeld, 1984)
- Metoda KMV (Crosbie si Bohn, 2003)
- Metoda verosimilitatii maxime pentru date transformate (Duan, 1994, 2000)
- Metoda reprezentarii dinamice (Duan si Fulop , 2006)

Datele folosite pentru estimarea modelelor structurale: **Societatea Transelectrica**

- cotatiile bursiere zilnice din perioada ian. 2008 – apr. 2011

<i>Nr. actiuni (mil.)</i>	<i>73.303</i>
<i>Datorii totala, P, (mil. RON)</i>	<i>1.689,62</i>
<i>Datorii pe termen scurt (mil. RON)</i>	<i>720,97</i>
<i>Cheltuieli financiare, C, (mil. RON)</i>	<i>137,87</i>
<i>Maturitatea medie, T, (ani)</i>	<i>6,29</i>
<i>Rata de compensare, δ, (%)</i>	<i>4,12</i>

Modele structurale de risc de credit

Metode de estimare a modelelor structurale

Metoda volatilitatii constante

Se rezolva sistemul:
$$E_t = g(V_t, \sigma_V) \quad \sigma_E = \sigma_V \cdot \frac{V_t}{E_t} \cdot \frac{\partial g}{\partial V}$$

Rezultatele estimarii parametrilor prin metoda volatilitatii constante

<i>Parametru</i>	<i>Merton</i>	<i>Leland</i>
<i>V</i>	2.496	2.929
<i>VB</i>	721	997,4
<i>σ</i>	0.3080	0.2525
<i>α̃</i>	1.2418	1,0773
<i>PD_{nr}</i>	0.0022	0.0006

Dezavantaje:

- volatilitatea randamentelor actiunilor este estimata ca o constanta;
- a doua ecuatie a sistemului este redundanta;
- nu permite determinarea distributiei parametrilor si deci nu putem testa semnificatia statistica a acestora;
- nu permite estimarea parametrului μ (rata asteptata a rentabilitatii activelor firmei), prin urmare putem determina doar probabilitatile neutrale la risc.

Modele structurale de risc de credit

Metode de estimare a modelelor structurale

Metoda KMV

- Date fiind observatiile istorice al capitalizarii bursiere $\{E1, E2, \dots, ET\}$ si o valoare initiala aleasa arbitrar a volatilitatii randemanetelor activelor $\sigma(0)$, algoritmul KMV presupune repetarea urmatorilor pasi pana cand valoare parametrului σ converge catre o valoare stabila:

1. Pentru $t= 1, \dots, T$ se determina valoare a activelor din:

$$V_t^{(j)}(\sigma^{(j)}) = g^{-1}(E_t; \sigma^{(j)})$$

2. Folosind valorile $\{V1, V2, \dots, VT\}$ din pasul anterior se calculeaza:

$$R_t^{(j)} = \ln\left(\frac{V_{t+1}^{(j)}}{V_t^{(j)}}\right)$$

$$\bar{R}^{(j)} = \frac{1}{T} \cdot \sum_{t=1}^T R_t^{(j)}$$

$$(\sigma^{(j+1)})^2 = \frac{1}{h \cdot T} \cdot \sum_{t=1}^T (R_t^{(j)} - \bar{R}^{(j)})^2$$

$$\mu^{(j+1)} = \frac{1}{h} \cdot \bar{R}^{(j)} + \frac{1}{2} \cdot (\sigma^{(j+1)})^2$$

<i>Parametru</i>	<i>Merton</i>	<i>Leland</i>	<i>Leland & Toft</i>
μ	-0,0545	-0,0859	-0,0451
σ	0,2589	0,2171	0,2142
$\tilde{\alpha}$	1,3773	1,0808	0,8797
<i>PD</i>	0,0000	0,0004	0,0162
<i>PDnr</i>	0,0000	0,0000	0,0021

Modele structurale de risc de credit

Metode de estimare a modelelor structurale

Metoda verosimilitatii maxime pentru date transformate (1)

- Ecuația de dinamica a activelor implica:

$$\ln(V_t) \mid \ln(V_{t_0}), \mu, \delta, \sigma \sim N(\ln(V_{t_0}) + \beta \cdot h, \sigma^2 \cdot h)$$

- Cunoscând funcția de densitate a valorii activelor putem determina funcția de densitate a acțiunilor astfel:

$$p(E_t \mid \theta) = p(\ln V_t \mid \ln V_{t-1}, \theta) \left/ \left| \frac{\partial g(\ln V_t)}{\partial \ln V_t} \right| \right.$$

- Funcția de log-verosimilitate a valorii acțiunilor devine:

$$\ln L(E_1, E_2, \dots, E_N \mid \theta) = \ln p(\ln V_1) + \sum_{t=2}^N \ln p(\ln V_t \mid \ln V_{t-1}, \theta) - \sum_{t=1}^N \ln \left(\left| \frac{\partial g(\ln V_t)}{\partial \ln V_t} \right| \right)$$

- Pentru modelele Shimko, Tejima & van Deventer și Bryis & Varenne estimăm separat parametrii structurii la termen a ratelor de dobândă fără risc folosind filtrul Kalman (Duan și Simonato (1995), Chen și Scott (1995), Bolder (2001))

