A state-space model for estimating Romania’s output gap and natural rate of unemployment

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Motivation

- Estimate cyclical position of the economy; important for:
  - measuring future inflationary pressures;
  - evaluating fiscal policy stance;
  - assessing structural reforms.
- Determine dominant approach for unemployment rate dynamics.
- Evaluate the effects of economic crisis.
- Assess performance of multivariate filter over univariate statistical filters.
Literature review (1)

Four groups of quantitative methods (Šrámková et al. [2010]):

- Univariate statistical filters:
  - Hodrick-Prescott (HP);
  - band-pass: Baxter-King (BK) and Christiano-Fitzgerald (CF);
  - Beveridge-Nelson (BN) decomposition.

- Production function approach

- Multivariate semi-structural methods:
  - multivariate HP and BN;
  - Blanchard-Quah structural VAR;
  - state-space models and Kalman filter.

- Direct measures and surveys
Natural Rate of Unemployment (NRU) vs. Non-Accelerating Inflation Rate of Unemployment (NAIRU) (Estrella and Mishkin [1999], Batini and Greenslade [2006]):

- NRU captures the long-run equilibrium determined by the structural characteristics of labour and goods markets;
- NAIRU is defined solely in relation to the level of unemployment that is consistent with a stable rate of inflation;
- The most intuitive distinction is to think of time horizon to which they refer;
- As the effects of shocks fade away, the NAIRU tends towards the NRU.
Two theories of equilibrium unemployment rate:

- **Structuralist approach (Phelps [1968]):** equilibrium rate of unemployment depends on labour market characteristics and other fundamental macroeconomic variables:
  - productivity trends (Brüh et al. [2011]),
  - replacement ratio, union density (Cassino and Thornton [2002]),
  - real interest rates (Logeay and Tober [2003]);

- **Persistence approach (Blanchard and Summers [1986]):** temporary shocks have very persistent effects on the equilibrium unemployment rate
  - hysteresis as an extreme case
Phillips curve (1)

- **price and wage settings** (Layard et al. [2005] imperfect competition model):
  \[
  p_t - w_t^e = \beta_0 - \beta_1 u_t - \beta_{11} \Delta u_t - \beta_2 (p_t - p_t^e) - q_t + z_p
  \]

  \[
  w_t - p_t^e = \alpha_0 - \alpha_1 u_t - \alpha_{11} \Delta u_t - \alpha_2 (w_t - w_t^e) + q_t + z_w
  \]

- **NRU** (fulfilled expectations):
  \[
  \bar{u}_t = \frac{\beta_0 + \alpha_0 + z_p + z_w}{\beta_1 + \alpha_1}
  \]

- **assumption 1**: equal price and wage surprises:
  \[
  p_t - p_t^e = w_t - w_t^e
  \]

- **assumption 2**: inflation rate follows a random walk:
  \[
  \pi_t = \pi_{t-1} + \nu_t \iff E_{t-1} \pi_t = \pi_{t-1} \iff p_t - p_t^e = \Delta \pi_t
  \]
Phillips curve (2)

- **Phillips curve:**

\[
\Delta \pi_t = -\theta_1 (u_t - \bar{u}_t) - \theta_{11} \Delta u_t
\]  \hspace{1cm} (2)

where \( \theta_1 = \frac{\beta_1 + \alpha_1}{2 + \beta_2 + \alpha_2} > 0 \) and \( \theta_{11} = \frac{\beta_{11} + \alpha_{11}}{2 + \beta_2 + \alpha_2} > 0 \).

- **unemployment rate dynamics:**

\[
u_t = \frac{\theta_1}{\theta_1 + \theta_{11}} \bar{u}_t + \frac{\theta_{11}}{\theta_1 + \theta_{11}} u_{t-1} - \frac{1}{\theta_1 + \theta_{11}} \Delta \pi_t.
\]  \hspace{1cm} (3)

- **stable inflation** \((\Delta \pi_t = 0):\)

\[
u_t = k \bar{u}_t + (1 - k) u_{t-1},
\]  \hspace{1cm} (4)

where \( k = \frac{\theta_1}{(\theta_1 + \theta_{11})}, 0 < k < 1 \).
Okun’s law and aggregate demand

- **Okun’s law:**

\[
y_t - \bar{y}_t = -\omega (u_t - \bar{u}_t)
\]  

(5)

- **Aggregate demand** (Berger and Everaert [2008]):

\[
y_t = \frac{1}{\lambda} (m_t - p_t) + \frac{1}{\mu} x_t + \gamma s_t
\]

- Add and subtract \(\bar{y}_t\), take first differences:

\[
\Delta y_t = \Delta \bar{y}_t - \frac{1}{\lambda} (\pi_t - \bar{\pi}_t) + \gamma \Delta s_t
\]

