Academy of Economic Studies

Doctoral School of Finance and Banking

An Analysis of Volatility
- A Non- normal Approach -

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TOPICS

I. Motivation

- The global financial crisis highlights the importance of estimating and forecasting the volatility of financial assets and implicitly the undertaken risk.

- *The estimation and the forecast of volatility* are two key points in areas such as:
  - defining the option prices;
  - forecasting the VaR;
  - establishing the optimal changes in capital, agreeing with the Basel Accord.

II. Objectives

- **Analyze the impact of the excess kurtosis vs. the impact of skewness of a financial time series in the context of financial crisis which started in 2008.**

- Showing that the Normal distribution is a not appropriate approach for modeling time series;

- Comparing NIG-GARCH vs. SSTD - APARCH using different methodologies of testing;

- Estimating the risk using a coherent measure and backtesting it.

III. Literature Review

The **ARCH model** was introduced by Engle (1982).
The **GARCH model** was developed independently by Bollerslev (1986) and Taylor (1986).

- *Ding* et al. (1993) introduced the Asymmetric Power ARCH model useful to characterize the 'long memory' property of the time series.

- *Forsberg and Bollerslev* (2002) using high frequency of data of returns for the ECU/USD and they constructed a NIG-GARCH model. They obtained accurate results regarding the prediction out-of-the sample of EUR returns using NIG- GARCH model.

- The NIG distribution was introduced *by Barndoff- Nielsen (1997)* as a special case of Generalized Hyperbolic distribution.

- *Christoffersen* et al. (2001) introduced the conditional coverage test.

- *Arztner (1998)* introduced CVaR as a coherent measure of risk.

VI. Data set

- The data set comprise the returns of the daily exchange rate EUR/RON between **1st January 2001 and 1st March 2013.**
- The returns :

$$r_t = (\ln(EUR/RON_t) - \ln(EUR/RON_{t-1}))*100$$

- The returns were tested for *normality*, *heteroskedasticity*, *stationarity and serial correlation.*
- **Results of tests:**
  - The return series **is not normal** distributed;
  - The series is **heteroskedastic**;
  - The series has **no unit root**;
  - The serial **presents serial correlation.**

VI. Data set (Cont'd)

	Breusch - Pagan Test	Goldfeld-Quandt Test	ARCH LM-Test
Statistic	0.1723	0.8731	390.7497
p-value	0.6781	0.9964	< 2.2e-16
H0	The residuals are homoskedastic.	The residuals are homoskedastic.	The residuals are homoskedastic.
Decision	Reject H0	Reject H0	Reject H0

Table 1: Homosketasticity test

	Augmented Dickey-Fuller Test	Kwiatkowski-Phillips-Schmidt-Shin (KPSS) Test
Statistic	-17.8915	0.3798
p-value	0.01	0.08585
H0	The series has unit root.	The series has unit root.
Decision	Reject H0	Reject H0*