- Ecuația observațiilor:

$$\begin{pmatrix} R_t(\tau_1) \\ R_t(\tau_2) \\ \dots \\ R_t(\tau_n) \end{pmatrix} = \begin{pmatrix} -\ln(A(\tau_1))/\tau_1 \\ -\ln(A(\tau_2))/\tau_2 \\ \dots \\ -\ln(A(\tau_n))/\tau_n \end{pmatrix} + \begin{pmatrix} B(\tau_1)/\tau_1 \\ B(\tau_2)/\tau_2 \\ \dots \\ B(\tau_n)/\tau_n \end{pmatrix} \cdot r_t + \begin{pmatrix} \varepsilon_{t,1} \\ \varepsilon_{t,2} \\ \dots \\ \varepsilon_{t,n} \end{pmatrix} \quad \varepsilon_t \sim N \left(\begin{pmatrix} 0 \\ 0 \\ \dots \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{\varepsilon 1}^2 & 0 & 0 & 0 \\ 0 & \sigma_{\varepsilon 2}^2 & 0 & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \sigma_{\varepsilon n}^2 \end{pmatrix} \right)$$

- Ecuația de tranziție a variabilei de stare:

$$r_{t+1} = G + F \cdot r_t + \xi_t \quad \xi_t \sim N(0, \sigma_\xi^2) \quad G = m \cdot (1 - e^{-k \cdot h}) \quad F = e^{-k \cdot h} \quad \sigma_\xi^2 = \frac{\sigma_r^2}{2 \cdot k} \cdot (1 - e^{-2 \cdot k \cdot h})$$

Modele structurale de risc de credit

Metode de estimare a modelelor structurale

Metoda verosimilitatii maxime pentru date transformate (2)

Parametru	Modelul Vasicek (1977)	
	Estimatie	Eroare Standard
m	0.1159	0.0006
k	0.3256	0.0001
σ_r	0.0909	0.0001
λ	-0.0394	0.0011
$\sigma_{\varepsilon 1}$	0.0262	0.0000
$\sigma_{\varepsilon 2}$	0.0226	0.0000
$\sigma_{\varepsilon 3}$	0.0144	0.0000
$\sigma_{\varepsilon 4}$	0.0072	0.0000
$\sigma_{\varepsilon 5}$	0.0039	0.0000
$\sigma_{\varepsilon 6}$	0.0015	0.0000
$\sigma_{\varepsilon 7}$	0.0003	0.0000
$\sigma_{\varepsilon 8}$	0.0016	0.0000

Evolutia variabilei de stare
(estimata prin filtrul Kalman)



Pentru a estima parametrii specifici ecuatiei de dinamica a ratei de dobanda s-au folosit ratele zilnice ROBOR din perioada ian. 2005 – mai. 2011 pentru urmatoarele scadente: ON, TM, 1W, 1M, 3M, 6M, 9M, 1Y

Modele structurale de risc de credit

Metode de estimare a modelelor structurale

Metoda verosimilitatii maxime pentru date transformate (3)

<i>Parametru</i>	<i>Merton</i>		<i>Leland</i>	
	<i>Estimatie (E.S.)</i>	<i>Interval de incredere (95%)</i>	<i>Estimatie (E.S.)</i>	<i>Interval de incredere (95%)</i>
μ	-0,0578 (0,0048)	[-0,0671 ; -0,0484]	-0,0858 (0,0042)	[-0,0940 ; -0,0775]
σ	0,2453 (0,0002)	[0,2448 ; 0,2457]	0,2176 (0,0002)	[0,2171 ; 0,2180]
$\tilde{\alpha}$	1,3786		1,0820	
<i>PD</i>	0,0000		0,0004	
<i>PDnr</i>	0,0000		0,0000	

<i>Parametru</i>	<i>Shimko, Tejima, van Deventer</i>		<i>Briys & Varenne</i>	
	<i>Estimatie (E.S.)</i>	<i>Interval de incredere (95%)</i>	<i>Estimatie (E.S.)</i>	<i>Interval de incredere (95%)</i>
μ	0,0127 (0,0065)	[-0,0001 ; 0,0253]	0,0258 (0,0050)	[0,0159 ; 0,0357]
σ	0,3293 (0,0032)	[0,3230 ; 0,3353]	0,3130 (0,0016)	[0,3099 ; 0,3161]
ρ	0,4060 (0,0066)	[0,3931 ; 0,4189]	0,6045 (0,0029)	[0,5988 ; 0,6102]
$\tilde{\alpha}$	1,3709		0,6193	
<i>PD</i>	0,0027		2,8695	
<i>PDnr</i>	0,0014		2,1823	

Modele structurale de risc de credit

Metode de estimare a modelelor structurale

Metoda reprezentarii dinamice (1)

- Duan si Fulop (2006) au propus ca modelele structurale sa fie scrise sub forma unui model dinamic de tip "state space"
- Duan si Fulop (2006) au folosit *filtre cu particule auxiliare* (propusa de Pitt si Shephard , 1999 si extinsa mai apoi de Pitt, 2002) pentru a estima modelul Merton (1974)
- Bruche (2007) a folosit o metoda verosimilitatii maxime simulate (Durbin, Koopman, 1997, 2000) pentru a estima trei modele: Merton (1974), Leland (1994) si Leland & Toft (1996).
- Huang si Yu (2010) au aplicat algoritmi MCMC pentru a estimata modelul Merton (1974)

Modele structurale de risc de credit

Metode de estimare a modelelor structurale

Metoda reprezentarii dinamice (2)