(6)

where \(\bar{\pi}_t = \Delta m_t + (\lambda/\mu) \Delta x_t - \lambda \Delta \bar{y}_t\)
\( \Delta \pi_t = -\theta_1(u_t - \bar{u}_t) - \theta_{11} \Delta u_t + \epsilon_t^\pi \) \hspace{1cm} (7a)

\( y_t - \bar{y}_t = -\omega(u_t - \bar{u}_t) + \epsilon_t^u \) \hspace{1cm} (7b)

\( \Delta y_t = \Delta \bar{y}_t - \frac{1}{\lambda}(\pi_t - \bar{\pi}_t) + \gamma \Delta s_t + \epsilon_t^y \) \hspace{1cm} (7c)
Transition equations

\[ \bar{y}_t = \bar{y}_{t-1} + \psi_{t-1} + \eta^1_t \]  \hfill (8a)
\[ \psi_t = \psi_{t-1} + \eta^2_t \]  \hfill (8b)
\[ \bar{u}_t = (1 + \delta)\bar{u}_{t-1} - \delta \bar{u}_{t-2} + \eta^3_t \]  \hfill (8c)
\[ \bar{\pi}_t = \bar{\pi}_{t-1} + \eta^4_t \]  \hfill (8d)
Adaptive RWMH and Geweke’s CD

- **posterior mode estimate:**
  \[
  \arg \max_{\theta} \log p(\theta | y) = \log p(y | \theta) + \log p(\theta) - \log p(y)
  \]

- **Adaptive RWMH:**
  \[
  \theta^* = \theta^{s-1} + z^s, \quad z^s \sim N(0, \Sigma^s)
  \]

  \[
  \alpha(\theta^{s-1}, \theta^*) = \min \left[ \frac{p(\theta = \theta^* | y)}{p(\theta = \theta^{s-1} | y)}, 1 \right] \rightarrow 23.4\%
  \]

- **Geweke’s convergence diagnostic:**
  \[
  CD = \frac{\widehat{E}[\theta_1] - \widehat{E}[\theta_2]}{\hat{\sigma}_1 / \sqrt{N_1} + \hat{\sigma}_2 / \sqrt{N_2}} \rightarrow N(0, 1)
  \]
Kalman filter

- **state-space form:**

  \[
  y_t = Z\alpha_t + Ax_t + \varepsilon_t, \varepsilon_t \sim N(0, H) \quad (9a)
  \]

  \[
  \alpha_t = T\alpha_{t-1} + \eta_t, \eta_t \sim N(0, Q) \quad (9b)
  \]

- **Kalman filter:** estimate the distribution of \(\alpha_t\), for \(t = 1, \ldots, T\), conditional on \(Y_t = \{y_1, \ldots, y_t\}\) and \(X_t = \{x_1, \ldots, x_t\}\);

- **Kalman smoother:** estimate the distribution of \(\alpha_t\), for \(t = 1, \ldots, T\), conditional on \(Y_T = \{y_1, \ldots, y_T\}\) and \(X_T = \{x_1, \ldots, x_T\}\).
Data and transformations

- $y_t$: quarterly seasonally adjusted real GDP (NIS), in log;
- $\pi_t$: q-o-q inflation rate, compounded from quarterly CPI as average of the corresponding monthly CPI (NIS), seasonally adjusted, in log and first difference;
- $u_t$: quarterly seasonally adjusted unemployment rate, ILO definition (Eurostat);
- $\Delta s_t$: quarterly real modification of RON against EUR, obtained from average daily nominal exchange rate (NBR), quarterly CPI defined above and a similar HICP (Eurostat); $\Delta s_t > 0$ indicates a real depreciation.

