The Academy of Economic Studies
The Faculty of Finance, Insurance, Banking and Stock Exchange
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Dynamics of Exchange Rates Using Inhomogenous Tick-by-tick Data. The Case of the EURRON Currency Pair.

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Contents

Motivation

- As financial markets continue to deepen and grow and significant advances in technology allow trading venues to conduct transactions with ever smaller latencies, insights from market microstructure will become more important
- The highest sampling frequency is tick-by-tick data; recent literature on models based on asymmetric information theory suggests that time has a role in the dissemination of information among inhomogeneous participants to trading
- Standard econometric techniques applied to high frequency data are sensitive to interpolation techniques
- Implications for institutional investors' trading book and risk management

Brief Literature Survey

- O'Hara (1995) note that "adjustment of prices to information depends on time"
- Engle and Russell (1998) proposes the Autoregressive Conditional Duration (ACD) model to account for duration clustering, standardized durations follow an exponential distribution; Engle (2000) extends the model to the Weibull distribution; Lunde (1999) proposes the Gamma distribution
- Jasiak (1998) propose the Fractionally Integrated ACD (FIACD) model
- Engle (2000) extend the models to the price adjustment process by using an ACD-GARCH framework

- Joint density and log-likelihood for $\{(x_i, z_i), i = 1, \dots, T\}$

$$f\left(x_i, z_i | \breve{x}_{i-1}, \breve{z}_{i-1}; \theta_f\right) = g(x_i | \breve{x}_{i-1}, \breve{z}_{i-1}; \theta_x) q(z_i | x_i, \breve{x}_{i-1}, \breve{z}_{i-1}; \theta_z)$$

$$\mathcal{L}(\theta_x, \theta_z | \mathcal{F}_T^{x,z}) = \sum_{i=1}^{T} [\log g(x_i | \breve{x}_{i-1}, \breve{z}_{i-1}; \theta_x) + \log q(z_i | x_i, \breve{x}_{i-1}, \breve{z}_{i-1}; \theta_z)]$$

- Durations are considered weakly exogenous

- Intraday seasonal component

$$s\left(\theta_s, m(t_i)\right) = \mathrm{e}^{h\left(\theta_s, m(t_i)\right)}$$

$$x_i = \tilde{x}_i s\left(\theta_s, m(t_i)\right)$$

$$\log x_i = \log \tilde{x}_i + h\left(\theta_s, m(t_i)\right)$$

- EACD(p,q) model proposed by Engle and Russell (1998)

$$\Psi_i = \omega + \alpha(L)X_i + \beta(L)\Psi_i$$

- FIACD(p,d,q) model proposed by Jasiak (1998)

$$[1-\beta(L)]\Psi_i = \omega + [1-\beta(L)-[1-\phi(L)](1-L)^d]X_i$$

- ACD-GARCH models proposed by Engle (2000)
  - Variance of the transaction versus variance per second $h_i = x_i\sigma_i^2$
  - ARMA(1,1)-GARCH(1,1) specification for $r_i = \tilde{r}_i x_i^{-1/2}$

$$r_i = \rho r_{i-1} + u_i + \phi u_{i-1}$$

$$u_i = \sigma_i z_i \text{ with } z_i \text{ i.i.d} \sim t_\nu(0,1)$$

$$\sigma_i^2 = \omega + \alpha u_{i-1}^2 + \beta\sigma_{i-1}^2$$

  - Conditional variance with external regressor

$$\sigma_i^2 = \omega + \alpha u_{i-1}^2 + \beta\sigma_{i-1}^2 + \gamma x_i^{-1}$$