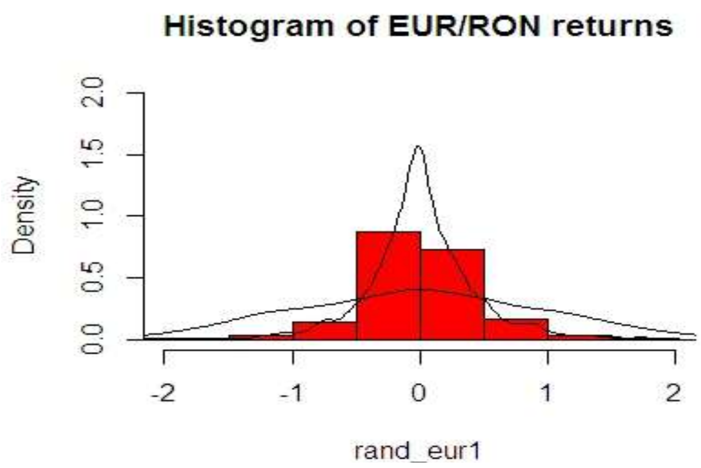
** for a treshold of 10%*

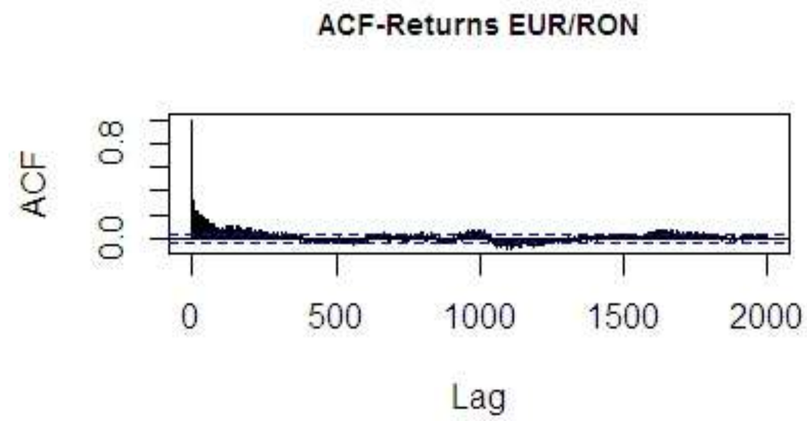
Table 2: Stationarity testing

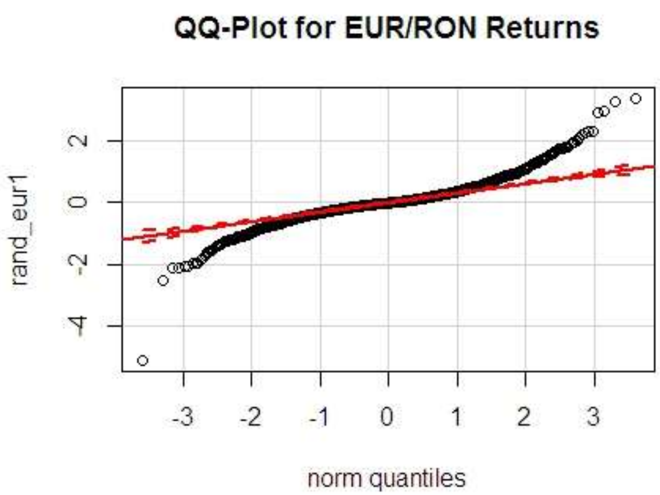
	Jarque- Bera Test	ShapiroTest - Shapiro-Wilk Test	D'Agostino Test	Lilliefors (Kolmogorov-Smirnov) Test	Pearson chi-square Test	Shapiro-Francia Test
Statistic	10538.1746	0.9102	-	0.092	786.2879	0.9086
p-value	2.20E-16	< 2.2e-16	-	< 2.2e-16	< 2.2e-16	< 2.2e-16
Chi2 | Omnibus	-	-	541.4908	-	-	-
Z3 | Skewness	-	-	4.1786	-	-	-
Z4 | Kurtosis	-	-	22.8917	-	-	-
p-value | Omnibus	-	-	< 2.2e-16	-	-	-
p-value | Skewness	-	-	2.93E-05	-	-	-
p-value | Kurtosis	-	-	2.20E-16	-	-	-
H0	The returns have a normal distribution.	The returns have a normal distribution.	The returns have a normal distribution.	The returns have a normal distribution.	The returns have a normal distribution.	The returns have a normal distribution.
Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

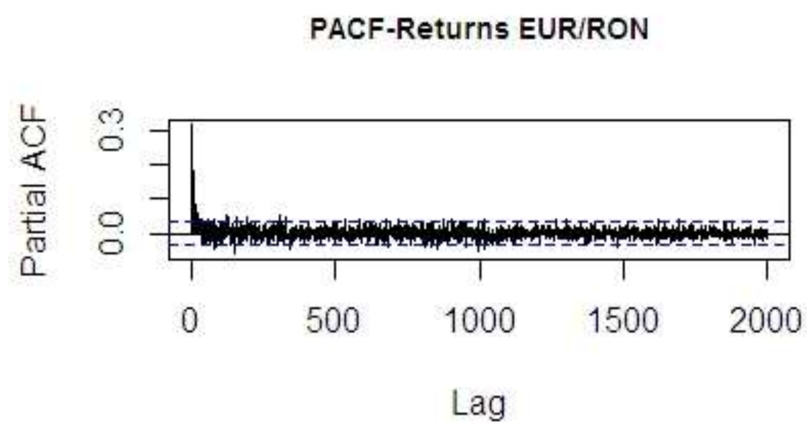
Table 3: Normality tests results

Data set (Cont'd)









V.

NIG-GARCH vs. SSTD - APARCH

Normal Inverse Gaussian Distribution

- Normal variance – mean mixture
- *Inverse Gaussian* distribution as mixing density.
- Density function is:

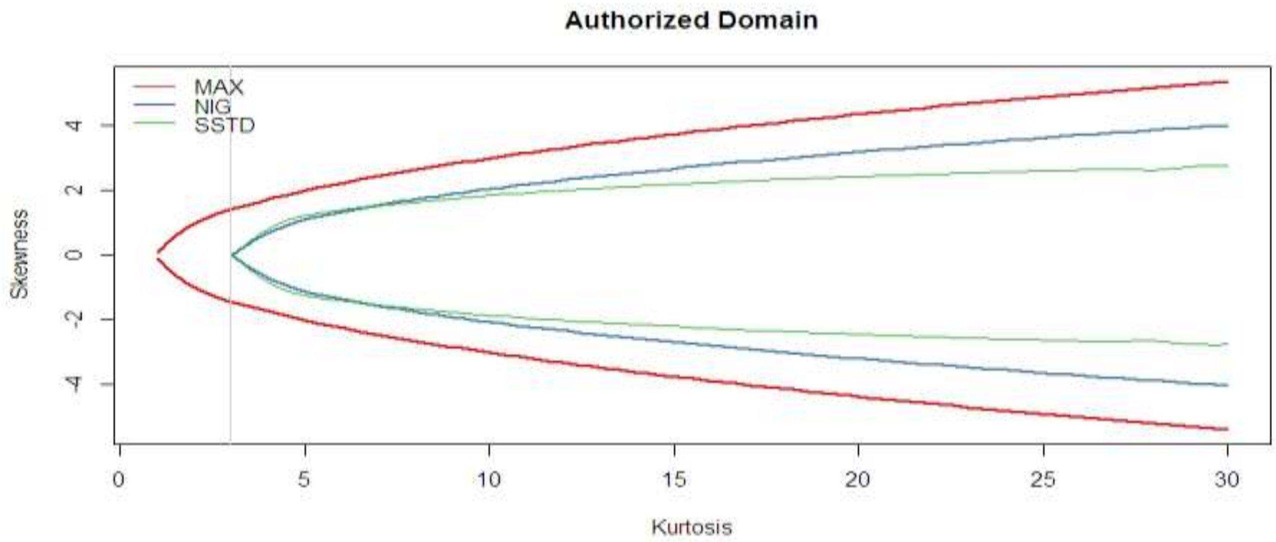
$$g(x;\alpha,\beta,\mu,\delta)=a(\alpha,\beta,\mu,\delta)q\left(\frac{x-\mu}{\delta}\right)^{-1}K_1\left\{\delta\alpha q\left(\frac{x-\mu}{\delta}\right)\right\}\exp(\beta x)$$

Skew Student t Distribution

- Normal mean-variance mixture distribution;
- The mixture density is *Inverse Gamma*;
- Density function is:

$$f_x(x)=\frac{2^{\frac{1-\nu}{2}}\delta^{\nu}|\beta|^{\frac{\nu+1}{2}}K_{\frac{\nu+1}{2}}\left(\sqrt{\beta^2(\delta^2+(x-\mu)^2)}\right)\exp(\beta(x-\mu))}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\pi}\left(\sqrt{\delta^2+(x-\mu)^2}\right)^{\frac{\nu+1}{2}}},\quad \beta\neq 0,$$

V. NIG-GARCH vs. SSTD – APARCH (Cont'd)



- NIG distribution is generally *more flexible* in accommodating varying combinations of skewness and kurtosis than the SSTD distribution;
- NIG distribution it a *strong option distribution* for modeling the financial series.

