A state-space model for estimating Romania's output gap and natural rate of unemployment

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Motivation

- Estimate cyclical position of the economy; important for:
  - measuring future inflationary pressures;
  - evaluating fiscal policy stance;
  - assessing structural reforms.
- Determine dominant approach for unemployment rate dynamics.
- Evaluate the effects of economic crisis.
- Assess performance of multivariate filter over univariate statistical filters.

Literature review (1)

Four groups of quantitative methods (Šrámková et al. [2010]):

- Univariate statistical filters:
  - Hodrick-Prescott (HP);
  - band-pass: Baxter-King (BK) and Christiano-Fitzgerald (CF);
  - Beveridge-Nelson (BN) decomposition.
- Production function approach
- Multivariate semi-structural methods:
  - multivariate HP and BN;
  - Blanchard-Quah structural VAR;
  - **state-space models and Kalman filter**.
- Direct measures and surveys

Literature review (2)

Natural Rate of Unemployment (**NRU**) vs. Non-Accelerating Inflation Rate of Unemployment (**NAIRU**) (Estrella and Mishkin [1999], Batini and Greenslade [2006]):

- NRU captures the long-run equilibrium determined by the structural characteristics of labour and goods markets;
- NAIRU is defined solely in relation to the level of unemployment that is consistent with a stable rate of inflation;
- The most intuitive distinction is to think of time horizon to which they refer;
- As the effects of shocks fade away, the NAIRU tends towards the NRU.

Literature review (3)

Two theories of equilibrium unemployment rate:

- **Structuralist approach (Phelps [1968])**: equilibrium rate of unemployment depends on labour market characteristics and other fundamental macroeconomic variables:
  - productivity trends (Brüha et al. [2011]),
  - replacement ratio, union density (Cassino and Thornton [2002]),
  - real interest rates (Logeay and Tober [2003]);
- **Persistence approach (Blanchard and Summers [1986])**: temporary shocks have very persistent effects on the equilibrium unemployment rate
  - hysteresis as an extreme case

Phillips curve (1)

- **price and wage settings** (Layard et al. [2005] imperfect competition model):

$$p_t - w_t^e = \beta_0 - \beta_1 u_t - \beta_{11}\Delta u_t - \beta_2(p_t - p_t^e) - q_t + z_p$$
$$w_t - p_t^e = \alpha_0 - \alpha_1 u_t - \alpha_{11}\Delta u_t - \alpha_2(w_t - w_t^e) + q_t + z_w$$

- **NRU** (fulfilled expectations):

$$\bar{u}_t = \frac{\beta_0 + \alpha_0 + z_p + z_w}{\beta_1 + \alpha_1} \quad (1)$$

- **assumption 1**: equal price and wage surprises:

$$p_t - p_t^e = w_t - w_t^e$$

- **assumption 2**: inflation rate follows a random walk:

$$\pi_t = \pi_{t-1} + v_t \Leftrightarrow E_{t-1}\pi_t = \pi_{t-1} \Leftrightarrow p_t - p_t^e = \Delta\pi_t$$

Phillips curve (2)

- **Phillips curve**:

$$\boxed{\Delta\pi_t = -\theta_1(u_t - \bar{u}_t) - \theta_{11}\Delta u_t} \tag{2}$$

where $\theta_1 = \frac{\beta_1+\alpha_1}{2+\beta_2+\alpha_2} > 0$ and $\theta_{11} = \frac{\beta_{11}+\alpha_{11}}{2+\beta_2+\alpha_2} > 0$.

- **unemploymment rate dynamics**:

$$u_t = \frac{\theta_1}{\theta_1+\theta_{11}}\bar{u}_t + \frac{\theta_{11}}{\theta_1+\theta_{11}}u_{t-1} - \frac{1}{\theta_1+\theta_{11}}\Delta\pi_t. \tag{3}$$

- **stable inflation** ($\Delta\pi_t = 0$):

$$u_t = k\bar{u}_t + (1-k)u_{t-1}, \tag{4}$$

where $k = \theta_1/(\theta_1+\theta_{11}), 0 < k < 1$.

Okun's law and aggregate demand

- **Okuns's law**:

$$\boxed{y_t - \bar{y}_t = -\omega(u_t - \bar{u}_t)} \tag{5}$$

- **aggregate demand** (Berger and Everaert [2008]):

$$y_t = \frac{1}{\lambda}(m_t - p_t) + \frac{1}{\mu}x_t + \gamma s_t$$

- add and subtract $\bar{y}_t$, take first differences:

$$\boxed{\Delta y_t = \Delta \bar{y}_t - \frac{1}{\lambda}(\pi_t - \bar{\pi}_t) + \gamma \Delta s_t} \tag{6}$$

where $\bar{\pi}_t = \Delta m_t + (\lambda/\mu)\Delta x_t - \lambda \Delta \bar{y}_t$

