ACADEMY OF ECONOMIC STUDIES
DOCTORAL SCHOOL OF FINANCE AND BANKING

ANALYZING THE PORTFOLIO IMPLICATIONS OF GOLD INVESTMENT USING EXTREME EVENTS, COPULAS FUNCTIONS AND VaR: A COMPARISON BETWEEN U.S. AND ROMANIA

MSc Student: Temneanu Amalia Elena
Supervisor: Prof. Phd Moisă Altăr

Contents:

Motivations for analyzing the gold - stocks dependence and quantifying the aggregate risk:

- March 2013 – April 2013: gold price decreased with approximately 20%. The last two times when gold encountered such high decreases were on August 2008 before the Lehman collapse and the start of the financial crisis and on September 2011 when the sovereign-debt crisis started. Therefore, we are interested in the **dependence of gold and stocks extreme events**.
- Stock exchange volatility and prices have a **negative correlation** which can increase due to the leverage effect (Black 1976).
- Gold price and its volatility (VIX index for gold) have an **unstable evolution during time** due to the fact that gold demand comes from industry (10%), jewellery (40%), investment (40%), official sector (10%) and, therefore, its price is influenced by many different factors.
- Basel Committee on Banking Supervision stated in its January 2013 Amendments to the Liquidity Coverage Ratio (LCR) that **gold** should be included on **the high quality liquid assets (HQLA) list** – what would be the impact on the other asset classes?
- June 2007 – June 2009: **gold increased 50%** (in USD value), while **S&P decreased 40%** and **BET decreased 60%.**
- **Stylized facts** about stock exchange and gold returns vs. normality assumption.
- Great criticism of VaR models and poor performance during the financial crisis - in the Consultative document - Fundamental Review of the Trading Book (May 2012) the Committee proposes **the use of ES instead of VaR.**

Objectives:

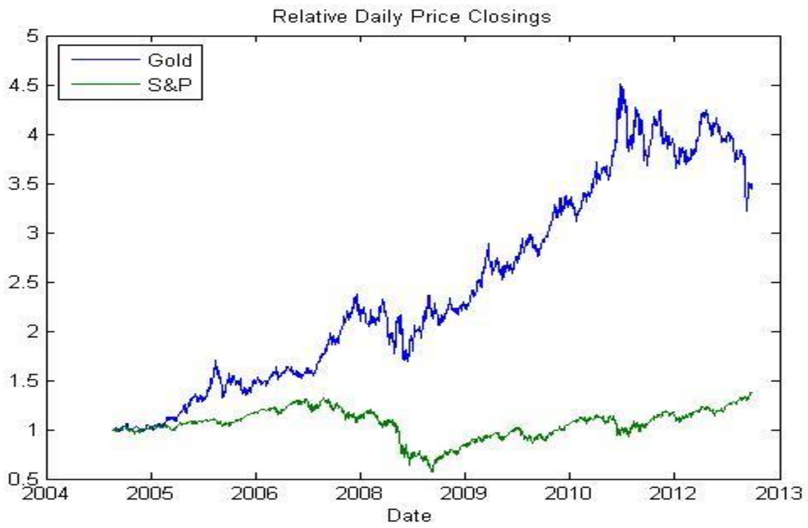
- Analyze the presence of **stylized facts** in our time series
- Choose the most suitable Copula function for our sample data and prove that incorporating **Extreme Value Theory** in the analysis provides better results regarding the dependence structure
- Analyze **the dependence between gold and equities** with the proper **Copula** function and find out if there is a stable relationship between them during time, if they share the same pattern of dependence structure when the global economy starts booming or crashing during the crisis, if these two markets are comonotonic or countermonotonic, symmetric or asymmetric and if the phenomenon of joint extreme values is present
- Compute **VaR** with Monte Carlo Method and Historical Simulation and perform backtesting
- Analyze the impact on **Conditional Value-at-Risk** of introducing gold in an equity portfolio
- Plot the **efficient frontiers** and compute the weights of the **minimum risk portfolios** using CVaR as a proxy for risk
- Analyze if it is better for a risk adverse investor to buy stocks in the Romanian market and diversify his portfolio with gold or in the US market and also diversify his portfolio with gold, in order to obtain **the highest risk-adjusted performance**

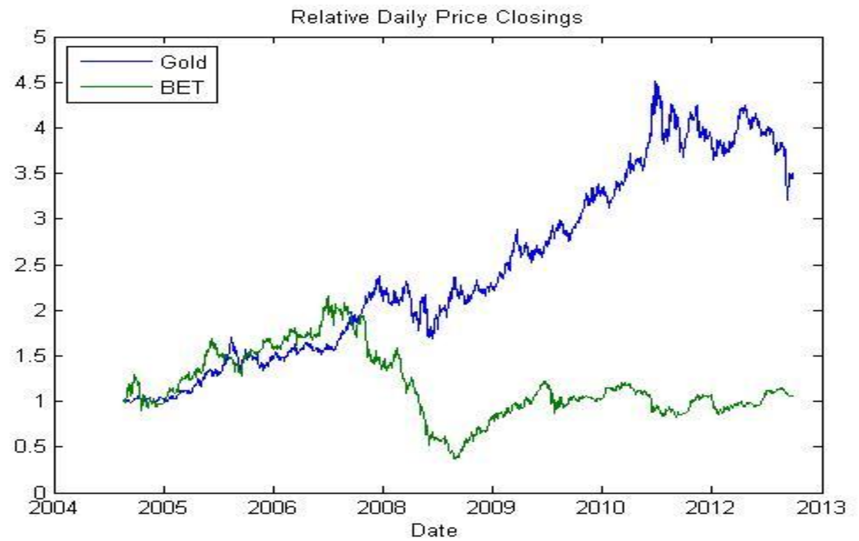
Methodology: Analyze the presence of stylized facts in our data

- Return series are not i.i.d. although they **show little serial correlation**
- Series of absolute or squared returns show **profound serial correlation**
- Conditional expected returns are close to zero
- Volatility appears to **vary over time**
- Return series are **leptokurtic** or heavy-tailed: the normal distribution presents an exponentially decreasing, while the empirical one shows a geometrical evolution - leptokurtosis effect (Fama - 1965)
- **Tail asymmetry:** generally, the probability of negative returns is bigger than the one for positive returns, most of the data series being characterized by a negative asymmetry coefficient - Skewness
- Extreme returns appear in **clusters** - financial markets reveal high volatility period and low volatility periods (Mandelbrot - 1963)

Data and results: Analyze the presence of stylized facts in our data

- Daily closing prices**:** S&P500, BET, Gold(spot rate) - Analyzed period: 06/01/2005 - 10/05/2013
- Source of data: Bloomberg
- Compute the daily log-returns: $R_t \approx \ln(S_t/S_{t-1})x100$

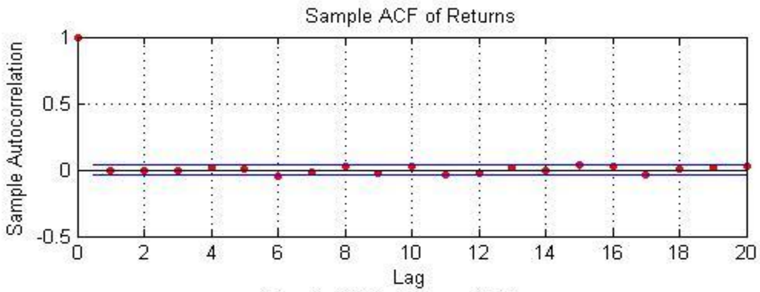


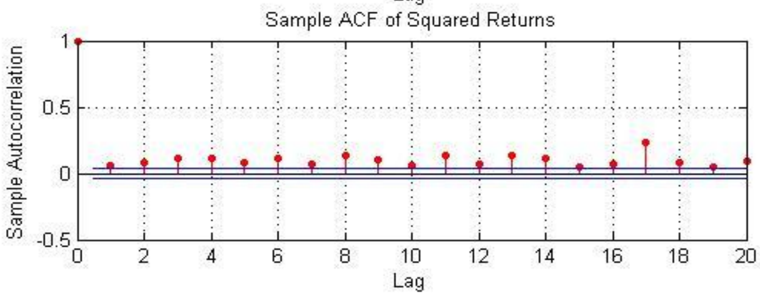


Test	H0	Significance Level (5%)	Test Statistic S&P	BET	Gold
ADF	Unit Root	-1.941600	-50.4592	-39.217	-40.948