V. NIG-GARCH vs. SSTD - APARCH (Cond't)

- **GARCH Model**

$$\sigma_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta \sigma_{t-1}^2$$

- **APARCH Model**

$$\sigma_t^\delta = \omega + \sum_{i=1}^{p} \alpha_i \left(|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i} \right)^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta$$

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V. NIG-GARCH vs. SSTD- APARCH –Methodology (Cond’t)

Schema:

- Rolling forecast with moving window of 1500 (1-ahead rolling forecast)
- Re-estimation at every 50 periods

LT=1500

t	t+LE -1	VaR(t+LE)
1	1646	VaR(1647)
2	1647	VaR(1648)
3	1648	VaR(1649)
⋮	⋮	⋮
1499	3145	VaR(3146)

LE=1646

LT=1500

Start Date	End Date	VaR Forecast Date
1/5/2001	6/4/2007	6/5/2007
1/6/2001	6/5/2007	6/6/2007
1/7/2001	6/6/2007	6/7/2007
⋮	⋮	⋮
11/12/2004	3/2/2010	3/3/2010
11/13/2004	3/3/2010	3/4/2010
11/14/2004	3/4/2010	3/5/2010
⋮	⋮	⋮
11/25/2007	2/26/2013	2/27/2013
11/26/2007	2/27/2013	2/28/2013
11/27/2007	2/28/2013	3/1/2013

LT - length of the training perimeter   *LE - length of the estimation perimeter*

Motivation:

- Capturing any time variation changes;
- The result is: *forecast conditional density, VaR 1-ahead forecast*

V. NIG-GARCH vs. SSTD - APARCH (Cond't)

- **NIG - GARCH (1,1) – conditional density forecast**

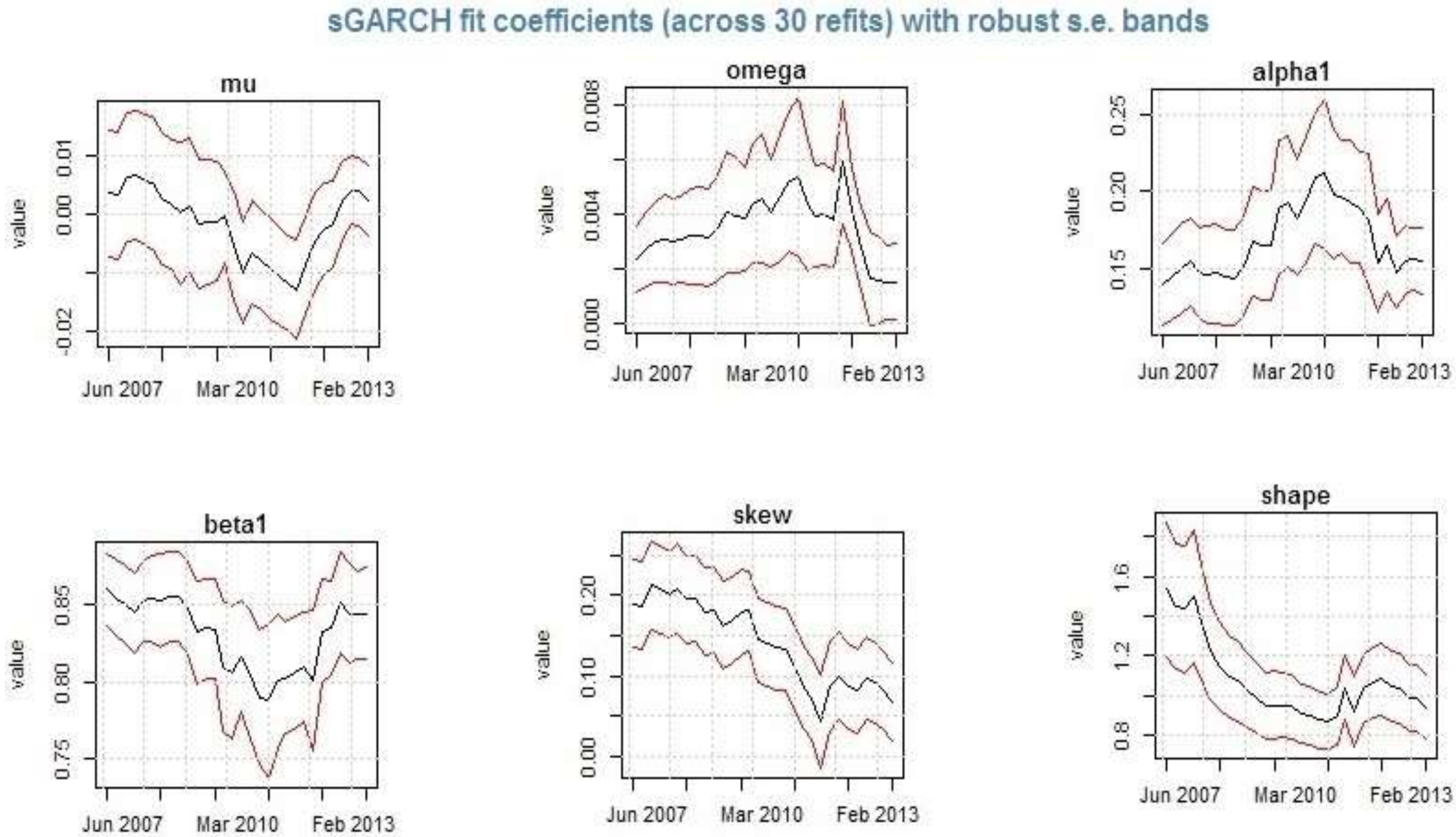
GARCH-NIG(1,1)					
Date	Mu	Sigma	Skew	Shape	Realized
2007-06-04	0.002	0.230	0.190	1.535	-0.132
2007-06-05	0.003	0.225	0.190	1.535	-0.138
2007-06-06	0.003	0.220	0.190	1.535	0.264
2007-06-07	0.006	0.232	0.190	1.535	0.064
2007-06-08	0.004	0.221	0.190	1.535	0.559
2007-06-11	0.007	0.296	0.190	1.535	-0.378
2007-06-12	0.000	0.313	0.190	1.535	-0.367
2007-06-13	0.002	0.325	0.190	1.535	0.352
2007-06-14	0.006	0.332	0.190	1.535	-0.619
2007-06-15	-0.001	0.390	0.190	1.535	-0.382
2013-02-18	0.010	0.274	0.068	0.945	-0.034
2013-02-19	-0.004	0.256	0.068	0.945	-0.064
2013-02-20	-0.002	0.239	0.068	0.945	-0.144
2013-02-21	-0.012	0.230	0.068	0.945	0.192
2013-02-22	0.028	0.229	0.068	0.945	-0.189
2013-02-25	-0.027	0.231	0.068	0.945	0.069
2013-02-26	0.020	0.219	0.068	0.945	0.014
2013-02-27	-0.002	0.205	0.068	0.945	-0.162
2013-02-28	-0.014	0.202	0.068	0.945	-0.107
2013-03-01	-0.004	0.194	0.068	0.945	-0.286