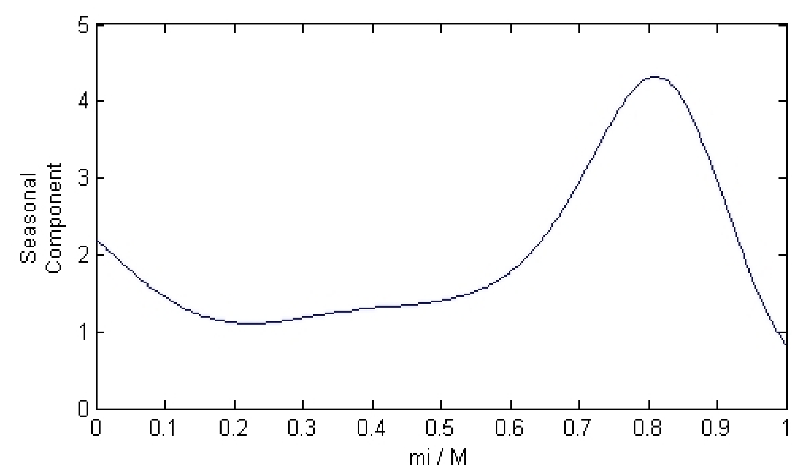
Data and duration seasonality

- EURRON price data provided by Olsen&Associates (timestamp, bid and ask quotes, institution, Olsen filter)
  - 3-Mar-2008 to 31-Mar-2011; 159 trading days after filtering; 360 average trades/day
- Duration seasonality estimated with a Fourier Flexible Form

$$h(\theta_s, m(t_i)) = \mu_1 \frac{m(t_i)}{M} + \mu_2 \left(\frac{m(t_i)}{M}\right)^2 + \sum_{p=1}^{P} \left(\delta_{c,p} \cos\left(2p\pi \frac{m(t_i)}{M}\right) + \delta_{s,p} \sin\left(2p\pi \frac{m(t_i)}{M}\right)\right)$$

Variable	Coefficient		Std. Error	t-Statistic
μ0	3.31	***	0.14	23.55
μ1	5.48	***	0.69	7.91
μ2	-6.46	***	0.67	-9.59
δ cos 01	0.79	***	0.06	14.08
δ sin 01	-0.85	***	0.04	-19.87
δ sin 02	-0.27	***	0.02	-15.12

R-squared	0.0219
Adjusted R-squared	0.0218
Akaike info criterion	3.8056
Schwarz criterion	3.8063
Durbin-Watson stat	1.2639



Long memory in durations - FIACD

I(0) and I(1) tests

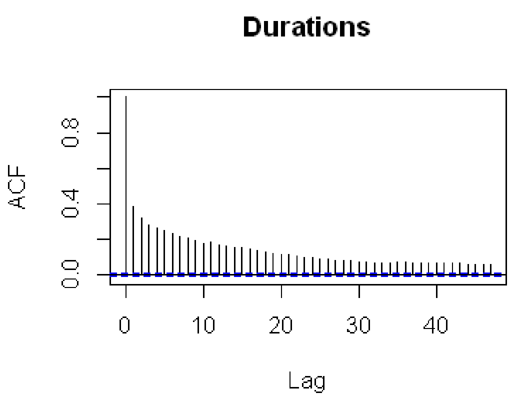
Test	Statistic	Critical Value (0.01)
Durations	-	-
KPSS		
Levels	7.324***	0.739
Trend	0.835***	0.216
ADF test	-28.114***	-4.040
PP test	-206.719***	-4.040
Fractionally Differenced Durations		
KPSS		
Levels	0.657**	0.739
Trend	0.092*	0.216

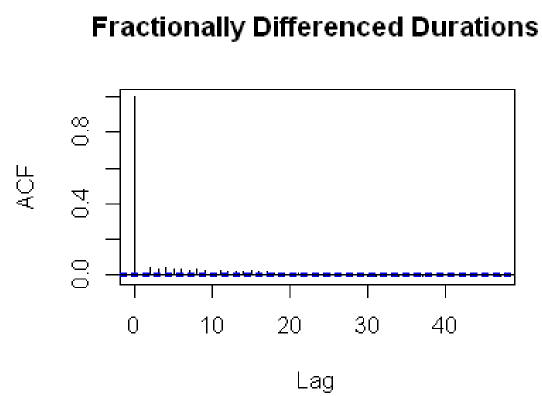
GPH estimator

Test	Statistic	Asymptotic SD	Standard Error Deviation
GPH *d*	0.2384	0.0434	0.0492

Ljung-Box Statistics

	Durations	FD Durations
Q(1)	8314.43***	1.64
Q(5)	26375.34***	313.88***
Q(20)	51365.94***	631.21***





ACD Models (1)