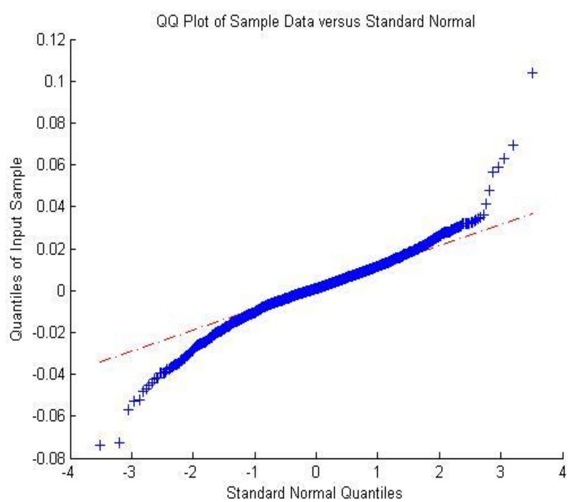
Series	S&P	BET	Gold
Mean	0.00016	-0.00002	0.00077
Std. Dev.	0.01390	0.02410	0.01330
Minimum	-0.09470	-0.16040	-0.07170
Maximum	0.10960	0.11570	0.10250
Skewness	-0.11500	-0.65630	-0.13140
Kurtosis	12.19	8.68	7.45
Jarque-Bera (5%) - critical value 5.9639	7126.5	2497.3	1461.4
P-value	0.0010	0.0010	0.0010
h	1	1	1
Observations	2024	2024	2024

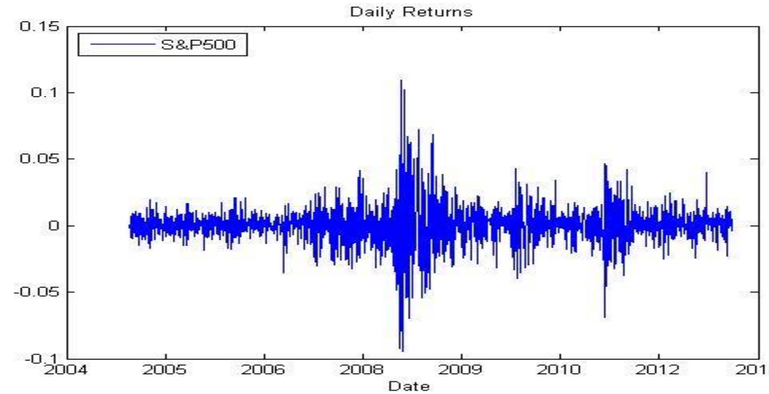
Data and results: Analyze the presence of stylized facts in our data

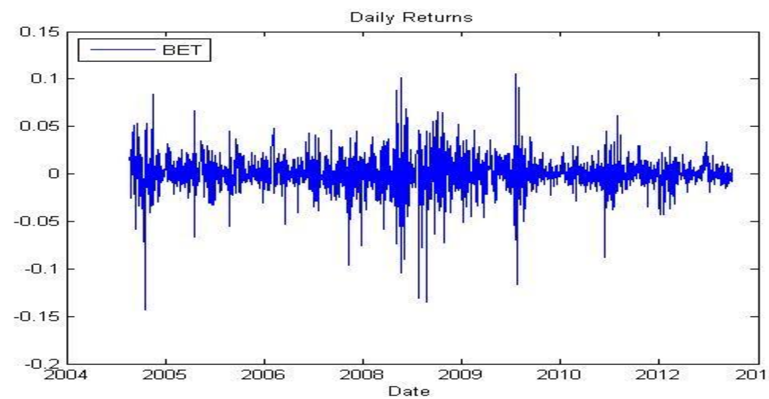


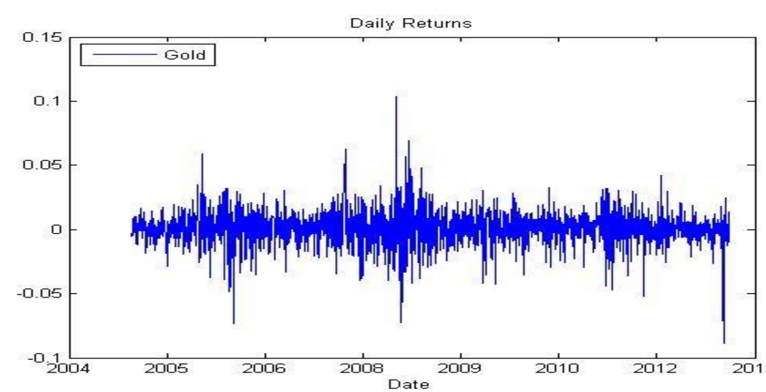


ACF and **QQ** plots of Gold Returns









Methodology: Copulas Functions

- Is there a **stable relationship** between gold and stocks during time? What **dependence structure** should be used to describe their evolution? Will the gold price and the stocks price share **the same pattern** of dependence structure when the global economy starts booming or crashing during the crisis? Are these two markets **comonotonic or countermonotonic**? Are these two markets **symmetric or asymmetric**? Is the phenomenon of **joint extreme values** present?
- All these aspects are major concerns in financial risk management and we will use **Copulas Functions** in order to properly analyze them.

- **Fisher (1997)* :** "Copulas are of interest to statisticians for two main reasons: Firstly, as a way of studying scale-free measures of dependence; and secondly, as a starting point for constructing families of bivariate distributions, sometimes with a view to simulation."

- **Sklar's Theorem (1959):** Let F be a joint distribution function with continuous margins $F_1, \ldots, F_d$. Then there exists a unique copula $C : [0, 1]^d \rightarrow [0, 1]$ such that, for all $x_1, \ldots, x_d$ in $\bar{R} = [-\infty, \infty]$

$$F(x_1, \ldots, x_d) = C(F_1(x_1), \ldots, F_d(x_d))$$

Conversely, if there are known the distribution functions for the d-dimensional joint distribution and marginal distributions, then the copula is given by the formula:

$$C(u_1,\ldots,u_d)=F(F^{-1}(u_1),\ldots,F^{-1}(u_d))$$

*the first update volume of the Encyclopedia of Statistical Sciences

Methodology: Copulas Functions

Elliptical Copulas

- **Gaussian copula** $C_P^{Gu}(u) = \Phi_P\left(\Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_d)\right)$

where: Φ denotes the standard univariate normal distribution, Φ_P denotes the joint distribution function of X, P is a correlation matrix

- **Student t copula** $C_{\upsilon,P}^{t}(u) = t_{\upsilon,P}\left(t_\upsilon^{-1}(u_1), \ldots, t_\upsilon^{-1}(u_d)\right)$

where: t_v is the distribution function f of a standard univariate t distribution, $t_{v,P}$ is the joint distribution function of the vector $X \sim t_d(v, 0, P)$, P is a correlation matrix

Archimedean Copulas

A continuous, strictly decreasing, convex function ϕ : [0, 1] → [0,∞] satisfying ϕ(1) = 0 is known as an Archimedean copula generator. It is known as a strict generator if ϕ(0)=∞. Therefore, the general form of an Archimedean copula with a generator function ϕ can be defined as following:

$$C(u_1, \ldots, u_d) = \varphi^{[-1]}\left(\varphi(u_1) + \ldots + \varphi(u_d)\right).$$

- **Clayton copula(1978)** $C_\theta^{Cl}(u_1, u_2) = \left(\max\left\{u_1^{-\theta} + u_2^{-\theta} - 1, 0\right\}\right)^{-\frac{1}{\theta}} \quad \theta \in [-1; \infty) \setminus \{0\}$

- **Gumbel copula(1960)** $C_\theta^{Gu}(u_1, u_2) = \exp\left[-\left((-\ln u_1)^\theta + (-\ln u_2)^\theta\right)^{1/\theta}\right] \quad \theta \in [1; \infty]$

Methodology: Copulas Functions

- **Frank copula(1979)** $C_{\alpha}^{F}(u_1,u_2) = -\frac{1}{\alpha}\ln\left(1+\frac{(e^{-\alpha u_1}-1)(e^{-\alpha u_2}-1)}{e^{-\alpha}-1}\right)$

- **Symmetrised Joe-Clayton copula**

$$C_{SJC}(u,v|\lambda_u,\lambda_l) = 0.5\times\left(C_{JC}(u,v|\lambda_u,\lambda_l)+C_{JC}(1-u,1-v|\lambda_u,\lambda_l)+u+v-1\right)$$

- **Time varying copulas** - a general form of the conditional dependence was introduced by Patton in 2006 for Archimedean and Elliptical copulas

$$\theta_t = \Lambda\left(\omega+\beta\theta_{t-1}+\alpha\frac{1}{m}\sum_{j=1}^{m}\left|u_{t-j}-v_{t-j}\right|\right), \quad \rho_t = \tilde{\Lambda}\left(\omega+\beta\cdot\rho_{t-1}+\alpha\cdot\frac{1}{m}\sum_{j=1}^{m}\Phi^{-1}(u_{t-j})\cdot\Phi^{-1}(v_{t-j})\right)$$

Estimating the parameters of the copula function

- **Canonical Maximum Likelihood(CML)** - the margins are fitted by an empirical CDF - obtain the uniform data - MLE for copula parameters.