V. NIG-GARCH vs. SSTD - APARCH (Cond't)

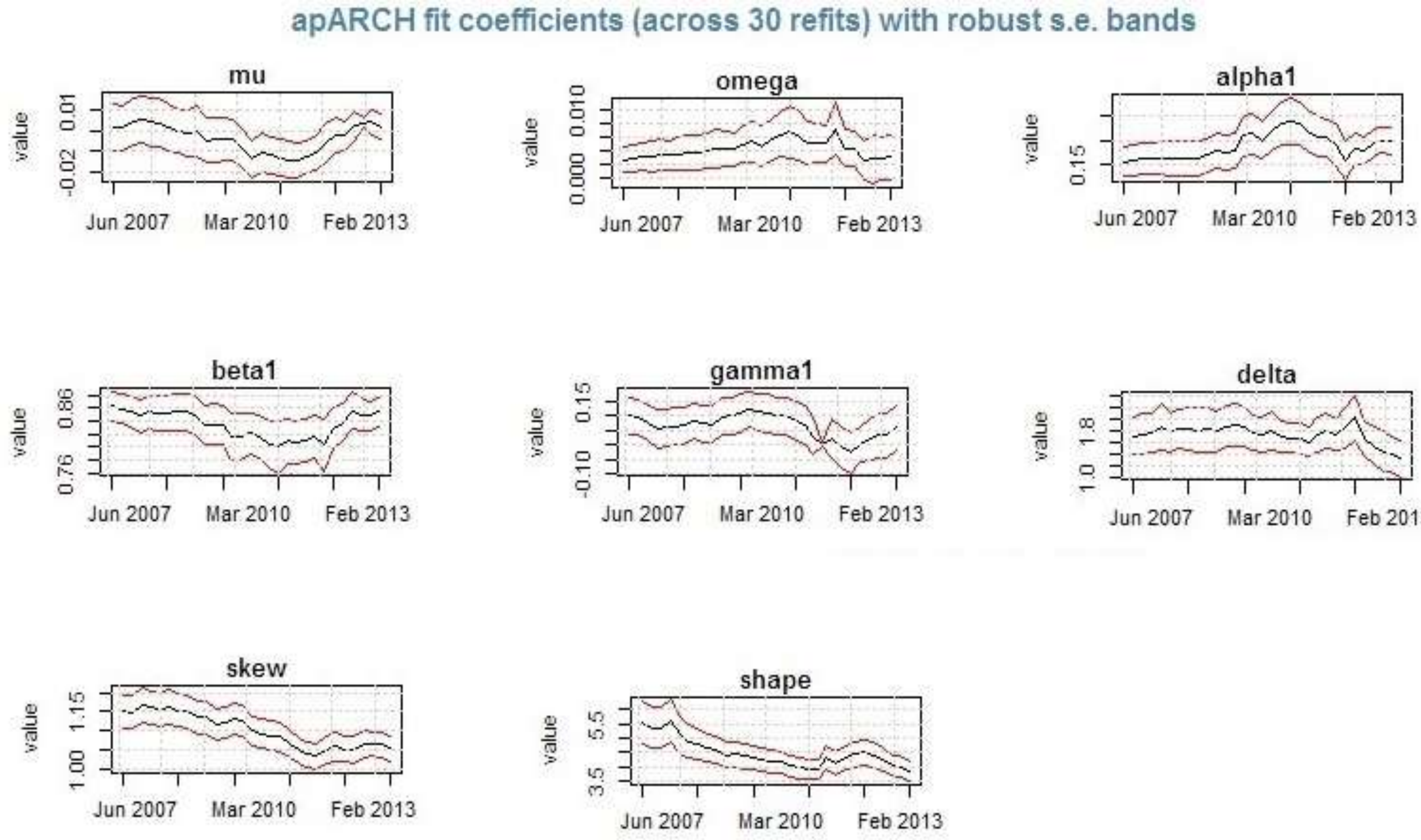
SSTD - APARCH (1,1) conditional density forecast