Parameter estimates of ACD models

	EACD(1,1)		WACD(1,1)		GACD(1,1)		
$\hat{\omega}_x$	**0.035**		**0.003**		**0.031**		*Est.*
	(0.023)		(0.040)		(0.028)		*Std. Err.*
	1.519		0.077		1.110		*t-stat.*
$\hat{\alpha}_x$	**0.041**	***	**0.055**	***	**0.041**	***	*Est.*
	(0.003)		(0.004)		(0.003)		*Std. Err.*
	16.178		13.953		13.261		*t-stat.*
$\hat{\beta}_x$	**0.907**	***	**0.880**	***	**0.907**	***	*Est.*
	(0.006)		(0.010)		(0.008)		*Std. Err.*
	139.716		91.492		114.740		*t-stat.*
$\hat{\gamma}_x$			**0.757**	***			*Est.*
			(0.002)				*Std. Err.*
			321.287				*t-stat.*
$\hat{\kappa}_x$					**0.679**	***	*Est.*
					(0.003)		*Std. Err.*
					198.692		*t-stat.*

- The Constrained Maximum Likelihood estimation procedure adopted converges for all three models considered, and the parameter estimates for α and ß are significant and add up to nearly one, however we would expect a sum much closer to one for a large sample if the long memory process had not been accounted for through FIACD

ACD Models (2)

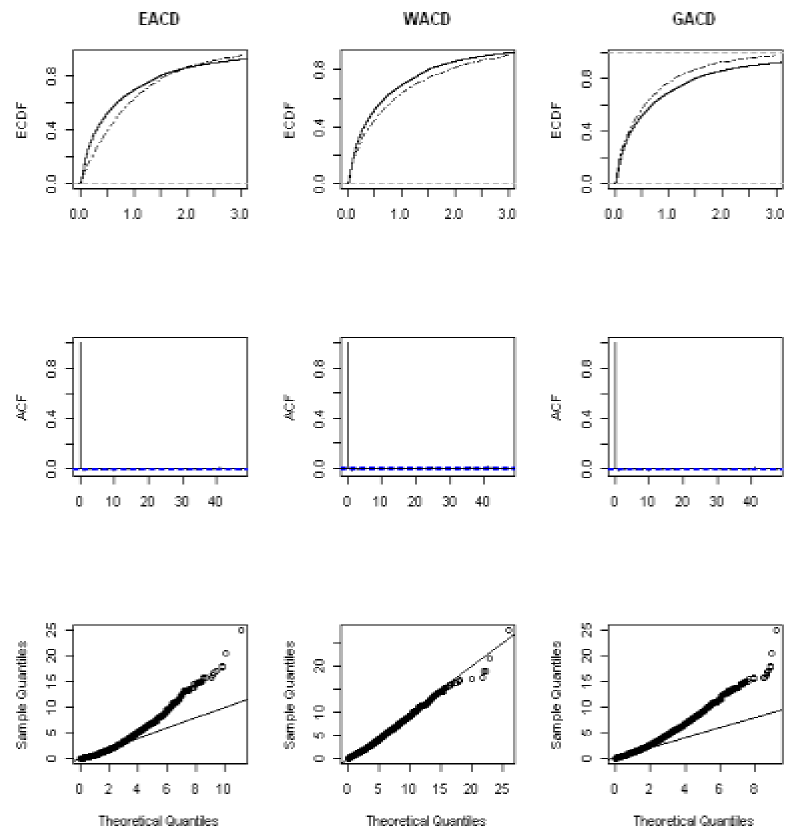
Diagnostic tests

Statistic	EACD(1,1)		WACD(1,1)		GACD(1,1)	
Q(1)	11.8916	***	24.4393	***	11.8916	***
Q(10)	24.2777	***	50.8226	***	24.2777	***
Log-Likelihood	-277985.10		-273313.6		-274663.1	
LR			9343	***	6644	***
KS	0.01426	***	0.0722	***	0.0813	***

- Q-Statistics still show significant serial correlation in standardized durations, however the values have decreased sharply
- Likelihood Ratio tests imply that both Weibull and Gamma distributions are a better fit than the exponential distribution
- Kolmogorov-Smirnov tests suggest that the specification can be improved by more flexible distributional assumptions

ACD Models (3)