- **Inferences for margins(IFM)** - the margins are issued from a semi-parametric GPD - Kernel

  Smooth distribution with shape and scale parameters also estimated by MLE - obtain the uniform data - MLE for copula parameters.

Methodology: ARMA-EGARCH

- As stated before the log-returns series are not IID (heteroskedasticity);

- In order to apply the **Extreme Value Theory** we need a proxy for IID observations, therefore we apply models for conditional mean and variance and compute the standardized residuals:

- Use the information criterions **AIC** and **BIC** - relative quality - in order to choose the proper model : **ARMA-GARCH** (symmetric shocks, Student t innovations, constrains on coefficients) **ARMA-GJR** (asymmetric shocks ,Student t innovations, constrains on coefficients) **ARMA-EGARCH** (asymmetric shocks ,Student t innovations, no constrains on coefficients)

- **ARMA-EGARCH** (Nelson -1991)* fits best our sample data: S&P, BET, Gold

Conditional **mean** equation: $$y_t = c + \sum_{t=1}^{m} \phi_i y_{t-i} + \sum_{j=1}^{m} \phi_j \varepsilon_{t-j}$$

Conditional **variance** equation : $$\log \sigma_t^2 = \alpha_0 + \alpha_1 \log \sigma_{t-1}^2 + \alpha_2 \left(\left| z_{t-1} \right| - E\left[\left| z_{t-1} \right| \right] \right) + \alpha_3 z_{t-1} \quad z_t = \varepsilon_t / \sigma_t$$

- If shocks are positive, the impact on volatility is $\alpha_2 + \alpha_3$ and if negative $\alpha_3 - \alpha_2$ (slope)

- **Test the presence of fat tails** in the filtered residuals series**

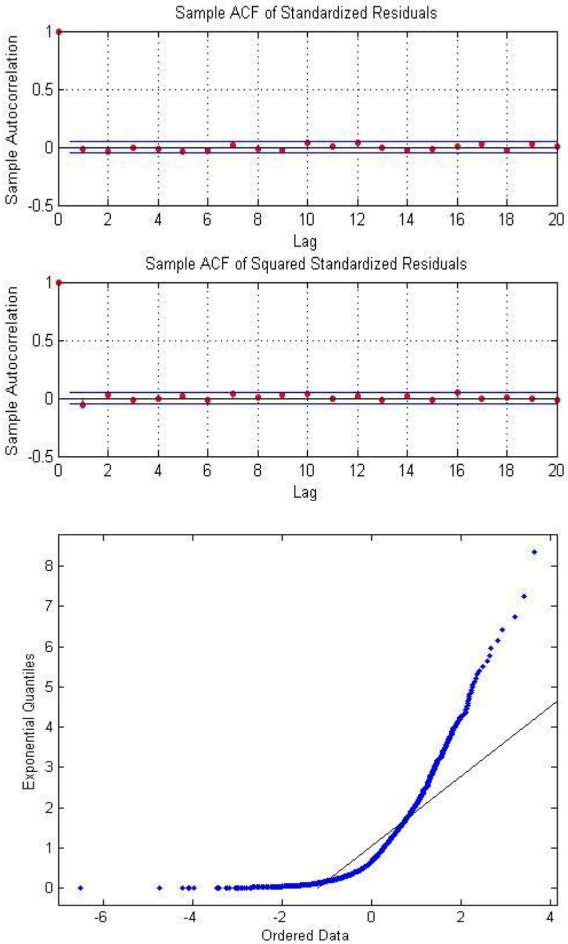
*The E-GARCH model is an asymm[illegible]GARCH model that has a better fit than symmetric GARCH for almost all financial assets. (Carol Alexander, 2008)

**Fat tails after standardization with GARCH [illegible] (Andersen, Bollerslev, Diebold and Labys ,1999)

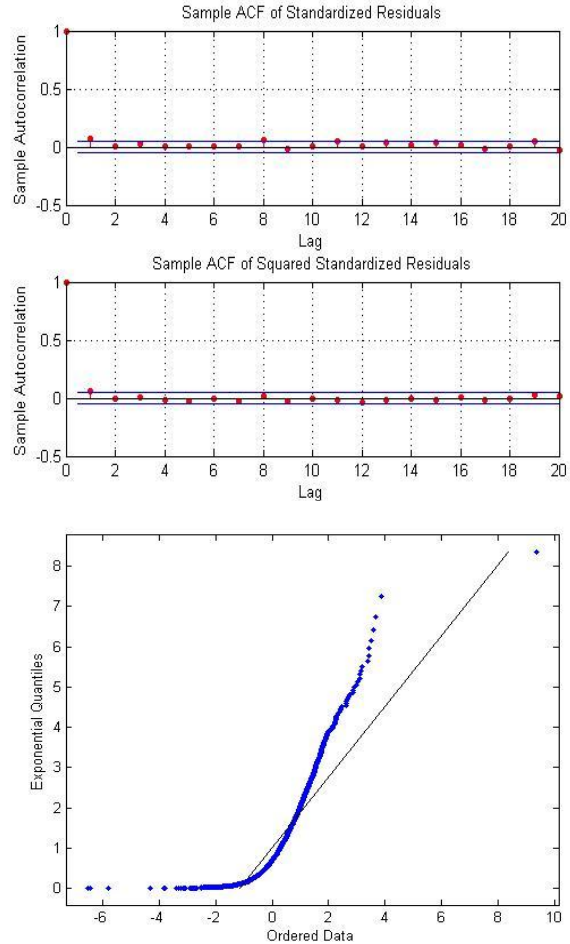
Data and results: ARMA-EGARCH

- Test the presence of **autocorrelation and heteroskedasticity** in standardized residuals
- Test the presence of **fat tails** in standardized residuals (QQ plots against the exponential)

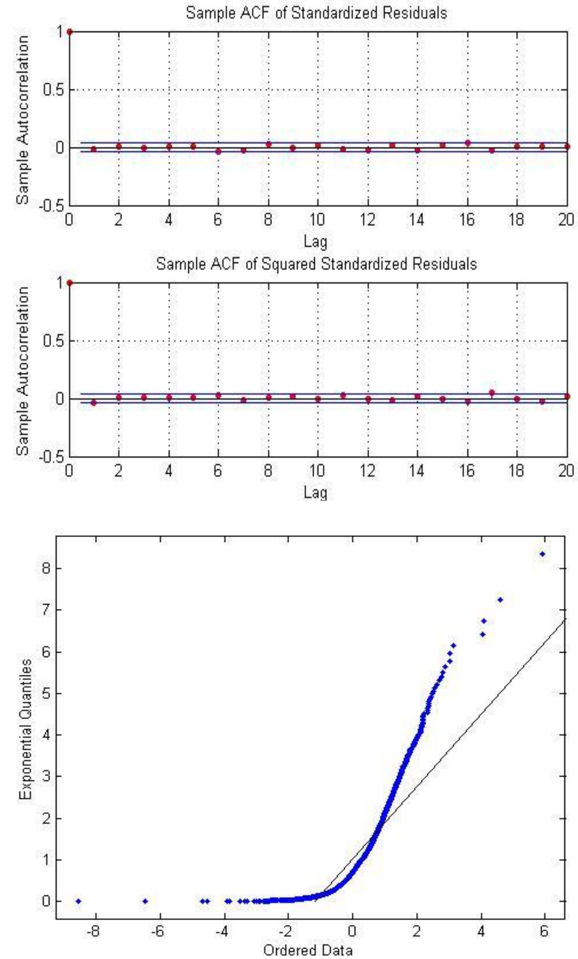
S&P500



BET



Gold



The exponential distribution **decays faster** in comparison with the empirical distribution, thus the presence of fat tails, we apply **EVT**

Methodology: Extreme Value Theory

Peaks over Threshold (POT)