Date	Mu	Sigma	Skew	Shape	Realized
2007-06-04	0.004	0.236	1.151	5.552	-0.132
2007-06-05	0.000	0.232	1.151	5.552	-0.138
2007-06-06	0.003	0.228	1.151	5.552	0.264
2007-06-07	-0.001	0.236	1.151	5.552	0.064
2007-06-08	0.004	0.223	1.151	5.552	0.559
2007-06-11	-0.003	0.285	1.151	5.552	-0.378
2007-06-12	0.007	0.314	1.151	5.552	-0.367
2007-06-13	-0.001	0.337	1.151	5.552	0.352
2007-06-14	0.002	0.342	1.151	5.552	-0.619
2007-06-15	0.004	0.414	1.151	5.552	-0.382
2013-02-18	0.011	0.293	1.053	3.840	-0.034
2013-02-19	-0.004	0.267	1.053	3.840	-0.064
2013-02-20	-0.003	0.247	1.053	3.840	-0.144
2013-02-21	-0.012	0.242	1.053	3.840	0.192
2013-02-22	0.027	0.246	1.053	3.840	-0.189
2013-02-25	-0.026	0.256	1.053	3.840	0.069
2013-02-26	0.018	0.241	1.053	3.840	0.014
2013-02-27	-0.001	0.218	1.053	3.840	-0.162
2013-02-28	-0.015	0.221	1.053	3.840	-0.107
2013-03-01	-0.005	0.212	1.053	3.840	-0.286

V. NIG-GARCH vs. SSTD - APARCH (Cond't)



V. NIG-GARCH vs. SSTD - APARCH (Cond't)



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NIG-GARCH vs. SSTD - APARCH (Cond’t)

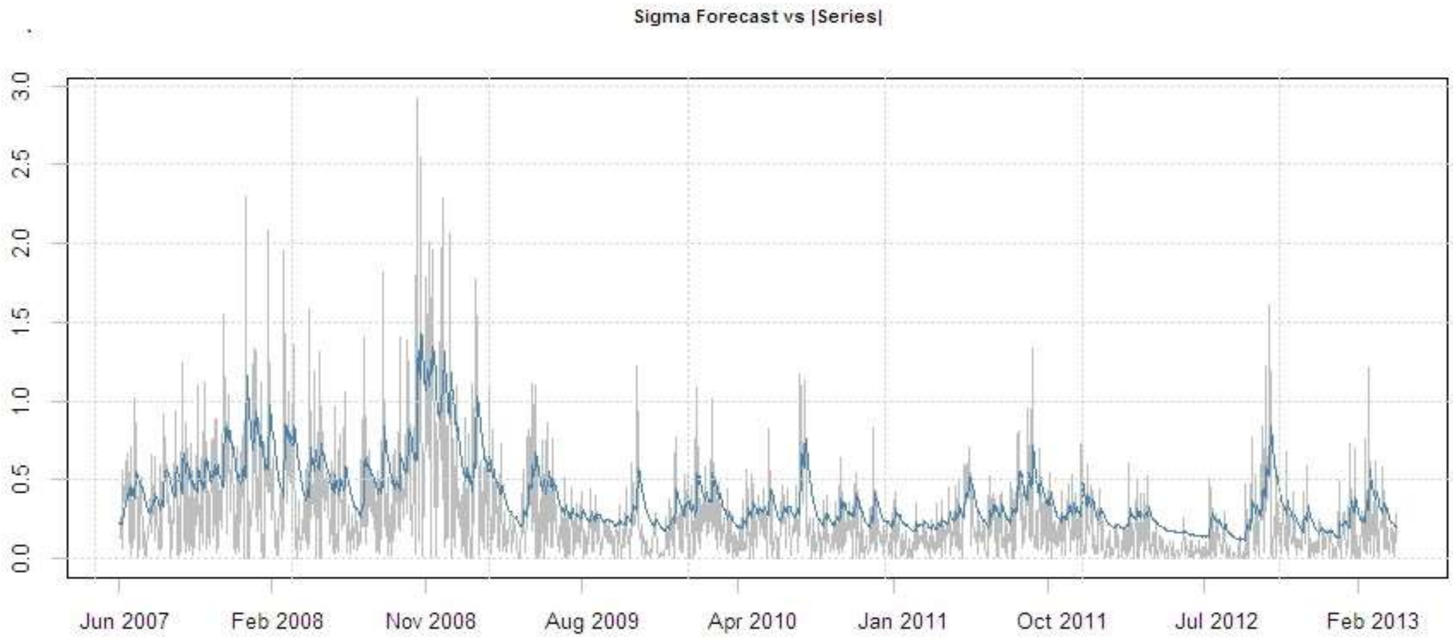


Figure : Sigma Forecast vs. Realized

V. NIG-GARCH vs. SSTD-APARCH - Backtesting

- **UNCONDTIONAL COVERAGE RATE (Kupiec)**
  - Examines how many times a financial institution's VaR is violated for a given period.

$$POF = 2 \cdot ln\left(\left(\frac{1-\hat{\alpha}}{1-\alpha}\right)^{T-I(\alpha)}\left(\frac{\hat{\alpha}}{\alpha}\right)^{I(\alpha)}\right)$$

- **CONDITIONAL COVERAGE RATE (Christoffersen)**
  - Tests if VaR violations are independent and correct.

$$LR_{cc} = -2\ln\left[L(p)/L\left(\hat{\Pi}_1\right)\right] \sim \chi_2^2$$

V. NIG-GARCH vs. SSTD-APARCH - Backtesting

TAIL TEST (Berkowitz)