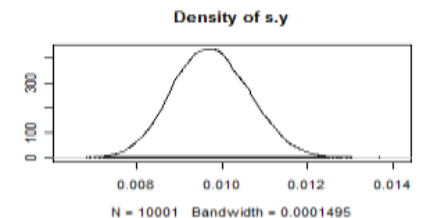
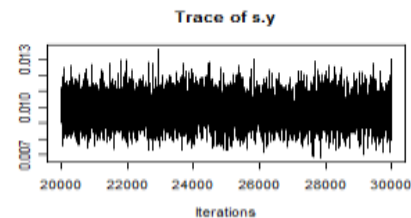
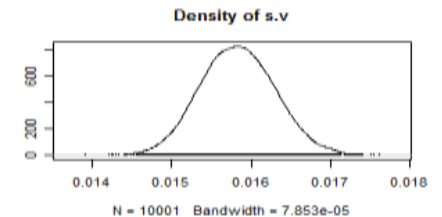
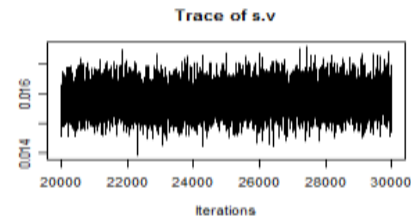
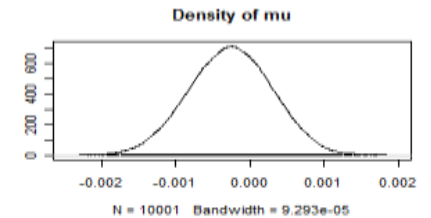
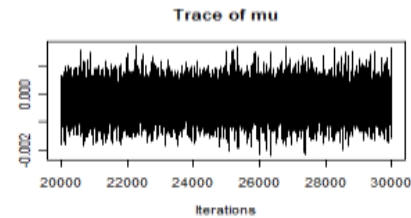
Modelul Merton

$$\begin{cases} y_t = \tilde{\alpha}_t + \ln[\Phi(d_1) - e^{-\tilde{\alpha}_t - t \cdot \Delta t} \cdot \Phi(d_2)] + \eta_t & \eta_t \sim N(0, s_y^2) \\ \tilde{\alpha}_{t+1} = \tilde{\alpha}_t + \beta \cdot h + \varepsilon_t & \varepsilon_t \sim N(0, h \cdot \sigma^2) \end{cases}$$

$$y_t = \ln\left(\frac{E_t}{V_B}\right) \quad \tilde{\alpha} = \ln\left(\frac{V_T}{V_B}\right)$$

$$V_B = D_T \quad \beta = \mu - \frac{\sigma^2}{2}$$

Parametru	Media	Dev.std	Interv. incredere (95%)
μ	-0.00026	0.00055	[-0.00134 ; 0,00081]
σ	0.01583	0.00047	[0.01495 ; 0,01676]
s_y	0.00975	0.00089	[0.00807 ; 0,01153]



Modele structurale de risc de credit

Metode de estimare a modelelor structurale

Metoda reprezentarii dinamice (3)

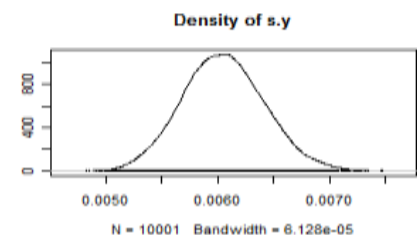
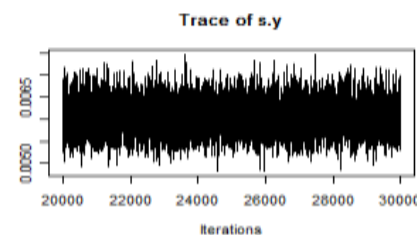
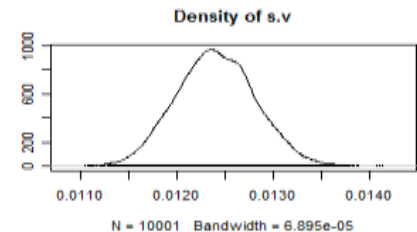
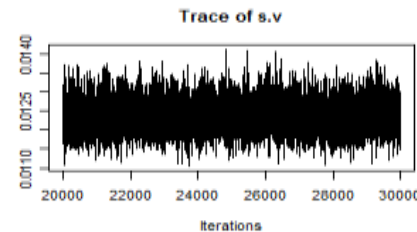
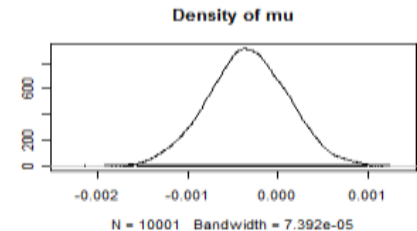
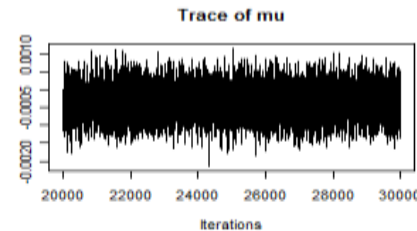
Modelul Leland

$$\begin{cases} y_t = \tilde{\alpha}_t + \ln[V_B + (b - V_B) \cdot e^{-(1+x)\tilde{\alpha}_t}] + \eta_t & \eta_t \sim N(0, s_y^2) \\ \tilde{\alpha}_{t+1} = \tilde{\alpha}_t + \beta \cdot h + \varepsilon_t & \varepsilon_t \sim N(0, h \cdot \sigma^2) \end{cases}$$

$$b = \frac{(1-\tau) \cdot C}{r} \quad y_t = \ln(E_t - b) \quad V_B = \frac{b \cdot r}{r + \frac{\sigma^2}{2}}$$

$$\tilde{\alpha} = \ln\left(\frac{V_T}{V_B}\right) \quad \beta = \mu - \frac{\sigma^2}{2} \quad x = 2 \cdot r / \sigma^2$$