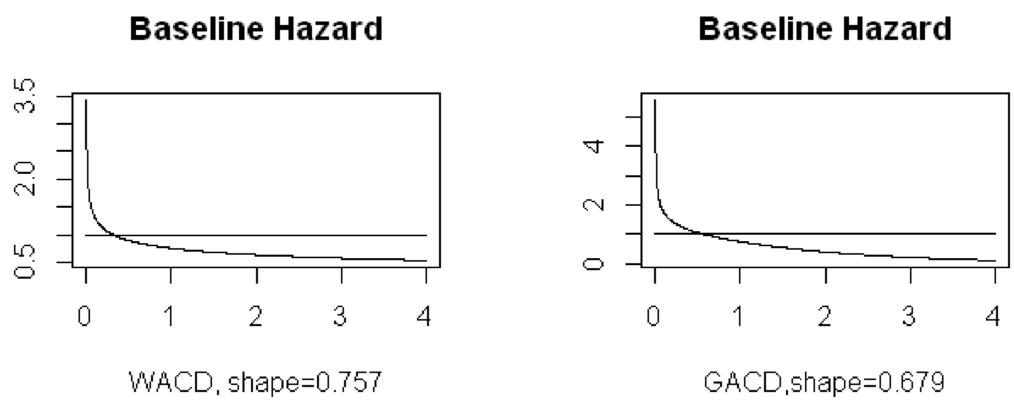
Empirical CDF versus theoretical CDF, ACF plots, QQ plots



- The models considered fit the data reasonably well, except for the tail behavior, however large deviations from theoretical quantiles represent less than 2.5% of the sample

ACD Models (4)

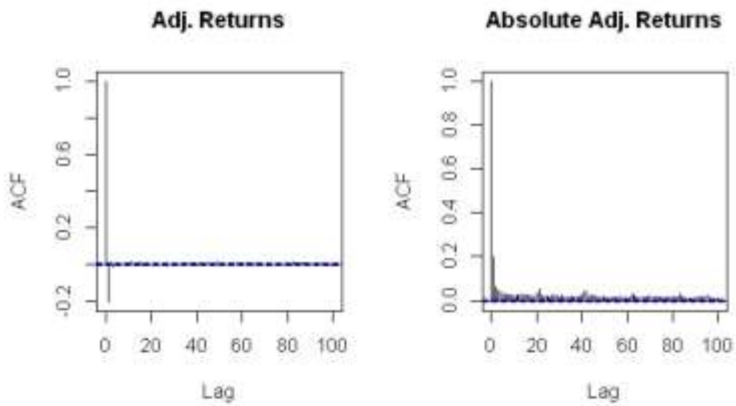
- Baseline hazard function for the Weibull and Standard Gamma distributions versus the constant exponential hazard



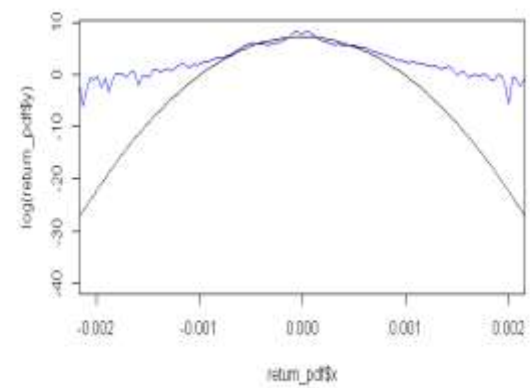
- The estimated parameters of the Standard Weibull and Standard Gamma distributions are robust to starting values for maximum likelihood estimation and imply a strong rejection of the constant hazard, but also of the non-monotonic hazard function – “early failures”

ACD-GARCH Framework (1)

ACF plots for adjusted returns and absolute adjusted returns



Log-PDF of returns against N(0,1)



- ACF of returns suggests MA(1), standatd Student t-distribution can be used to model heavy tails
- Engle (2000) refers to the model as Ultra-High Frequency (UHF)-GARCH

ACD-GARCH Framework (2)