Data covers 2000Q1:2011Q4 period
<table>
<thead>
<tr>
<th>Coeff. Distribution</th>
<th>Prior mean</th>
<th>Prior st.dev</th>
<th>Prior mode</th>
<th>Posterior mean</th>
<th>Posterior st.dev</th>
<th>95% BCI</th>
<th>Geweke’s CD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_1$</td>
<td>Gamma</td>
<td>0.5</td>
<td>0.15</td>
<td>1.082</td>
<td>0.104</td>
<td>0.70 — 1.67</td>
<td>-0.26*</td>
</tr>
<tr>
<td>$\theta_{11}$</td>
<td>Normal</td>
<td>0.5</td>
<td>0.5</td>
<td>1.725</td>
<td>0.075</td>
<td>1.54 — 2.53</td>
<td>-2.29***</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Normal</td>
<td>2.5</td>
<td>0.15</td>
<td>2.015</td>
<td>0.006</td>
<td>1.61 — 2.07</td>
<td>0.53*</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Normal</td>
<td>0.5</td>
<td>0.2</td>
<td>1.000</td>
<td>0.002</td>
<td>0.82 — 1.13</td>
<td>-1.71**</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Normal</td>
<td>1</td>
<td>0.4</td>
<td>0.832</td>
<td>0.108</td>
<td>0.16 — 1.33</td>
<td>-0.80*</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Beta</td>
<td>0.925</td>
<td>0.05</td>
<td>0.937</td>
<td>0.041</td>
<td>0.81 — 0.98</td>
<td>-0.33*</td>
</tr>
<tr>
<td>$\sigma(\varepsilon_y)$</td>
<td>InvGamma</td>
<td>0.4</td>
<td>0.04</td>
<td>0.427</td>
<td>0.033</td>
<td>0.36 — 0.50</td>
<td>0.17*</td>
</tr>
<tr>
<td>$\sigma(\varepsilon_u)$</td>
<td>InvGamma</td>
<td>0.1</td>
<td>0.009</td>
<td>0.224</td>
<td>0.025</td>
<td>0.20 — 0.26</td>
<td>-0.21*</td>
</tr>
<tr>
<td>$\sigma(\varepsilon_\pi)$</td>
<td>InvGamma</td>
<td>0.5</td>
<td>0.04</td>
<td>0.649</td>
<td>0.048</td>
<td>0.57 — 0.74</td>
<td>-0.20*</td>
</tr>
<tr>
<td>$\sigma(\eta^1)$</td>
<td>InvGamma</td>
<td>0.008</td>
<td>0.004</td>
<td>0.006</td>
<td>0.002</td>
<td>0.003 — 0.020</td>
<td>0.03*</td>
</tr>
<tr>
<td>$\sigma(\eta^2)$</td>
<td>InvGamma</td>
<td>0.013</td>
<td>0.004</td>
<td>0.013</td>
<td>0.004</td>
<td>0.009 — 0.029</td>
<td>-0.04*</td>
</tr>
<tr>
<td>$\sigma(\eta^3)$</td>
<td>InvGamma</td>
<td>0.02</td>
<td>0.003</td>
<td>0.022</td>
<td>0.003</td>
<td>0.016 — 0.030</td>
<td>0.10*</td>
</tr>
<tr>
<td>$\sigma(\eta^4)$</td>
<td>InvGamma</td>
<td>0.2</td>
<td>0.025</td>
<td>0.310</td>
<td>0.050</td>
<td>0.25 — 0.40</td>
<td>0.12*</td>
</tr>
</tbody>
</table>
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Prior and posterior distributions (2)
Unobserved components (2)

Output ($y$) gap decomposition

Unemployment rate ($u$) gap decomposition

Inflation rate ($\pi$) gap decomposition

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Reliability of estimated states (1)

Unemployment rate gap

Output gap

Inflation rate gap

Kalman

HP, lambda=1600

CF, Low=6, High=32
Reliability of estimated states (2)

End-point problem

Kalman
HP, lambda=1600
HP, lambda=1600, EC forecasts
CF, Low=6, High=32
CF, Low=6, High=32, EC forecasts

Output gap

NAIRU

Estimated
DG ECFIN

Estimated
DG ECFIN
Correlation coefficients between gaps and lags/leads of inflation change

Unemployment gap – quarterly inflation change

Output gap – quarterly inflation change

Unemployment gap – annual inflation change

Output gap – annual inflation change

Kalman  HP, lambda=1600  CF, Low=6, High=32
Conclusions

- Estimated current cyclical position (positive unemployment gap and strong negative output gap) indicates need for deeper structural reforms in order to:
  - boost economic activity;
  - equilibrate labour market.

- Current inflation rate seems to be at equilibrium as NBR met inflation target for 2011.

- Model captures effects of economic crisis:
  - growth rate of potential output halves, from 5.0% in 2001-2008 to 2.4% in 2009-2011;
  - NRU is still on an upward shape.

- Persistence effects are important for unemployment rate dynamics.

- Gaps are (potentially) useful in forecasting inflation.


Annex: Testing assumptions

<table>
<thead>
<tr>
<th>OLS relation</th>
<th>Net wages’ inflation = 1.86 + 0.94 · CPI inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob.</td>
<td>[0.01] [0.00]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wald test</th>
<th>Net wages’ inflation = const. + 1 · CPI inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^2$ Prob.</td>
<td>[0.73]</td>
</tr>
</tbody>
</table>

*Tab. A1. Wald test for coefficients restriction (1)*

<table>
<thead>
<tr>
<th>OLS relation</th>
<th>CPI inflation at $t$ = 0.13 + 0.91 · CPI inflation at $t-1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob.</td>
<td>[0.54] [0.00]</td>
</tr>
</tbody>
</table>

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<tr>
<th>Wald test</th>
<th>CPI inflation at $t$ = 0 + 1 · CPI inflation at $t-1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^2$ Prob.</td>
<td>[0.09]</td>
</tr>
</tbody>
</table>