- Given a random vector $X_1,X_2,\ldots,X_n$ with a distribution function $F(x) = P(X_i \leq x)$ and a predetermined high threshold u, then an exceedance above the threshold u ($y = X_i - u$) occurs when $X_i > u$ for any $i = 1,n$. We are interested in estimating **the conditional excess distribution function F_u** defined as:

$$F_u(y) = P(X - u \leq y \mid X > u), \qquad 0 \leq y \leq x_F - u$$

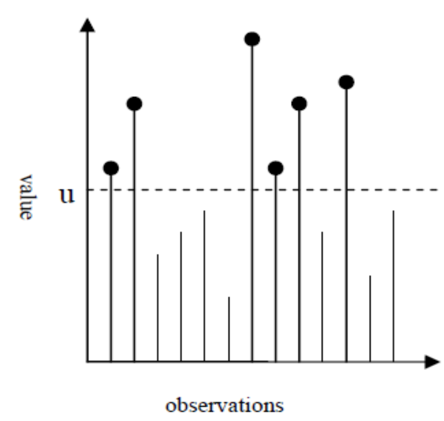
$$F_u(y) = \frac{F(u+y) - F(u)}{1 - F(u)} = \frac{F(x) - F(u)}{1 - F(u)}$$

Balkema and de Haan (1974) Pickands (1975)

- For a large class of underlying distribution functions F, for a sufficiently high threshold u, the conditional excess distribution function F_u is well **approximated** by:

$$F_u(y) \approx G_{\xi,\sigma}(y), \qquad u \to \infty$$

$$G_{\xi,\sigma}(y) = \begin{cases} 1 - (1 + \frac{\xi}{\sigma} y)^{-1/\xi}, \xi \neq 0 \\ 1 - e^{-y/\sigma}, \xi = 0 \end{cases} \qquad \text{for } y \in \begin{cases} [0, (x_F - u)], \xi \geq 0 \\ [0, -\frac{\sigma}{\xi}], \xi < 0 \end{cases}$$



$G_{\xi,\sigma}$ is the **Generalized Pareto Distribution (GPD)**

Data and results: Extreme Value Theory

- We use graphical methods to choose the threshold "u": **Mean Excess plot** and **Hill plot** (Embrecths 1997)
- The **Mean Excess function** of a **Generalized Pareto Distribution**: $e(u) = \frac{\sigma + \xi u}{1 - \xi}$
- The sample Mean Excess function and the sample Mean Excess plot:

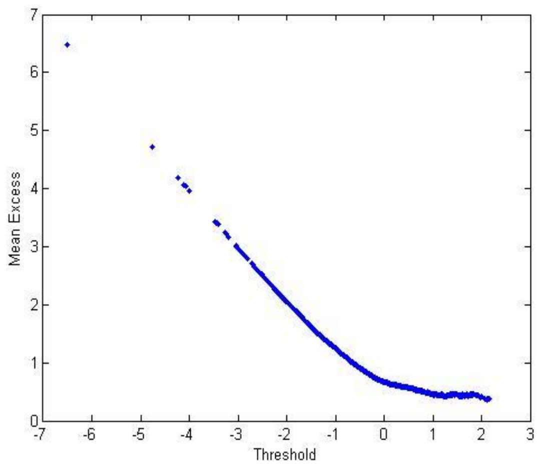
$$e_n(u) = \frac{\sum_{i=1}^{n} (X_i - u)}{\sum_{i=1}^{n} I_{\{X_i > u\}}}$$

$$\left\{ \left(X_{k,n}, e_n\left(X_{k,n}\right) \right) \middle| k = 1, \ldots, n \right\}$$

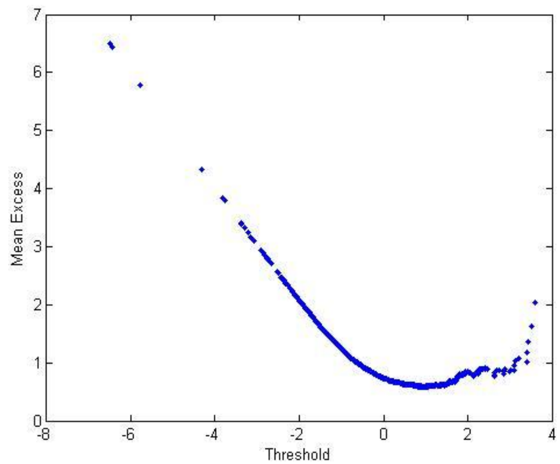
- We build the plots against the whole distribution and choose the threshold "u" in the region where **the curve becomes linear** and, therefore, the data is approximated by the **GPD**

Data and results: Extreme Value Theory

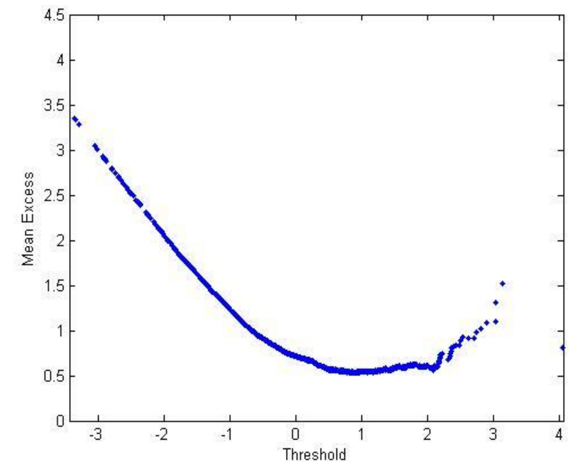
S&P right tail



BET right tail



Gold right tail



	S&P	BET	Gold
Threshold	4%	8%	4%

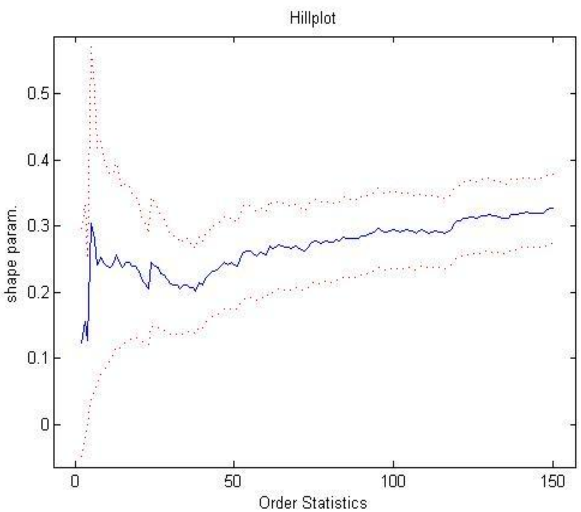
We can notice that the sample mean excess plot is seldom perfectly linear, particularly towards the right-hand end, where we are averaging a small number of large excesses. In fact we have omitted the final few points from consideration, as they can severely distort the picture.

Therefore, we expect that the estimated tail index for the right tail of S&P distribution to be approximately zero and for the right tails of BET and Gold distributions to be positive.

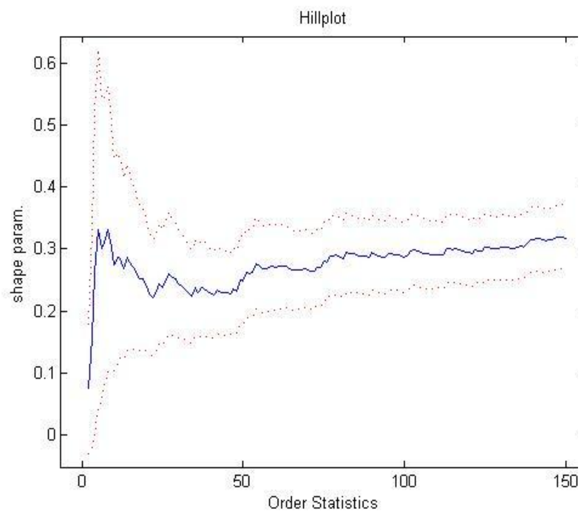
Data and results: Extreme Value Theory

- In case of fat tails distributions which decay like a power function we can use Hill Estimator for the shape parameter ξ
- The sample **Hill estimator** and the sample **Hill plot**: $\xi_{k,n}^{Hill} = \frac{1}{k}\sum_{i=1}^{k}\log X_{n+1-i,n} - \log X_{n-k,n}$ $\left\{\left(k, \hat{\xi}_{k,n}\right), 1 \le k \le n\right\}$
- The shape parameter has less bias for less data and larger variance for less data, therefore, we find a **stable region** on the graph in order to minimise the MSE

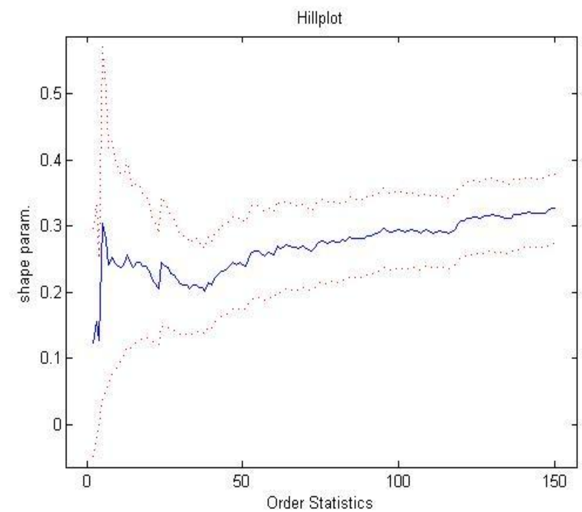
S&P right tail



BET right tail



Gold right tail



Tail	S&P $_{right}$	S&P $_{left}$	BET $_{right}$	BET $_{left}$	Gold $_{right}$	Gold $_{left}$
Stability interval of Hill estimator	80 obs 130 obs	70 obs 110 obs	100 obs 150 obs	110 obs 160 obs	80 obs 110 obs	80 obs 120 obs