GARCH(1,1) model parameter estimates

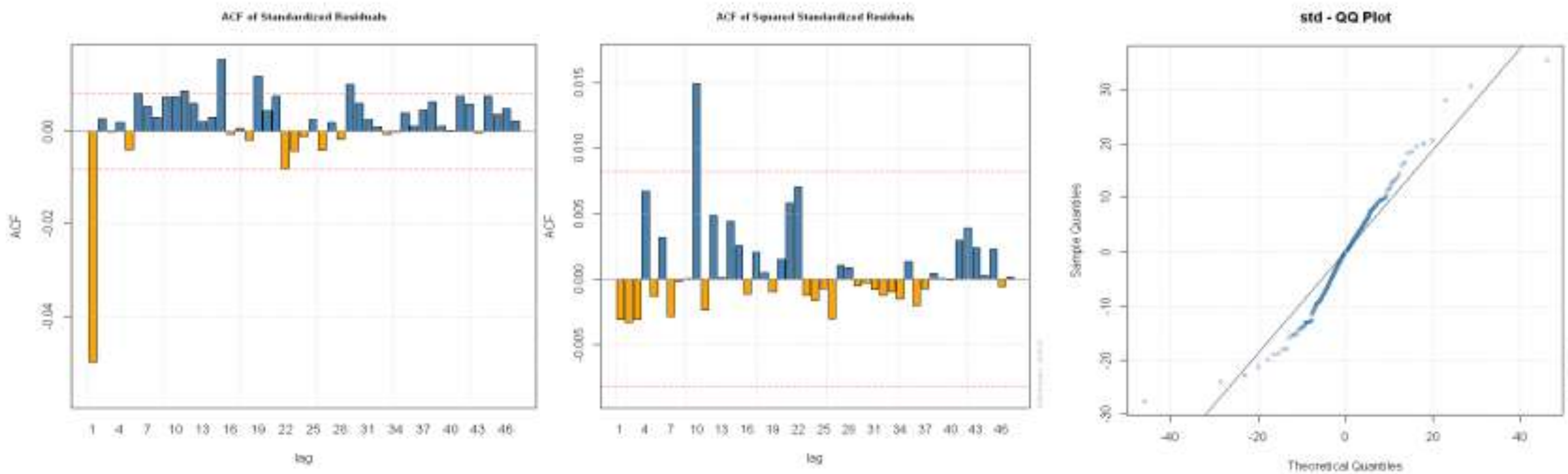
	Estimate	Std. Err.	t Stat.	p-value	Robust Std. Err.	Robust t Stat.	Robust p-value
$\hat{\mu}_r$	0.00007*	0.00003	2.09	0.036	0.00004	1.93	0.053
MA(1)	-0.20428***	0.00378	-54.04	0.000	0.00543	-37.62	0.000
$\hat{\omega}_r$	0.00034***	0.00001	28.21	0.000	0.00002	17.02	0.000
$\hat{\alpha}_r$	0.92992***	0.03018	30.81	0.000	0.03813	24.39	0.000
$\hat{\beta}_r$	0.06908***	0.01053	6.56	0.000	0.02424	2.85	0.004
$\hat{\nu}_r$	2.31107***	0.01144	202.09	0.000	0.00952	242.88	0.000

- In the mean equation, we observe the significant "bid-ask bounce" effect
- The mean equation constant is not significant at 1%, as in many studies such as Engle (2000), Liu (2010), etc.
- We observe the "IGARCH effect" in the conditional duration equation
- The shape parameter estimate confirms the pronounced heavy tail distribution of returns, also suggests that inference should be made considering robust statistics; improvements can be made by using a mixture of distributions
- The following is true:

$$E\left(r_i^2 \middle| \mathcal{F}_{i-1}\right) = E\left(h_i \middle| \mathcal{F}_{i-1}\right) = E\left(x_i \sigma_i^2 \middle| \mathcal{F}_{i-1}\right) = \Psi_i \sigma_i^2$$

ACD-GARCH Framework (3)

GARCH(1,1) model diagnostic plots



- ACF of residuals and squared residuals show that the model is potentially misspecified (as confirmed by portmanteau tests), however the correlations are relatively low
- The Sudent t-distribution manages to account relatively well for the heavy tail behavior

ACD-GARCH Framework (4)

IGARCH(1,1) model parameter estimates

	Estimate	Std. Err.	t Stat.	p-value	Robust Std. Err.	Robust t Stat.	Robust p-value
$\hat{\mu}_r$	0.00007**	0.00003	2.09	0.036	0.00003	2.23	0.026
MA(1)	-0.20429***	0.00384	-53.24	0.000	0.00421	-48.58	0.000
$\hat{\omega}_r$	0.00034***	0.00001	29.26	0.000	0.00001	23.46	0.000
$\hat{\alpha}_r$	0.93089***	0.00777	119.87	0.000	0.01049	88.72	0.000
$\hat{\beta}_r = 1 - \hat{\alpha}_r$	0.06911***	-	-	-	-	-	-
$\hat{v}_r$	2.31089***	0.00955	241.97	0.000	0.01029	224.70	0.000