Parametru	Media	Dev.std	Interv. incredere (95%)
μ	-0.00033	0.00044	[-0.00121 ; 0,00052]
σ	0.01240	0.00041	[0.01162 ; 0,01321]
s_y	0.00605	0.00036	[0.00536 ; 0,00680]



Modele structurale de risc de credit

Metode de estimare a modelelor structurale

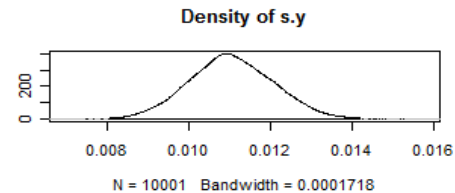
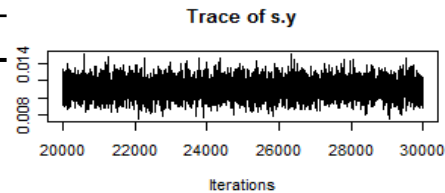
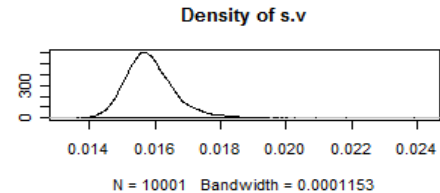
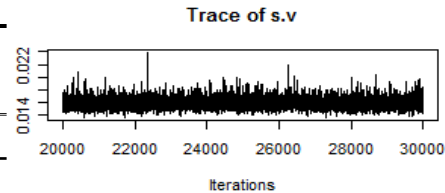
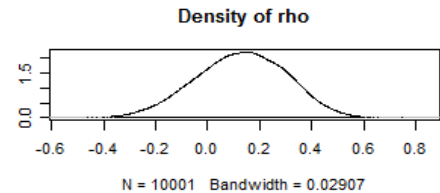
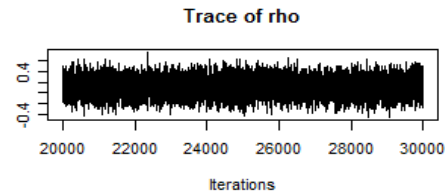
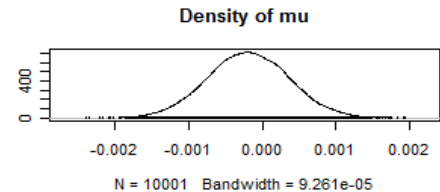
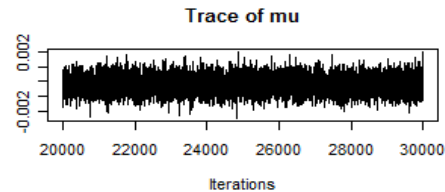
Metoda reprezentarii dinamice (4)

Modelul Shimko, Tejima & van Deventer

$$\begin{cases} y_t = \tilde{\alpha}_t + \ln[\Phi(d_1) - e^{-\tilde{\alpha}_t} \cdot P_t \cdot \Phi(d_2)] + \eta_t & \eta_t \sim N(0, s_y^2) \\ \tilde{\alpha}_{t+1} = d_t + \tilde{\alpha}_t + \varepsilon_t & \varepsilon_t \sim N(0, h \cdot \sigma^2 \cdot (1 - \rho^2)) \end{cases}$$

$$y_t = \ln\left(\frac{E_t}{V_B}\right) \quad \tilde{\alpha} = \ln\left(\frac{V_T}{V_B}\right) \quad d_t = \beta \cdot h + \sigma \cdot \sqrt{h} \cdot \rho \cdot \xi_t$$

$$\beta = \mu - \frac{\sigma^2}{2}$$



Parametru	Media	Dev.std	Interv. incredere (95%)
μ	-0.00019	0.00055	[-0.00126 ; 0,00090]
ρ	0.12780	0.17300	[-0,21990 ; 0,44970]
σ	0.01587	0.00078	[0,01460 ; 0,01766]
s_y	0.01104	0.00103	[0,00906 ; 0,01308]

Modele de risc de credit pentru portofolii de credite

- Abordarea structurala:
 - Modelul ASFR/ Vasicek (1987)
 - Modelul Moody's KMV
 - Modelul CreditMetrics
- Abordarea actuariala:
 - CreditRisk+
- Abordarea multi-factoriala/ econometrica:
 - CreditPortfolioView
- Koyluoglu si Hickman (1998) arata ca modelele de mai sus, sunt asemanatoare din punct de vedere conceptual, desi la prima vedere par diferite datorita metodelor matematice diferite, folosite pentru determinarea distributiei pierderilor.
 - Introducerea factorilor de risc sistematic pentru a surprinde corelatiile dintre activele firmelor (corelatiile dintre evenimentele de nerambursare)
 - Definirea evenimentului de nerambursare – variabila aleatoare Bernoulli;
 - Determinarea pierderii din portofoliul dd credite:

$$L = \sum_{i=1}^n L_i = \sum_{i=1}^n E_i \cdot LGD_i \cdot D_i$$

Modele de risc de credit pentru portofolii de credite

Modelul ASFR

➤ Ipoteze:

- Portofoliul este perfect granular;
- Exista un singur factor de risc sistematic:

$$R_i = \sqrt{\rho} \cdot Y + \sqrt{1-\rho} \cdot Z_i \quad Y \sim N(0,1) \quad Z_i \sim N(0,1)$$