Data and results: Extreme Value Theory

- Estimate the parameters of the **Generalized Pareto Distribution**:

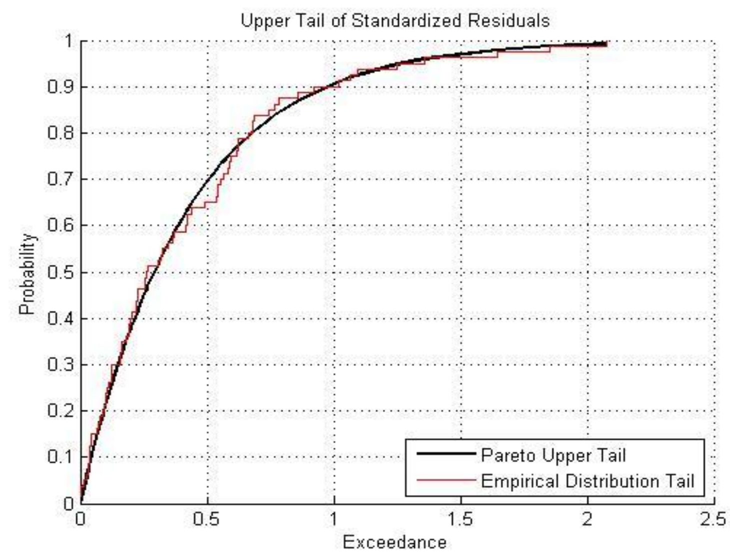
Tail	S&P $_{right}$	S&P $_{left}$	BET $_{right}$	BET $_{left}$	Gold $_{right}$	Gold $_{left}$
ξ ML estimates	0.01205	0.1635	0.0044	0.1682	0.11457	0.1696
σ ML estimates	0.422	0.5317	0.5829	0.589	0.0531	0.617

- The stock indices **S&P** and **BET** have **the left tail heavier than the right one** due to the fact that their volatility and prices have a negative correlation which is increased by the leverage effect (Black 1976).

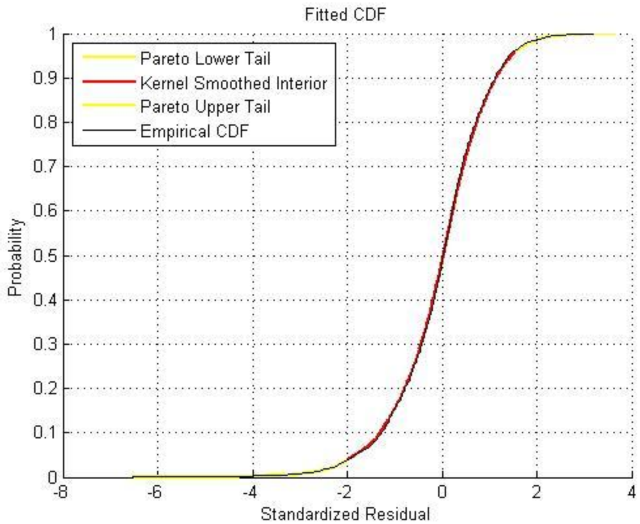
- The shape parameters estimated for the **Gold** residuals are significantly different from zero and even if the left tail is heavier than the right one, the difference is not significant. Actually, this result is as expected due to the fact that gold prices and its volatility have an unstable evolution during time.

Upper Tail of S&P standardized residuals →

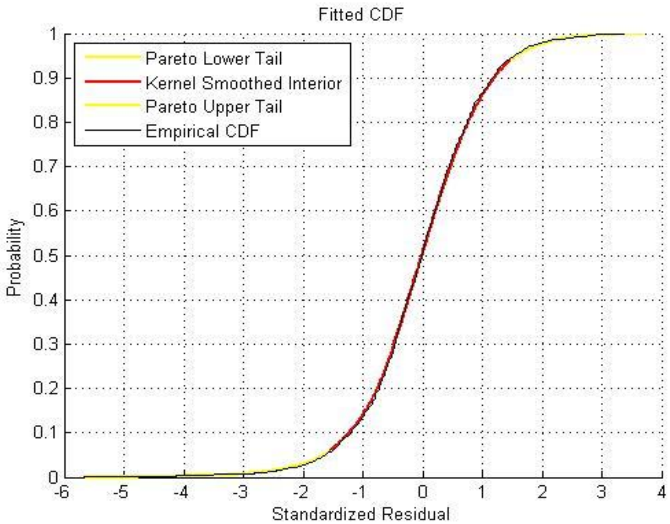
GPD provides a good fit when compared to the empirical distribution



Data and results: Extreme Value Theory

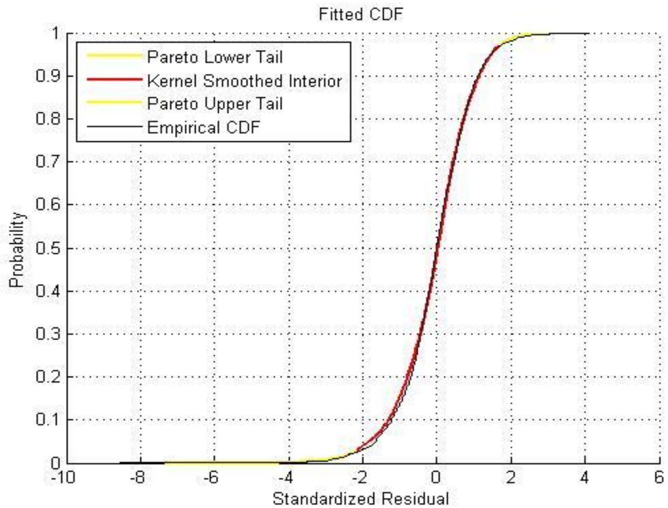


S&P500 CDF



BET CDF

Fit the semi-parametric distribution using the estimated parameters with the **Peaks over Threshold** framework and a non-parametric **Gaussian Kernel Smooth** in the interior.



Gold CFD

Data and results: Copulas Functions

- Estimate the Copula parameters through: **CML** and **IFM**
- Use the information criteria **AIC** and **BIC** - relative quality - in order to choose the proper function

Copula Function IFM(S&P+Gold)	Information Criteria AIC	BIC
Normal	-35.793	-30.180
Clayton	-44.929	-39.316
Rotated Clayton	-25.952	-20.339
Gumbel	-39.107	-33.495
Rotated Gumbel	-54.155	-48.542
Student t	**-76.628**	**-65.402**
Plackett	-43.028	-37.415
Frank	-39.713	-34.100
Symmetrised Joe-Clayton	-51.303	-40.077
Time-varying Normal	-94.202	-77.364
Time-varying rotated Gumbel	-93.888	-77.050
Time-varying SJC	-78.431	-44.754
Time-varying Student t	-93.170	-76.332

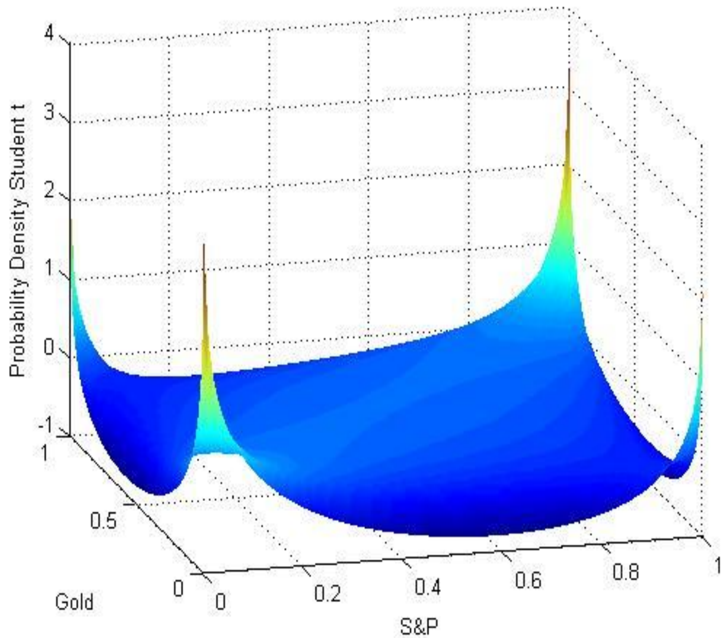
Copula Function CML(S&P+Gold)	Information Criteria AIC	BIC
Normal	-35.592	-29.979
Clayton	-44.491	-38.878
Rotated Clayton	-25.283	-19.670
Gumbel	-38.361	-32.748
Rotated Gumbel	-53.687	-48.074
Student t	**-76.430**	**-65.205**
Plackett	-43.003	-37.390
Frank	-39.710	-34.097
Symmetrised Joe-Clayton	-51.216	-39.990
Time-varying Normal	-93.320	-76.481
Time-varying rotated Gumbel	-93.056	-76.218
Time-varying SJC	-78.219	-44.542
Time-varying Student t	-90.472	-73.634