Measurement equations

$$\Delta\pi_t = -\theta_1(u_t - \bar{u}_t) - \theta_{11}\Delta u_t + \epsilon_t^{\pi} \tag{7a}$$

$$y_t - \bar{y}_t = -\omega(u_t - \bar{u}_t) + \epsilon_t^{u} \tag{7b}$$

$$\Delta y_t = \Delta\bar{y}_t - \frac{1}{\lambda}(\pi_t - \bar{\pi}_t) + \gamma\Delta s_t + \epsilon_t^{y} \tag{7c}$$

Transition equations

$$\bar{y}_t = \bar{y}_{t-1} + \psi_{t-1} + \eta_t^1 \quad (8a)$$

$$\psi_t = \psi_{t-1} + \eta_t^2 \quad (8b)$$

$$\bar{u}_t = (1 + \delta)\bar{u}_{t-1} - \delta \bar{u}_{t-2} + \eta_t^3 \quad (8c)$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \eta_t^4 \quad (8d)$$

Adaptive RWMH and Geweke's CD

- **posterior mode estimate**:

$$\arg\max_{\theta} \log p(\theta|y) = \log p(y|\theta) + \log p(\theta) - \log p(y)$$

- **Adaptive RWMH**:

$$\theta^* = \theta^{s-1} + z^s, z^s \sim N(0, \Sigma^s)$$

$$\alpha(\theta^{s-1}, \theta^*) = \min\left[\frac{p(\theta = \theta^*|y)}{p(\theta = \theta^{s-1}|y)}, 1\right] \longrightarrow 23.4\%$$

- **Geweke's convergence diagnostic**:

$$CD = \frac{\widehat{E[\theta_1]} - \widehat{E[\theta_2]}}{\widehat{\sigma_1}/\sqrt{N_1} + \widehat{\sigma_2}/\sqrt{N_2}} \longrightarrow N(0,1)$$

Kalman filter

- **state-space form**:

$$y_t = Z\alpha_t + Ax_t + \varepsilon_t, \varepsilon_t \sim N(0, H) \quad (9a)$$

$$\alpha_t = T\alpha_{t-1} + \eta_t, \eta_t \sim N(0, Q) \quad (9b)$$

- **Kalman filter**: estimate the distribution of α_t, for $t = 1, ..., T$, conditional on $Y_t = \{y_1, ..., y_t\}$ and $X_t = \{x_1, ..., x_t\}$;
- **Kalman smoother**: estimate the distribution of α_t, for $t = 1, ..., T$, conditional on $Y_T = \{y_1, ..., y_T\}$ and $X_T = \{x_1, ..., x_T\}$.

Data and transformations

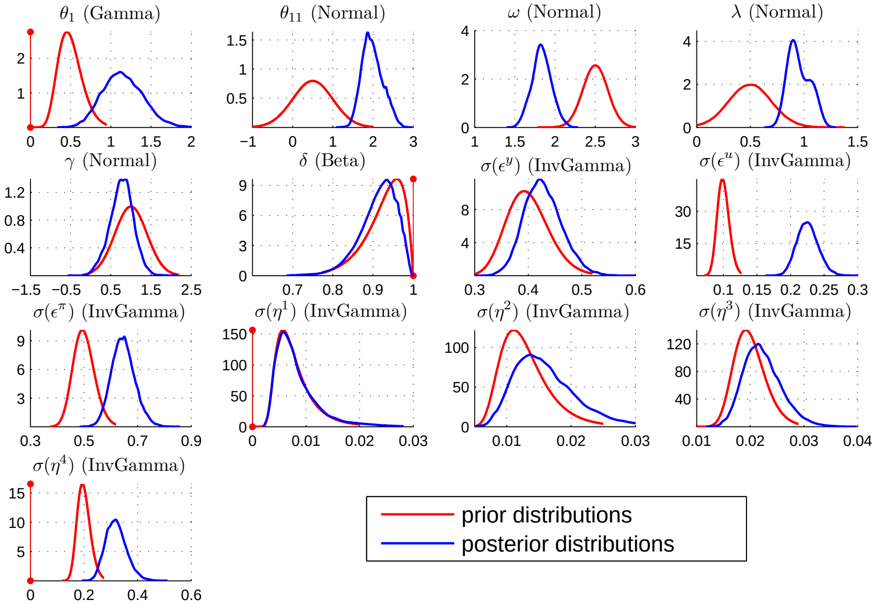
- y_t: quarterly seasonally adjusted real GDP (NIS), in log;
- π_t: q-o-q inflation rate, compounded from quarterly CPI as average of the corresponding monthly CPI (NIS), seasonally adjusted, in log and first difference;
- u_t: quarterly seasonally adjusted unemployment rate, ILO definition (Eurostat);
- Δs_t: quarterly real modification of RON against EUR, obtained from average daily nominal exchange rate (NBR), quarterly CPI defined above and a similar HICP (Eurostat); $\Delta s_t > 0$ indicates a real depreciation.