➤ Probabilitatea de nerambursare

- neconditionata: $\tilde{P}_i = P[R_i < c_i] = \Phi(c_i)$
- conditionata: $P_i(Y) = \Phi\left(\frac{\Phi^{-1}(\tilde{P}_i) - \sqrt{\rho} \cdot Y}{\sqrt{1-\rho}}\right)$

➤ Gordy(2003) arata ca:

$$L_{\%} - E[L_{\%}|Y] \rightarrow 0 \quad a.s. \quad q_z(L_{\%}) - E[L_{\%}|q_{1-z}(Y)] \rightarrow 0 \quad a.s$$

➤ Pierderea portofoliului respectiv EL se determina astfel:

$$L_{\%} = \sum_{i=1}^n w_i \cdot LGD_i \cdot E[D_i|Y] = \sum_{i=1}^n w_i \cdot LGD_i \cdot P_i(Y) \quad EL_{\%} = E[L_{\%}] = \sum_{i=1}^n w_i \cdot LGD_i \cdot \tilde{P}_i$$

➤ VaR respectiv UL se determina astfel:

$$\begin{aligned} VaR_z(L_{\%}) &= \sum_{i=1}^n w_i \cdot LGD_i \cdot P_i(q_{1-z}(Y)) \\ &= \sum_{i=1}^n w_i \cdot LGD_i \cdot \Phi\left(\frac{\Phi^{-1}(\tilde{P}_i) + \sqrt{\rho} \cdot \Phi^{-1}(z)}{\sqrt{1-\rho}}\right) \end{aligned} \quad UL_{\%} = VaR_z(L_{\%}) - EL_{\%}$$

Modele de risc de credit pentru portofolii de credite

Riscul de concentrare sectoriala (1)

- Ipoteze:
 - Exista un singur factor de risc pentru fiecare sector;
 - Factorii sistematici sunt independentii;
 - Portofoliul de credite pentru fiecare sector este perfect granular;
- Putem aplica modelul ASFR pentru fiecare sector pentru a determina pierderea neanticipata (UL). Astfel, consideram vectorul pierderilor neanticipate aferente sectoarelor (ul) exogen.
- Pierderea neanticipata pentru intreg portofoliul:

$$UL_{\%}^P = \frac{1}{\sum_{j=1}^S E_j} \sum_{j=1}^S E_j \cdot ul_{\%}^j = w' \cdot ul_{\%}$$

- Indice de concentrare Herfindahl – Hirschman:

$$IHH = \frac{(S-1)}{S} \cdot w' \cdot w - \frac{1}{S-1}$$

Modele de risc de credit pentru portofolii de credite

Riscul de concentrare sectoriala (2)

- Obținem următorul program de optimizare:

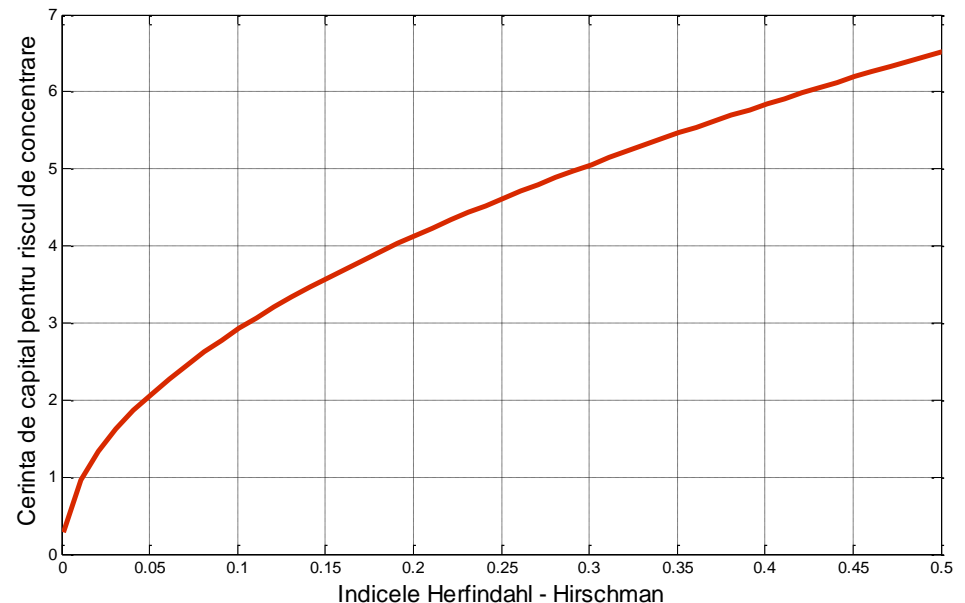
$$\begin{cases} \max_w \{UL_{\%}^P\} \\ \text{cu constrangerile :} \\ \frac{(S-1)}{S} \cdot w' \cdot w - \frac{1}{S-1} = IHH \\ w' \cdot 1_S = 1 \end{cases}$$

- Cu soluția:

$$UL_p^* = \lambda_1 \cdot \left(\frac{1}{S} + \frac{S-1}{S} \cdot IHH \right) + \lambda_2$$

$$\lambda_1 = \sqrt{D} \quad \lambda_2 = \frac{1}{S} \cdot (B - \sqrt{D})$$

$$B = \sum_{s=1}^S UL_s \quad C = \sum_{s=1}^S UL_s^2 \quad D = \frac{B^2 - S \cdot C}{1 - S \cdot h}$$