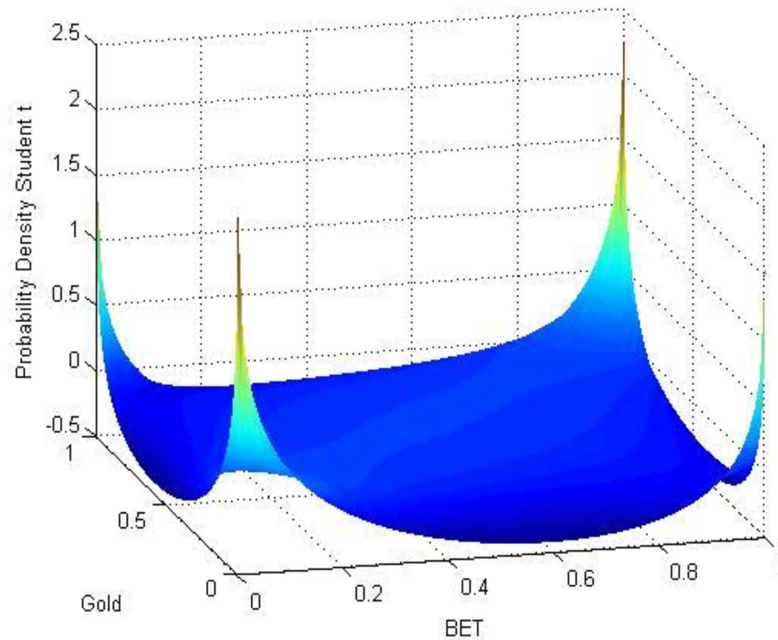
Copula Function IFM(BET+Gold)	Information Criteria AIC	BIC
Normal	-78.482	-72.869
Clayton	-88.058	-82.445
Rotated Clayton	-72.163	-66.550
Gumbel	-101.957	-96.344
Rotated Gumbel	-113.398	-107.786
Student t	**-207.448**	**-196.222**
Plackett	-92.629	-87.016
Frank	-88.997	-83.384
Symmetrised Joe-Clayton	-113.209	-101.984
Time-varying Normal	-110.419	-93.580
Time-varying rotated Gumbel	-95.558	-78.719
Time-varying SJC	-104.705	-71.028
Time-varying Student t	-93.862	-77.023

Copula Function CML(BET+Gold)	Information Criteria AIC	BIC
Normal	-78.038	-72.425
Clayton	-87.189	-81.576
Rotated Clayton	-70.225	-64.612
Gumbel	-99.248	-93.635
Rotated Gumbel	-111.635	-106.022
Student's t	**-207.199**	**-195.973**
Plackett	-92.533	-86.920
Frank	-88.928	-83.315
Symmetrised Joe-Clayton	-112.574	-101.348
Time-varying Normal	-108.384	-91.545
Time-varying rotated Gumbel	-95.149	-78.311
Time-varying SJC	-103.601	-69.924
Time-varying Student t	-93.118	-76.280

Data and results: Copulas Functions

Student t Copula recorded the lowest AIC and BIC values for both portfolios - it can capture both central and tail dependence (Stefano Demarta & Alexander J. McNeil, 2004)





Data and results: Copulas Functions

The estimated Student t Copula parameters:

Time series	Correlation Coeff.	DOF	Kendall's tau
S&P+Gold	0.149	6.495	0.159
BET+Gold	0.069	11.599	0.073

- **BET+Gold:** the correlation coefficient is almost zero, therefore there is no linear relationship between the two variables. The Kendall's tau coefficient has also an approximately zero value, therefore the probability that large values of BET returns are paired with large values of Gold returns is almost equal to the probability that large values of BET returns are paired with small values of Gold returns.

- **S&P+Gold:** the correlation coefficient has a small value, thus the linear relationship between S&P and Gold is positive and weak. Moreover, the Kendall's tau coefficient denotes a small similiarity of the orderings of the data when ranked by each of the quantities.

- We can also state that the equities prices and gold prices are neither comonotonic or countermonotonic.

Data and results: Copulas Functions

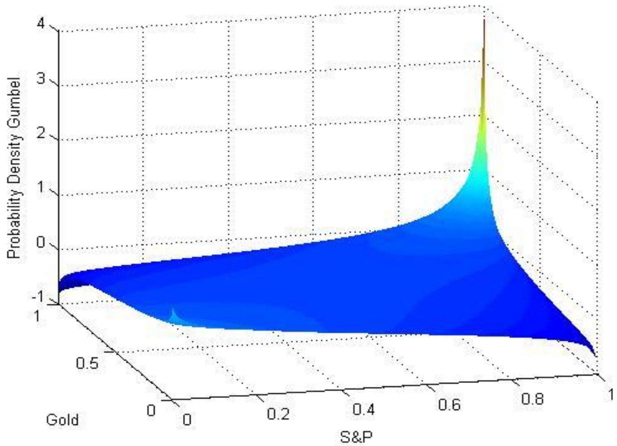
Very low tail dependence coefficients for both analyzed portfolios, the phenomenon of joint extreme values **is not present**

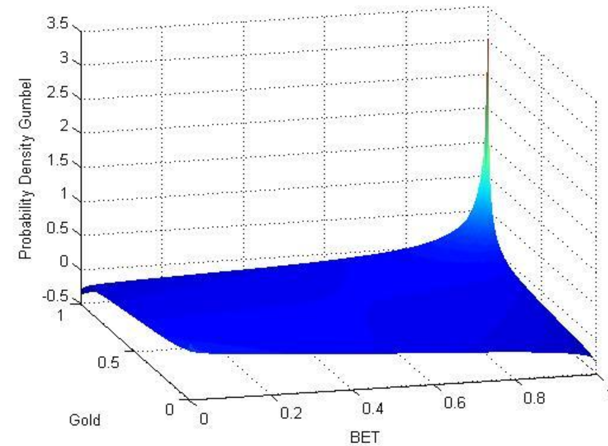
Copula Function	S&P+Gold		BET+Gold	
	Lower	Upper	Lower	Upper
Normal	0.000	0.000	0.000000	0.000000
Clayton	0.021	0.000	0.000028	0.000000
Rotated Clayton	0.000	0.008	0.000000	0.000039
Gumbel	0.000	0.114	0.000000	0.053388
Rotated Gumbel	0.125	0.000	0.051629	0.000000
Student t	0.048	0.048	0.005500	0.005500
Plackett	0.000	0.000	0.000000	0.000000
Frank	0.000	0.000	0.000000	0.000000
SJC	0.056	0.008	0.000292	0.001225

Normal copula has **zero** tail dependence, Student t copula has **symmetric** tail dependence, Clayton copula has **zero upper** tail dependence, Rotated Clayton copula has **zero lower** tail dependence, Gumbel copula has **zero lower** tail dependence, Rotated Gumbel copula has **zero upper** tail dependence, Frank copula has **zero** tail dependence, SJC copula parameters are the tail dependence coefficients, but in reverse order

Data and results: Copulas Functions

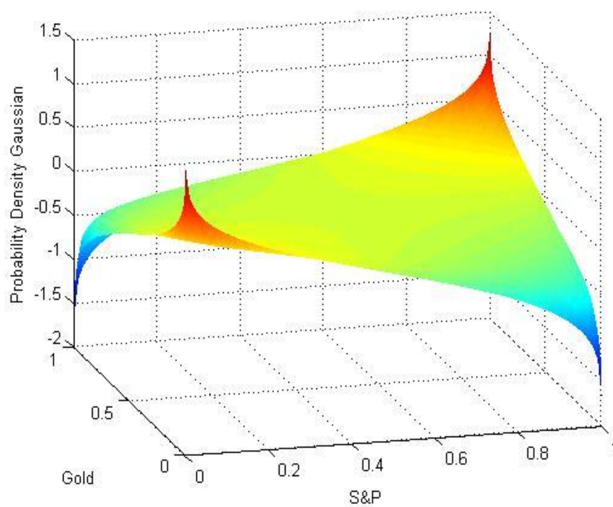
Exception: **Copula Gumbel** and **Rotated Gumbel** which focus mostly on dependence in the tails and overestimate it - too pessimistic on diversification benefits