- The results of the IGARCH(1,1) specification have a similar interpretation to the ones of the standard GARCH(1,1) model
- IGARCH effect becomes an important issue

ACD-GARCH Framework (5)

- Under IGARCH, the conditional variance is analogous to a random walk process with drift
- Nelson(1990) however points out that if $E(\log(\beta + \alpha z_i^2)) < 0$ then the unconditional variance process is strictly stationary and ergodic, but not weakly stationary; the estimated parameters of the IGARCH(1,1) model satisfy this condition (the mean is –0.299) and $\omega > 0$
- Under these conditions, the unconditional variance is finite almost surely though increasing
- A shock to the square of the noise process is persistent in probability in the unconditional and conditional variance, but not almost surely

ACD-GARCH Framework (6)

IGARCH(1,1) model with external regressor parameter estimates

$$\sigma_i^2 = \omega + \alpha u_{i-1}^2 + \beta \sigma_{i-1}^2 + \gamma x_i^{-1}$$

	Estimate	Std. Err.	t Stat.	p-value	Robust Std. Err.	Robust t Stat.	Robust p-value
$\hat{\mu}_r$	0.00013***	0.00003	5.14640	0.00000	0.00003	5.27540	0.00000
MA(1)	-0.16454***	0.00315	-52.29840	0.00000	0.00372	-44.29360	0.00000
$\hat{\omega}_r$	0.00008***	0.00001	15.72660	0.00000	0.00001	14.64090	0.00000
$\hat{\alpha}_r$	0.93405***	0.00215	434.84404	0.00000	0.00290	344.59210	0.00000
$\hat{\beta}_r = 1 - \hat{\alpha}_r$	0.06596***	-	-	-	-	-	-
$\hat{\gamma}_r$	0.01036***	0.00043	24.12380	0.00000	0.00047	22.04890	0.00000
$\hat{\nu}_r$	2.14554***	0.00576	372.33460	0.00000	0.00613	349.81730	0.00000

- Current durations are informative; the reciprocal of the actual durations process enteres the conditional volatility equation
- The positive estimate of γ is consistent with the model of inhomogeneous agents of Easley and O'Hara (1992) – "no trade means no news"
- Duration seasonality can be interpreted as also accounting for an intraday pattern of volatility

ACD-GARCH Framework (6)

Diagnostic tests

GARCH(1,1)	IGARCH(1.1)	IGARCH(1.1) with ext. reg
LogLikelihood : 157166.8	LogLikelihood : 157167.4	LogLikelihood : 165518.6
Information Criteria	Information Criteria	Information Criteria
------------------------------------	------------------------------------	------------------------------------
Akaike -5.5129	Akaike -5.5129	Akaike -5.8058
Bayes -5.5119	Bayes -5.5119	Bayes -5.8047
Shibata -5.5129	Shibata -5.5129	Shibata -5.8058
Hannan-Quinn -5.5126	Hannan-Quinn -5.5126	Hannan-Quinn -5.8055
		LR test 16702.4***

- Including external regressors improves the specification considering information criterion and the likelihod ratio test

Conclusion

- We find evidence that the joint process of durations and prices for the EURRON currency pair has a complex structure which has implications for forecasting and risk management applications at the intraday level
- Considering durations weakly exogenous, the nonparametric "duration clustering" property is explained by long memory and decreasing conditional intensity functions of standardized durations
- Durations between transactions exhibit an intraday seasonal pattern
- Duration-adjusted returns exhibit the familiar "heavy tails" and "duration clustering" properties, as well as microstructure-specific effects such as "bid-ask bounce" and persistence of shocks to the noise process of the IGARCH(1,1) specification in the forecast distribution of long term and conditional variance
- We find evidence that the data is consistent with the model of inhomogeneous agents

Further Improvements

- The models considered are sensitive to structural change, which is a promising area for further research
- Duration seasonality can be improved by modeling interday seasonality
- More flexible distributions for standardized durations
- Simultaneous estimation of more sophisticated models where durations are endogeneous; Granger causality tests

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