Modele liniare generalizate mixte in managementul riscului de credit

- Introduse in managementul riscului de credit de Frey si McNeil (2001);
- Folosite de Hamerle, Liebig si Scheule (2004), McNeil si Wendin (2006, 2007) si Stefanescu, Tunaru si Turnbull (2009)
- Utile pentru simulari de criza ("*stress tests*") cu variabile macroeconomice (Jakubik si Schmieder, 2008).
- Abordare generala de determinare a distributiei pierderilor din portofoliul de credite (modele precum ASFR, CreditMetrics, CreditPortfolioView, CreditRisk+ pot fi considerate cazuri particulare)
- In general, distributia pierderilor se detremina prin simulari MonteCarlo

Modele liniare generalizate mixte in managementul riscului de credit

Modelul Bernoulli cu mix de distributii (1)

$$\left\{ \begin{array}{l} D_i | Y \sim \text{indep. Bernoulli} (P_i) \\ P_i = F_i(Y) \\ Y \sim \Theta_Y \end{array} \right. \quad \left\{ \begin{array}{l} n_k | Y \sim \text{indep. Binomial} (P_k, N_k) \\ P_k = F_k(Y) \\ Y \sim \Theta_Y \end{array} \right.$$

- Probabilitate conditionata de nerambursare:

$$P[D_i = 1 | Y] = E[D_i | Y] = P_i$$

- Probabilitate neconditionata de nerambursare:

$$P[D_i = 1] = \int P[D_i = 1 | Y] \cdot dP[Y \leq y] = \int F_i(Y) \cdot dP[Y \leq y] = E_Y[P_i] = \tilde{P}_i$$

- Corelatia intre evenimentele de nerambursare:

$$\rho_D = \text{corr}(D_i, D_j) = \frac{\text{cov}(D_i, D_j)}{\sqrt{\text{var}(D_i)} \cdot \sqrt{\text{var}(D_j)}}$$

$$\text{var}(D_i) = \text{var}(E[D_i | Y]) + E[\text{var}(D_i | Y)] = \tilde{P}_i \cdot (1 - \tilde{P}_i)$$

$$\text{cov}(D_i, D_j) = E_Y[P_i \cdot P_j] - \tilde{P}_i \cdot \tilde{P}_j$$

$$E_Y[P_i \cdot P_j] = \int F_i(Y) \cdot F_j(Y) \cdot dP[Y \leq y]$$

Modele liniare generalizate mixte in managementul riscului de credit

Modelul Bernoulli cu mix de distributii (2)

- Definirea probabilitatii conditionate de nerambursare

$$R_{itk} = \beta'_k \cdot x_t + \sigma_k \cdot Y_t + \gamma \cdot Z_{itk}$$

$$Y_t \sim N(0, 1)$$

$$P[Z_{itk} \leq \varepsilon] = F(\varepsilon) \quad E[Z_{itk}] = 0 \quad \text{var}(Z_{itk}) = v_z^2$$

$$\begin{aligned} P_i &= P[D_i = 1|Y] = P[R_{itk} \leq c_{itk} | Y_t] = \\ &= P[\beta'_k \cdot x_t + \sigma_k \cdot Y_t + \gamma \cdot Z_{itk} \leq c_{itk} | Y_t] \\ &= P\left[Z_{itk} \leq \frac{c_{itk} - \beta'_k \cdot x_t - \sigma_k \cdot Y_t}{\gamma} \mid Y_t\right] \\ &= F(\mu_{itk} + \lambda'_k \cdot x_t + \eta_k \cdot Y_t) \end{aligned}$$

$$\begin{aligned} \mu_{itk} &= \frac{c_{itk}}{\gamma} \\ \lambda_k &= -\frac{1}{\gamma} \cdot \beta_k \\ \eta_k &= -\frac{\sigma_k}{\gamma} \end{aligned}$$

- Determinarea corelatiilor intre randamentele activelor

$$\rho_{R, \text{int ra}_{\text{grup}}} = \text{corr}(R_{itk}, R_{jtk}) = \frac{\sigma_k^2}{\sigma_k^2 + \gamma^2 \cdot v_z^2} = \frac{\eta_k^2}{\eta_k^2 + v_z^2} \quad \rho_{R, \text{inter}_{\text{grup}}} = \text{corr}(R_{itk}, R_{jtm}) = \frac{\sigma_k \cdot \sigma_m}{\sqrt{\sigma_k^2 + \gamma^2 \cdot v_z^2} \cdot \sqrt{\sigma_m^2 + \gamma^2 \cdot v_z^2}} = \frac{\eta_k \cdot \eta_m}{\sqrt{\eta_k^2 + v_z^2} \cdot \sqrt{\eta_m^2 + v_z^2}}$$

- Estimarea modelelor LGM

- Metoda cvasi-verosimilitatii cu penalizari (Breslow si Clayton, 1993) - functia glmmPQL din pachetul MASS din R
- Metoda verosimilitati maxime simulate (Durbin, Kopman, 1997);
- Algoritmi MCMC (Clayton, 1996) - pachetul GLMMGibbs din R, WinBUGS

Modele liniare generalizate mixte in managementul riscului de credit

Analiza sectoriala a evenimentelor de nerambursare in Romania

- Date:
 - Creditele totale si cele restante (cel putin o zi de intarziere la plata ratelor/dobanzilor scadente) grupate dupa judet (41 judete) si dupa moneda (2 grupe: ron si alte valute) publicate de BNR in cadrul raportului "Credite si depozite in profil teritorial"
 - Date trimestriale din perioada 2003 T1 – 2011 T1

- Modele estimate:

Modelul 1

$$P_i = \Phi[\mu_0 + \mu_{1,r} + \mu_{2,s} + \eta \cdot Y_t]$$

Modelul 3

$$P_i = \Phi[\mu_0 + \mu_{1,r} + \eta_s \cdot Y_t]$$

$$Y_t = \delta \cdot Y_{t-1} + \phi_s \cdot \zeta_t, \quad \zeta_t \sim N(0,1) \quad \eta_s = \frac{\phi_s^2}{1 - \delta^2}$$

Modelul 2

$$P_i = \Phi[\mu_0 + \mu_{1,r} + \eta_s \cdot Y_t]$$

Modelul 4

$$P_i = \Phi[\mu_0 + \mu_{1,r} + \lambda \cdot PIB_t + \eta_s \cdot Y_t]$$

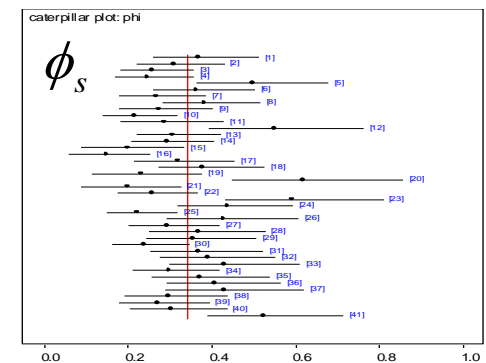
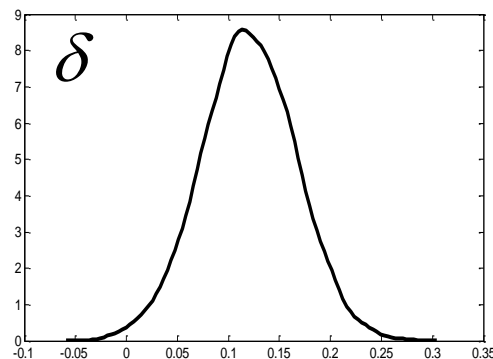
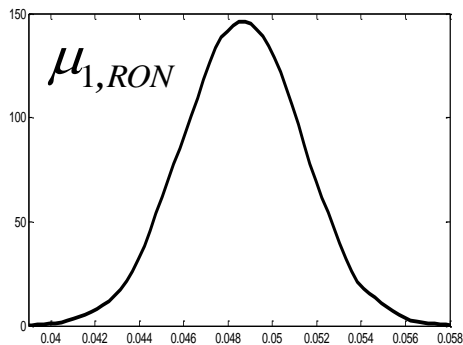
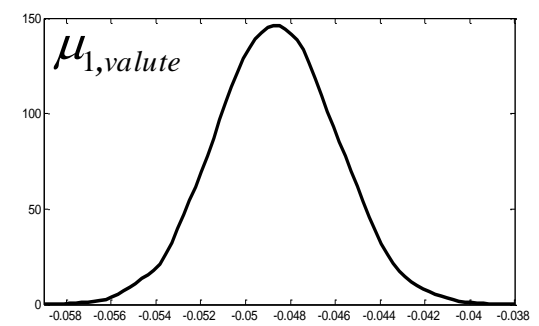
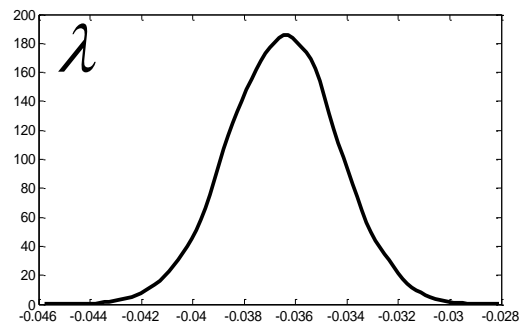
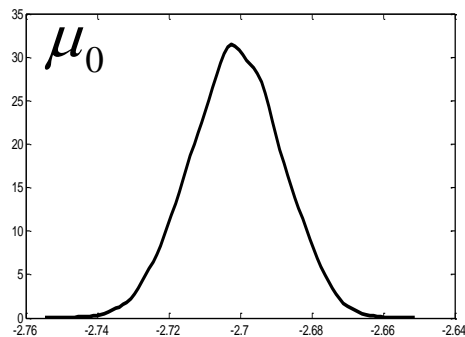
$$Y_t = \delta \cdot Y_{t-1} + \phi_s \cdot \zeta_t, \quad \zeta_t \sim N(0,1) \quad \eta_s = \frac{\phi_s^2}{1 - \delta^2}$$