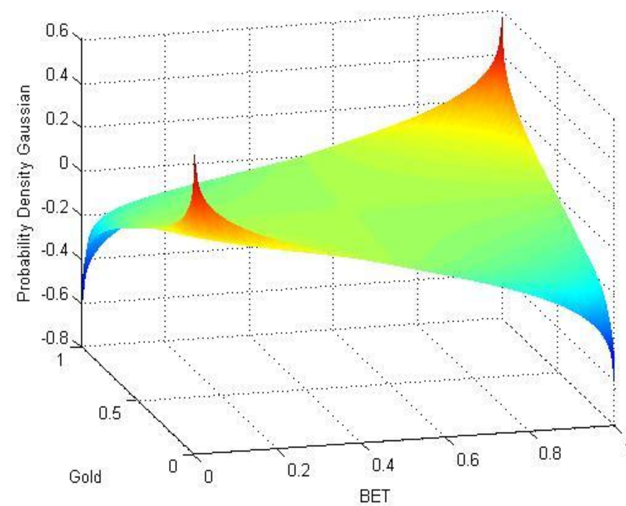




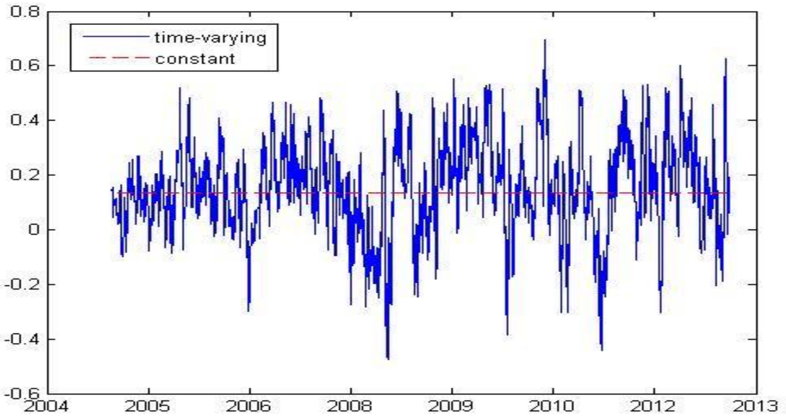
Normal Copula has zero tail dependence →

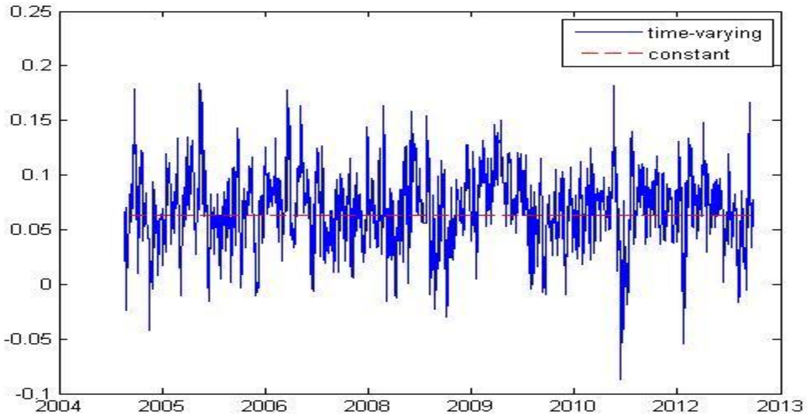
and therefore is too optimistic





Data and results: Copulas Functions





S&P/Gold Kendall's tau **BET/Gold Kendall's tau**

- The time varying Kendall's tau rank correlation coefficient between S&P and Gold ranges in **[-0.5, 0.7].** The **minimum value** is reached at the end of year 2008, when Lehman Brothers collapsed (September 2008) and the financial crisis started spreading. On November 2008, the Federal Reserve announced the start of QE1 which had an immediate positive impact on gold price. Another negative correlation spike is reached on August 2011 when gold increased at its all-time high of $1,917.90 per ounce, at the end of QE2 and the S&P plummeted approximately 13% after the US's AAA rating had been downgraded for the first time in history.
- All in all there are periods with positive rank correlation values and negative rank correlation values, with an average of **0.159** for the analysed sample period. We can conclude that there is **no stable** relation based on the rank correlation between gold and equities in US. Moreover, **the co-movement is not symmetric** and there cannot be identified a pattern of dependence structure when the global economy starts booming or crashing during the crisis, but only negative spikes occurring during extreme events.
- The time varying Kendall's tau coefficient between BET and Gold is **more stable** in comparison with the US picture, it ranges in **[-0.09, 0.18]** with an average value of **0.073.** Therefore, we conclude that it has an evolution in **tight range** around the mean with **an approximately symmetric co-movement**. As in the previous picture, there cannot be identified a pattern of dependence structure when the global economy starts booming or crashing during the crisis.
- The lowest level is reached on August 2011, a period with a negative evolution of BET index due to European sovereign debt crisis. On comparison to US stock market-gold co-movement, between BET and Gold there are fewer negative spikes during financial turmoil periods.

Methodology: Value-at-Risk

Define the portfolio and identify its risk factors:

- Portfolio1: $(w_1 \times R_{S\&P}) + (w_2 \times R_{Gold})$
- Portfolio2: $(w_1 \times R_{BET}) + (w_1 \times R_{USDRON}) + (w_2 \times R_{Gold})$

Monte Carlo method:

- Set the basic parameters for the VaR model: **the confidence level** and **the risk horizon**. Therefore, we choose to estimate Value-at-Risk for 1%, 5% and 10% quantile over a 1-day and 10-days holding periods
- Calibrate the filtered returns of each risk factor independently based on **EVT** and then calibrate the dependence between variables with the **Student t Copula**.
- **Simulate** dependent uniform variables (number of trials × time horizon), transform the uniform variables to standardized residuals by applying the inversion of the semi-parametric marginal CDF. These residuals are independent in time but dependent at any point in time.
- Using the simulated standardized residuals as the I.I.D. input noise process, **reintroduce the autocorrelation and heteroskedasticity.**
- Compute **the portfolio cumulated returns** by multiplying the simulated returns with the specified weights.
- Estimate the α quantile of the simulated portfolio return distribution in order to obtain the **100α% VaR**. Then, estimate the 100α% ETL as the average of the returns less than the α quantile.

Data and results: Value-at-Risk

Compute the **ΔCVaR** in relative terms as difference between CVaR for diversified portfolios and CVaR for equity portfolios:

*time horizon: 1 day

Portfolio	1	2	3	4
S&P w_i	1	0.95	0.9	0.85
Gold w_i	0	0.05	0.1	0.15
VaR(90%)	-1.43	-1.38	-1.32	-1.27
VaR(95%)	-2.19	-2.10	-2.00	-1.92
VaR(99%)	-4.28	-4.22	-4.02	-3.84
ΔCVaR(90%)		-2.5%	-7.0%	-11.1%
ΔCVaR(95%)		-1.7%	-6.4%	-10.8%
ΔCVaR(99%)		-0.4%	-5.3%	-10.1%

*time horizon: 1 day

Portfolio	1	2	3	4
BET/USDRON w_i	1	0.95	0.9	0.85
Gold w_i	0	0.05	0.1	0.15
VaR(90%)	-2.48	-2.33	-2.18	-2.04
VaR(95%)	-3.52	-3.30	-3.10	-2.91
VaR(99%)	-6.25	-5.87	-5.54	-5.21
ΔCVaR(90%)		-7.1%	-12.6%	-18.1%
ΔCVaR(95%)		-7.4%	-12.9%	-18.2%
ΔCVaR(99%)		-9.4%	-14.6%	-19.8%

*time horizon: 10 days

Portfolio	1	2	3	4
S&P w_i	1	0.95	0.9	0.85
Gold w_i	0	0.05	0.1	0.15
VaR(90%)	-5.53	-5.14	-4.88	-4.65
VaR(95%)	-7.48	-6.98	-6.63	-6.30
VaR(99%)	-11.91	-11.04	-10.52	-9.97
ΔCVaR(90%)		-7.0%	-11.6%	-15.9%
ΔCVaR(95%)		-7.1%	-11.7%	-16.0%
ΔCVaR(99%)		-6.0%	-10.8%	-15.3%

*time horizon: 10 days

Portfolio	1	2	3	4
BET/USDRON w_i	1	0.95	0.9	0.85
Gold w_i	0	0.05	0.1	0.15
VaR(90%)	-7.50	-7.01	-6.54	-6.09
VaR(95%)	-10.43	-9.81	-9.16	-8.56
VaR(99%)	-16.73	-15.72	-14.73	-13.74
ΔCVaR(90%)		-7.9%	-12.2%	-18.1%
ΔCVaR(95%)		-7.5%	-12.1%	-17.9%
ΔCVaR(99%)		-6.1%	-12.0%	-17.8%

Therefore, **gold is a better diversifier for an investor in Romanian stock market** in comparison with an investor in US stock market when we use as a proxy for risk the CVaR (extreme risk).