- Tests if the mean and the variance of the violations equals those implied by the model, $LR = -2\left(L(0,1,0) - L(\hat{\mu},\hat{\sigma}^2,\hat{\rho})\right)$

Where

$$L = -\frac{1}{2}\log(2\pi) - \frac{1}{2}\log\left[\frac{\sigma^2}{1-\rho^2}\right] - \frac{\left(z_t - \frac{\mu}{1-\rho}\right)^2}{\frac{2\sigma^2}{1-\rho^2}} - \frac{T-1}{2}\log(\sigma^2) - \sum_{t=2}^{T}\left(\frac{(z_t - \mu - \rho z_{t-1})^2}{2\sigma^2}\right)$$

DURATION TEST (Christoffersen and Pelletier)

- Hazard function

$$\lambda(d) = \frac{\Pr(D=d)}{1 - \sum_{j<d}\Pr(D=j)}$$

V. NIG-GARCH vs. SSTD-APARCH - Results

I.

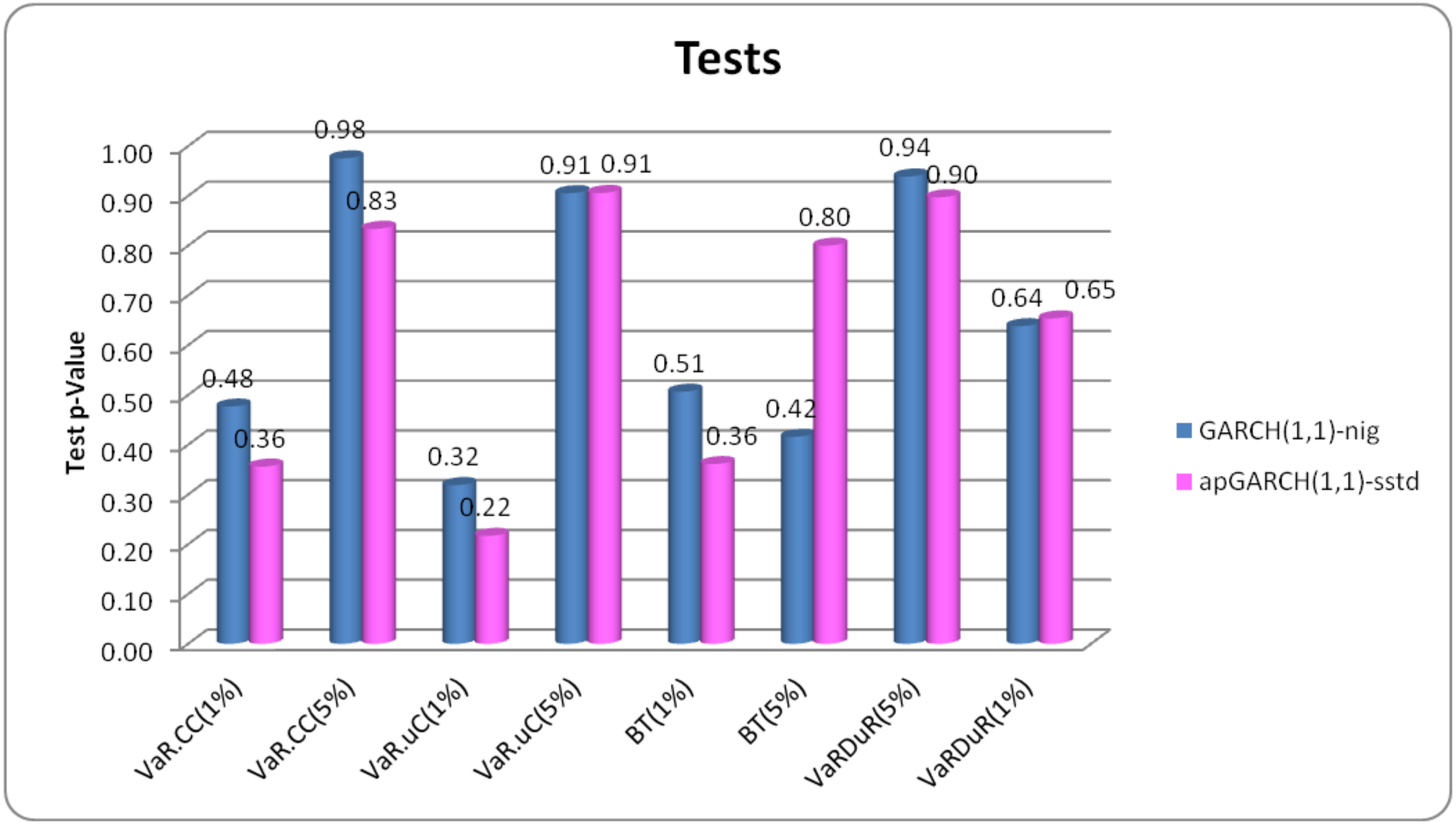
Model / Indicator	MSE	MAE
GARCH(1,1)-nig	0.1799	0.2727
apGARCH(1,1)-sstd	0.1804	0.2728

II.

Information Criterion	GARCH-NIG (1,1)	APARCH-SSTD (1,1)
Akaike	-8.637	-8.387
Bayes	-8.615	-8.368
Shibata	-8.637	-8.387
Hannan-Quinn	-8.629	-8.380

V. NIG-GARCH vs. SSTD-APARCH - Results

III.