Modele liniare generalizate mixte in managementul riscului de credit

Analiza sectoriala a evenimentelor de nerambursare in Romania

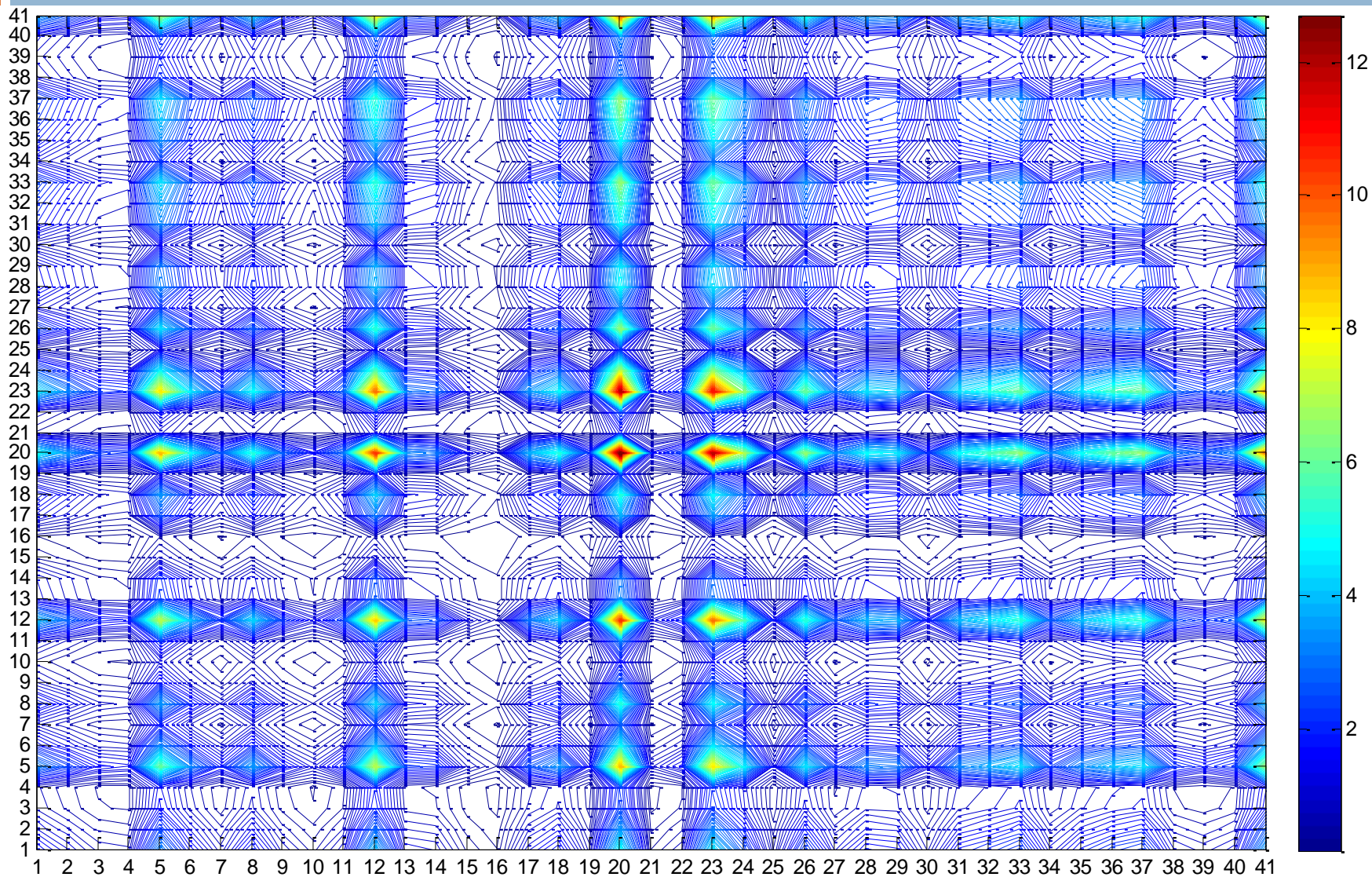
	<i>Modelul 1</i>	<i>Modelul 2</i>	<i>Modelul 3</i>	<i>Modelul 4.0</i>	<i>Modelul 4.1</i>	<i>Modelul 4.2</i>	<i>Modelul 4.3</i>
<i>DIC</i>	<i>20.714,1</i>	<i>13.215,7</i>	<i>13.203,7</i>	<i>13.151,8</i>	<i>13.163,7</i>	<i>13.174,8</i>	<i>13,176.9</i>

Distributiile a posteriori pentru parametrii modelului 4.0



Modele liniare generalizate mixte in managementul riscului de credit

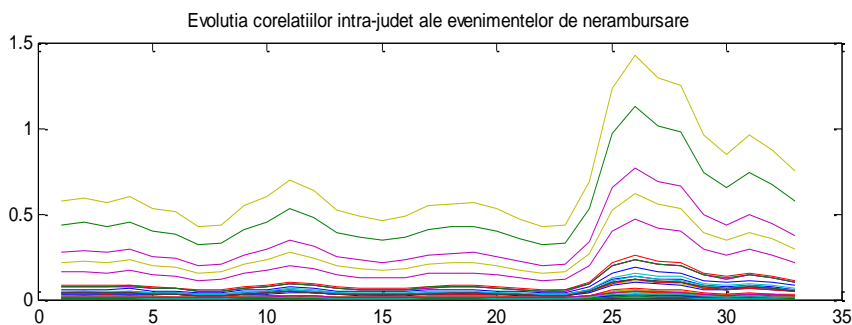
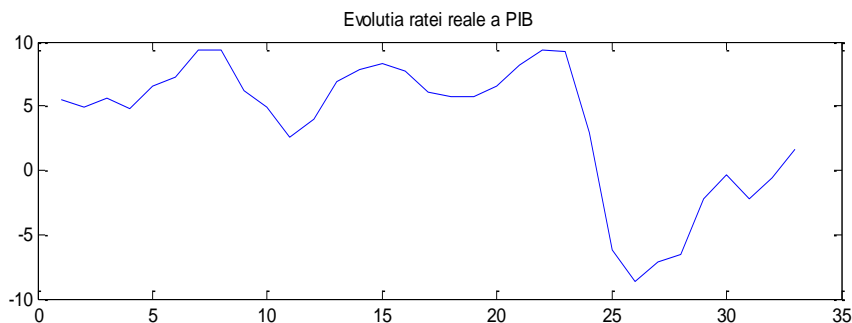
Matricea corelatiilor activelor



Modele liniare generalizate mixte in managementul riscului de credit

Analiza sectoriala a evenimentelor de nerambursare in Romania

Corelatiile evenimentelor de neramburdare



Distributia cumulata a corelatiilor dintre judete