*Tab. A2. Wald test for coefficients restriction (2)*

<table>
<thead>
<tr>
<th>t-Statistic</th>
<th>-2.42</th>
<th>Prob.</th>
<th>[0.14]</th>
</tr>
</thead>
</table>

*Tab. A3. Augmented Dickey-Fuller test for quarterly inflation rate*
Annex: Reduced-form measurement equations

\[
\begin{pmatrix}
y_t \\
u_t \\
\pi_t
\end{pmatrix}
= 
\begin{pmatrix}
y^*_t \\
u^*_t \\
\pi^*_t
\end{pmatrix}
+ 
\begin{pmatrix}
\omega \lambda \\
-\frac{\lambda}{\theta_{11}} \\
\omega \theta_{11} \\
-\omega \omega_{11} \\
-\omega \\
\omega \lambda \gamma \\
\frac{\alpha}{\alpha} \\
\frac{\alpha}{\alpha} \\
\frac{\alpha}{\alpha}
\end{pmatrix}
\times
\begin{pmatrix}
\frac{\alpha}{(\theta_1 + \theta_{11}) \lambda} \\
\frac{\alpha}{\theta_{11}} \\
\frac{\alpha}{\omega \theta_{11} \lambda} \\
\frac{1}{\alpha} \\
\frac{\alpha}{\alpha} \\
\frac{\alpha}{(\theta_1 + \theta_{11}) \lambda \gamma}
\end{pmatrix}
\times
\begin{pmatrix}
y_{t-1} - y^*_{t-1} \\
u_{t-1} - u^*_{t-1} \\
\pi_{t-1} - \pi^*_{t-1} \\
\Delta s_t
\end{pmatrix}
+ 
\begin{pmatrix}
y_t \\
u_t \\
\pi_t
\end{pmatrix}
\]

where \( \alpha = \theta_1 + \theta_{11} + \lambda \omega > 0 \)
Annex: Neighbourhoods around the optimum

\begin{align*}
\text{THETA1} & \times 10^5 \\
1.04, 1.06, 1.08, 1.1, 1.12 \\
\text{THETA11} & \times 10^5 \\
1.65, 1.7, 1.75, 1.8 \\
\text{OMEGA} & \times 10^5 \\
1.95, 2, 2.05, 2.1 \\
\text{LAMBDA} & \times 10^5 \\
0.96, 0.98, 1, 1.02, 1.04 \\
\text{GAMMA} & \times 10^5 \\
0.8, 0.82, 0.84, 0.86 \\
\text{DELTA} & \times 10^5 \\
0.9, 0.92, 0.94, 0.96, 0.98 \\
\text{std_eps_y} & \times 10^5 \\
0.62, 0.64, 0.66, 0.68 \\
\text{std_eps_u} & \times 10^5 \\
0.41, 0.42, 0.43, 0.44 \\
\text{std_eps_p} & \times 10^5 \\
0.215, 0.22, 0.225, 0.23 \\
\text{std_shock_1} & \times 10^5 \\
5.6, 5.8, 6 \\
\text{std_shock_2} & \times 10^5 \\
0.0125, 0.013, 0.0135 \\
\text{std_shock_3} & \times 10^5 \\
0.021, 0.0215, 0.022, 0.0225 \\
\text{std_shock_4} & \times 10^{-3} \\
0.3, 0.31, 0.32
\end{align*}
Annex: Convergence of the chains

Coefficients convergence, each 500th simulation, no burning

THETA1
mean=1.150, std=0.249
mode=1.281, median=1.138

THETA11
mean=1.987, std=0.259
mode=1.692, median=1.960

OMEGA
mean=1.835, std=0.116
mode=1.771, median=1.830

LAMBDA
mean=0.954, std=0.093
mode=0.851, median=0.924

GAMMA
mean=0.771, std=0.292
mode=0.624, median=0.780

DELTA
mean=0.913, std=0.046
mode=0.926, median=0.919

std_eps_y
mean=0.428, std=0.035
mode=0.408, median=0.425

std_eps_u
mean=0.225, std=0.016
mode=0.220, median=0.225

std_shock_1
mean=0.008, std=0.004
mode=0.005, median=0.007

std_shock_2
mean=0.016, std=0.005
mode=0.009, median=0.015

std_shock_3
mean=0.022, std=0.004
mode=0.023, median=0.022

std_shock_4
mean=0.320, std=0.039
mode=0.290, median=0.318

Acceptance rate

Geweke Z, 0.2vs0.5, 2000th
Annex: Trend components

NRU

Inflation rate trend

Potential output

Kalman
HP, lambda=1600
CF, Low=6, High=32
Actual
Annex: Alternative measures for equilibrium unemployment rate

Alternative measures for unemployment rate

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