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V. NIG-GARCH vs. SSTD-APARCH - Results

	GARCH-NIG(1,1)	APARCH-SSTD(1,1)
1%		
Expected Exceed:	15	15
Actual VaR Exceed:	19	20
Unconditional Coverage (Kupiec)*		
LR.uc p-value:	**0.319**	0.217
Reject Null:	NO	NO
Conditional Coverage (Christoffersen)**		
LR.cc p-value:	0.477	0.356
Reject Null:	NO	NO
5%		
Expected Exceed:	75	75
Actual VaR Exceed:	74	76
Unconditional Coverage (Kupiec)*		
LR.uc p-value:	0.905	**0.906**
Reject Null:	NO	NO
Conditional Coverage (Christoffersen)**		
LR.cc p-value:	**0.976**	0.834
Reject Null:	NO	NO
MSE	**0.1799**	0.1804
MAE	**0.2727**	0.2728
DAC	**0.5333**	0.5327

VI. Conditional Value – at – Risk

- It is a *coherent* and *convex* measure.

- It can be write like an weighted average between VaR and upper CVaR.

$$CVaR = \lambda\, VaR + (1-\lambda)\, CVaR^{+}, \qquad 0 \leq \lambda \leq 1$$

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VI. Conditional Value – at – Risk

Expected Shortfall Test proposed by Mc Neil and Frey (2000)

- We have:
- Residuals: $R_{t+1} = \dfrac{X_{t+1} - S_q^t}{\sigma_{t+1}}$
- Exeedance residuals: $r_{t+1} = \dfrac{x_{t+1} - \hat{S}_q^t}{\hat{\sigma}_{t+1}}$,

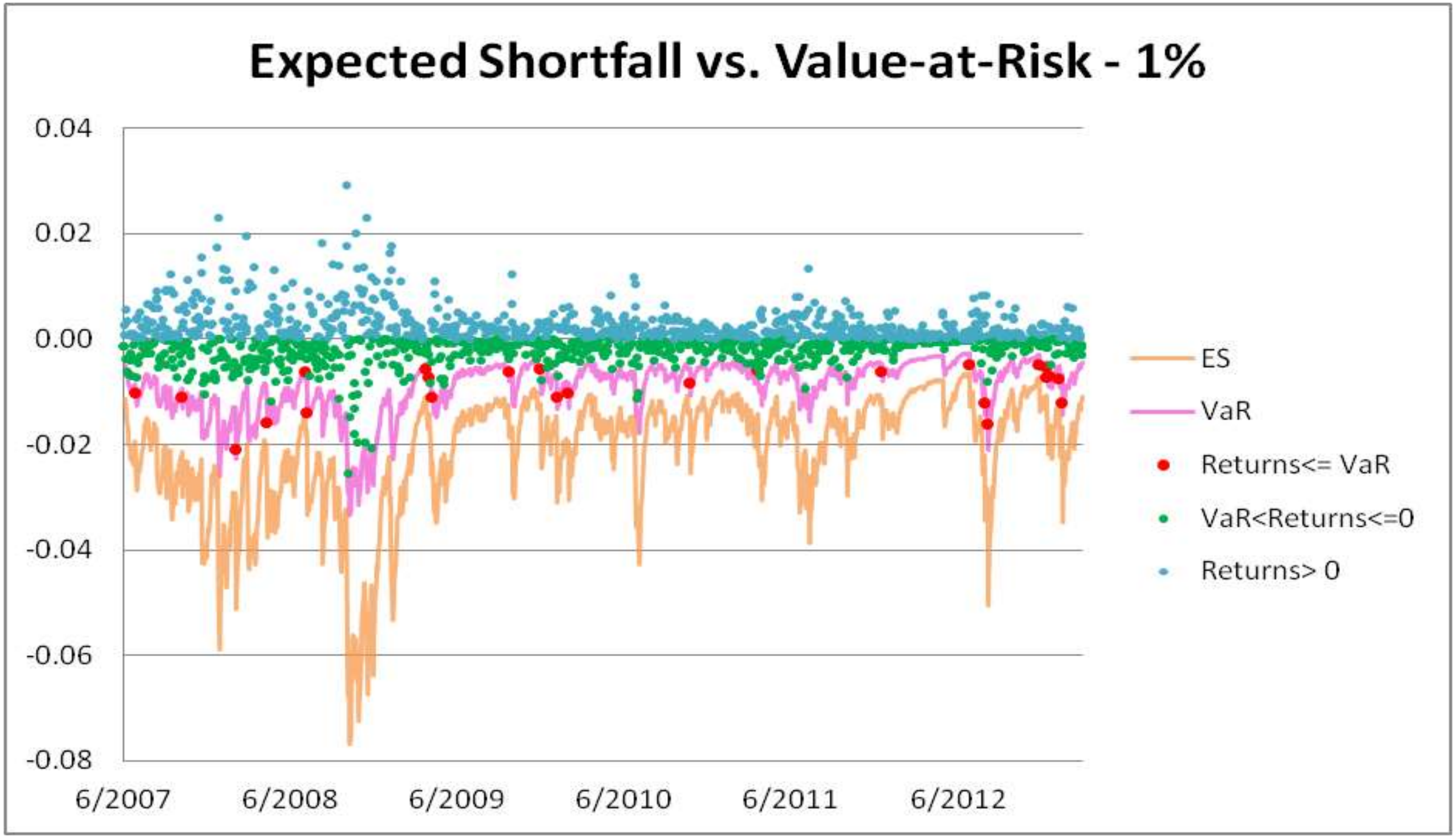
•H0: correctly estimate the dynamics of the process (μ_{t+1} and σ_{t+1}) and these residuals should be i.i.d. with mean 0.