Data and results: Value-at-Risk

Backtesting (25/04/2012 -10/05/2013 , 250 observations):

- Compute the out-of-sample data with 1 day window length and compare the number of empirical violations with VaR confidence levels (1% 5% 10%).
- Compute a Bernoulli test to estimate the confidence intervals.

	VaR(99%)	VaR(95%)	VaR(90%)
Confidence intervals	<2.5%	<7.2%	<12.4%
P1 GARCH-EVT-COPULAS	1.20%	5.20%	10.40%
P1 HISTORICAL SIMULATION	2.40%	6%	11.20%
P2 GARCH-EVT-COPULAS	0.80%	5.60%	10.80%
P2 HISTORICAL SIMULATION	2%	6.40%	12%

Basel II Reglemantations for Backtesting	
VaR(99%) 250 observations	green ≤ 4 exceptions
	yellow [5,9] exceptions
	red ≥ 10 exceptions

Therefore, we get better results when compute the aggregate risk with a GARCH-EVT-COPULAS model and **Monte Carlo Simulation** method in comparison with the simplest and most frequently used Historical Simulation method.

Data and results: Conditional Value-at-Risk

*time horizon: 1 day

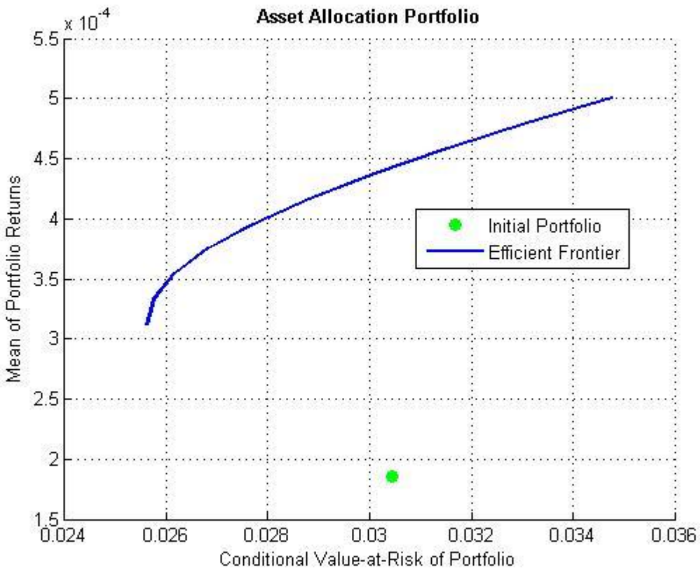
Portfolio	Minimum CVaR
S&P w_i	0.48
Gold w_i	0.52
Expected Return	0.00031
CVaR(95%)	0.02562
Conditional Sharpe Ratio	0.01220

Portfolio	Minimum CVaR
USDRON/BET w_i	0.34
Gold w_i	0.66
Expected Return	0.00109
CVaR(95%)	0.02609
Conditional Sharpe Ratio	0.04180

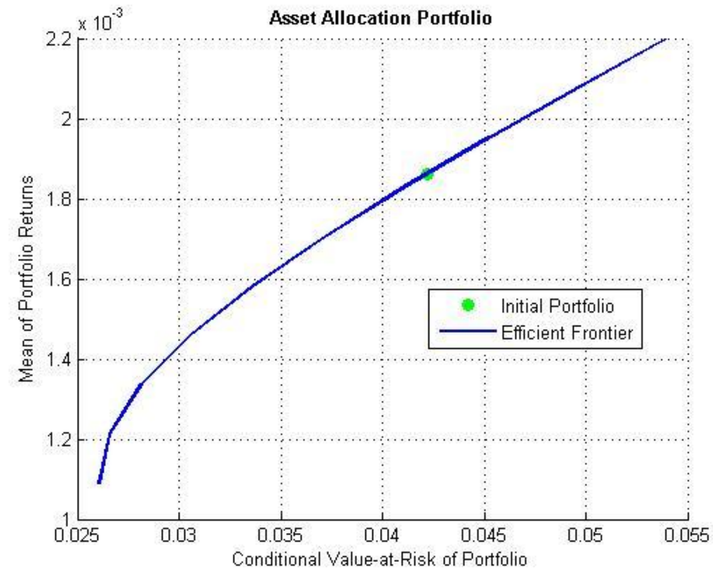
When you invest in S&P500 index and Gold you have to choose approximately **equal risk contributions** in order to obtain the minimum CVaR portfolio. This is an expected result due to the fact that our times series have a similar evolution of their volatilities during the analysed period and, moreover, similar higher moments (skewness and kurtosis).

Analysing the portfolio formed by BET and Gold, the Minimum-CVaR optimization monotonically **underweights** the allocation in equities (34%) because of its more extreme negative skewness and higher kurtosis, and it **overweighs** the allocation in gold (66%) because of its more attractive combined skewness and kurtosis.

Data and results: Conditional Value-at-Risk



S&P500+Gold



BET+Gold

The expected Conditional Sharpe ratio of BET+Gold minimum CVaR portfolio is **3.5 times higher** than the one of S&P+Gold minimum CVaR portfolio.

Therefore, if we are risk adverse investors and look for the minimum CVaR portfolio, we would choose to invest **in the Romanian Stock market** in order to obtain a **higher risk-adjusted performance**.

Initial Portfolio	Weights
USDRON/BET w_i	0.80
S&P w_i	0.80
Gold w_i	0.20

Conclusions:

- We should carefully analyze the **stylized facts** in order to choose the most proper models for our sample data.
- When applying the **ARMA-EGARCH** model for a different period of the same data, the parameters estimates does not change greatly, therefore the model is properly chosen.
- When fitting the copulas functions to data we get more accurate results for the semi-parametric approach in comparison with the CML method, thus the use of **Extreme Values Theory – POT respectively – improves our estimates**.
- We can conclude that there is **no stable** relation based on the rank correlation between gold and equities in US. Moreover, **the co-movement is not symmetric** and there cannot be identified a pattern of dependence structure when the global economy starts booming or crashing during the crisis, but only negative spikes occurring during extreme events. Gold and BET has an evolution in **tight range** around the mean with an approximately symmetric co-movement. As in the previous picture, there cannot be identified a pattern of dependence structure when the global economy starts booming or crashing during the crisis. It depends on the economic context, FED decisions, investors' sentiment and specific factors that separately influence both markets.
- Therefore, we should actively **analyse gold-stocks evolution** in order to achieve the targeted level of diversification of a mixed portfolio.
- The phenomenon of **joint extreme values is not present** between S&P and Gold time series , BET and Gold time series respectively because the tail dependence coefficients computed with different Copulas functions are not significantly different from zero.

Conclusions:

- The **backtesting** results of VaR computed with Monte Carlo method and ARMA-EGARCH-EVT-COPULAS for calibration of the model to historical data are in accordance to the Basel II requirements. Moreover, we get better results in comparison with Historical Simulation method.
- If we increase the amount invested in gold and decrease the amount invested in equities there is a **higher impact** on Conditional Value-at-Risk computed for the **BET index** portfolio in comparison with the S&P index portfolio for 1-day and 10-days time horizons, at a 90%, 95%, and 99% confidence levels. The greatest decrease of CVaR is encountered at a 99% confidence level, for 1-day horizon. Therefore, **gold is a better diversifier for an investor in Romanian stock market** in comparison with an investor in US stock market when we use as a proxy for risk the CVaR (extreme risk).
- When you invest in S&P500 index and Gold you have to choose approximately **equal** risk contributions in order to obtain the minimum CVaR portfolio. On the other hand, analysing the portfolio formed by BET and Gold, the Minimum-CVaR optimization monotonically **underweights** the allocation in equities (34%) and **overweighs** the allocation in gold (66%).
- If we are risk adverse investors and look for the minimum CVaR portfolio, we would choose to invest in the Romanian Stock market in order to obtain a higher risk-adjusted performance.
- All in all, it is a vital condition to **actively manage the dependence structure of gold and stocks** relation and actively measure the aggregate risk in order to achieve a targeted degree of diversification or a level of risk adjusted performance.

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