VI. Conditional Value – at – Risk - Results

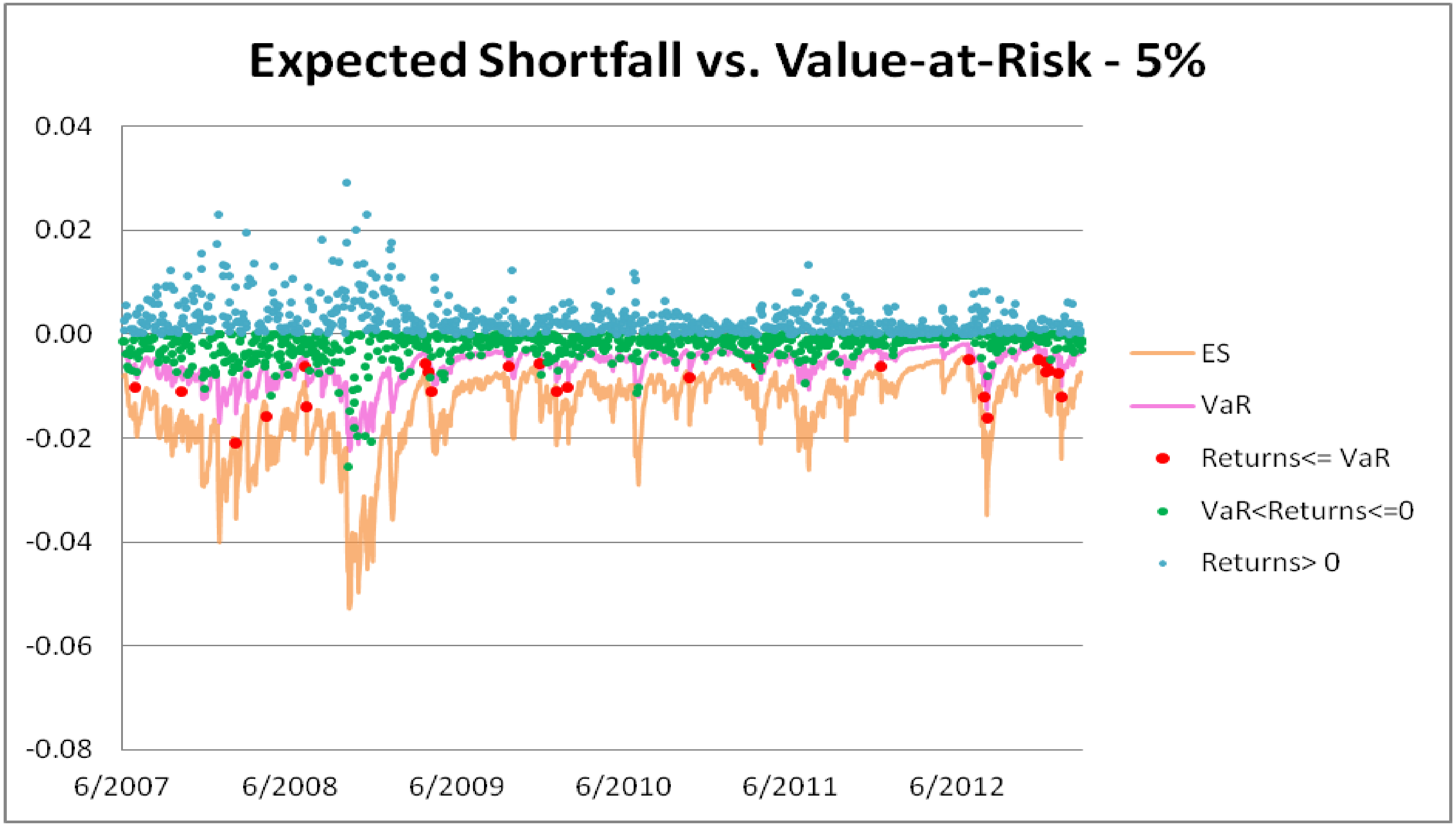
- The ES for NIG-GARCH(1,1) for the percentiles of 1% and 5% are ***not underestimated***.

Expected Shortfall Test of McNeil and Frey	5%	1%
Expected exceed	75	15
Actual Exceed	75	25
H0	Mean of Excess Violations of VaR is zero.	Mean of Excess Violations of VaR is zero.
p-value	0.221	0.750
Decisions	Fail to Reject H0	Fail to Reject H0

VI. Conditional Value – at – Risk - Results



VI. Conditional Value – at – Risk - Results



VII. Conclusions

- Normal distribution is not appropriate for modeling time series volatility.

- My conclusion underline the idea already presented by Wilhelmsson (2006) and I proved that phenomenon of '*fat tails' has a higher impact on the exchange rate EUR/RON than the skewness* of the financial returns. This finding can help investors in doing more accurate previsions regarding the evolution of EUR/RON returns.

- The backtest was based on MSE, MAE/ the informational criteria /and tests based on **VaR violations** (*unconditional coverage test, conditional coverage test, tail test and duration test*). The general conclusion was that the **NIG - GARCH (1, 1) performed better on financial series**.

- ES represents a *coherent measure* of the risk and we proved that ES for EUR/RON returns is not underestimated.

VIII. References

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Abbreviations

APARCH *– Asymmetric Power Autoregressive Conditional Heteroskedastic model*
BT(1%/5%) *is the p-value of the Tail test for the quartiles of 1% respectively 5%.*
CVaR *– Conditional Value-at-Risk*
ES *– Expected Shortfall*
GARCH - *Generalized Autoregressive Conditional Heteroskedastic model*
MSE *– Mean Square Errors*
MAE *– Mean Absolute Errors*
NIG- *Normal Inverse Gaussian distribution*
SSTD *– Skew Student's t distribution*
VaR *– Value- at Risk*
VaR.CC(1%/5%) - *p-value of the Conditional Coverage test for the quartiles of 1% respectively 5%.*
VaR.Dur(1%/5%) - *p-value of the Duration test for the quartiles of 1% respectively 5%.*
VaR.uC(1%/5%) -*p-value of the Unconditional Coverage test for the quartiles of 1% respectively 5%.*

THANK YOU!