Data covers 2000Q1:2011Q4 period

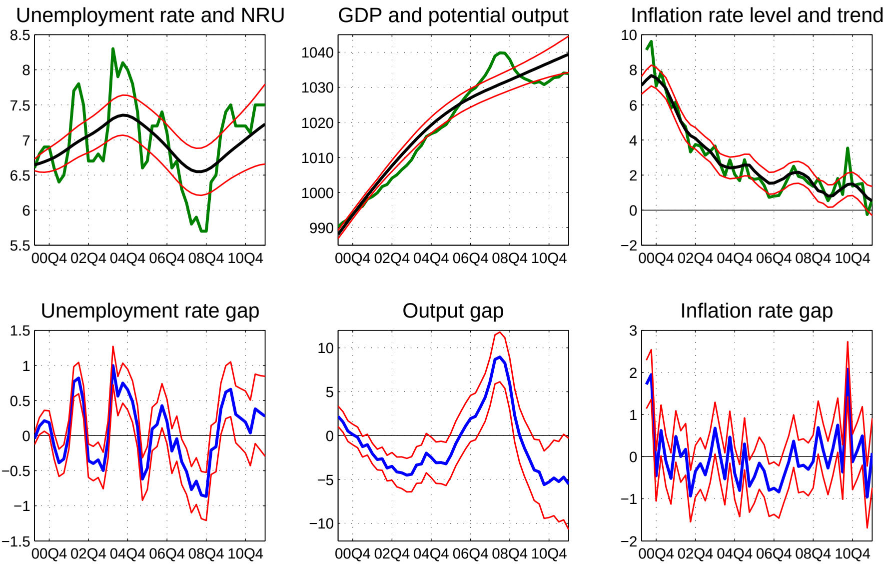
Prior and postrior distributions (1)

Coeff.	Distribution	Prior mean	Prior st.dev.	Posterior mode	Posterior st.dev.	95% BCI	Geweke's CD
θ_1	Gamma	0.5	0.15	1.082	0.104	0.70 — 1.67	-0.26*
θ_{11}	Normal	0.5	0.5	1.725	0.075	1.54 — 2.53	-2.29***
ω	Normal	2.5	0.15	2.015	0.006	1.61 — 2.07	0.53*
λ	Normal	0.5	0.2	1.000	0.002	0.82 — 1.13	-1.71**
γ	Normal	1	0.4	0.832	0.108	0.16 — 1.33	-0.80*
δ	Beta	0.925	0.05	0.937	0.041	0.81 — 0.98	-0.33*
$\sigma(\epsilon^y)$	InvGamma	0.4	0.04	0.427	0.033	0.36 — 0.50	0.17*
$\sigma(\epsilon^u)$	InvGamma	0.1	0.009	0.224	0.025	0.20 — 0.26	-0.21*
$\sigma(\epsilon^\pi)$	InvGamma	0.5	0.04	0.649	0.048	0.57 — 0.74	-0.20*
$\sigma(\eta^1)$	InvGamma	0.008	0.004	0.006	0.002	0.003 — 0.020	0.03*
$\sigma(\eta^2)$	InvGamma	0.013	0.004	0.013	0.004	0.009 — 0.029	-0.04*
$\sigma(\eta^3)$	InvGamma	0.02	0.003	0.022	0.003	0.016 — 0.030	0.10*
$\sigma(\eta^4)$	InvGamma	0.2	0.025	0.310	0.050	0.25 — 0.40	0.12*

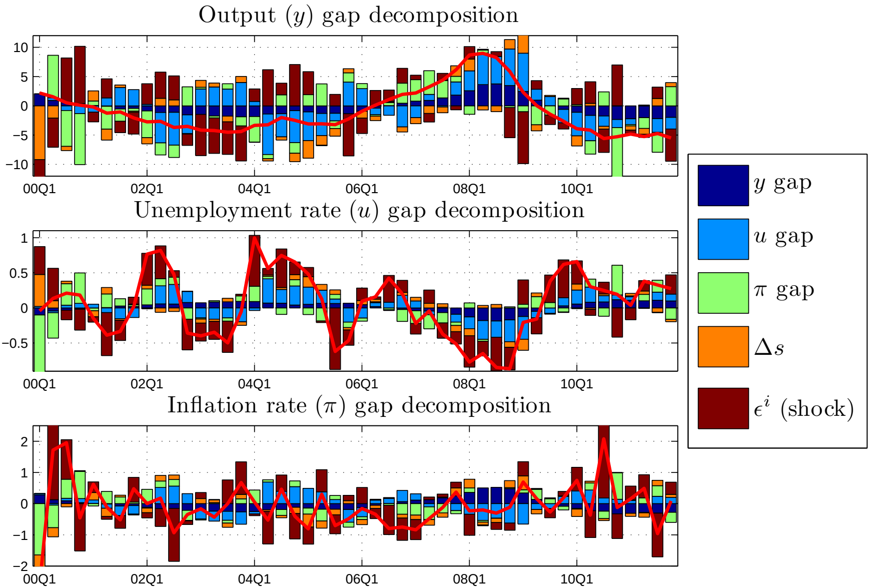
Prior and postrior distributions (2)



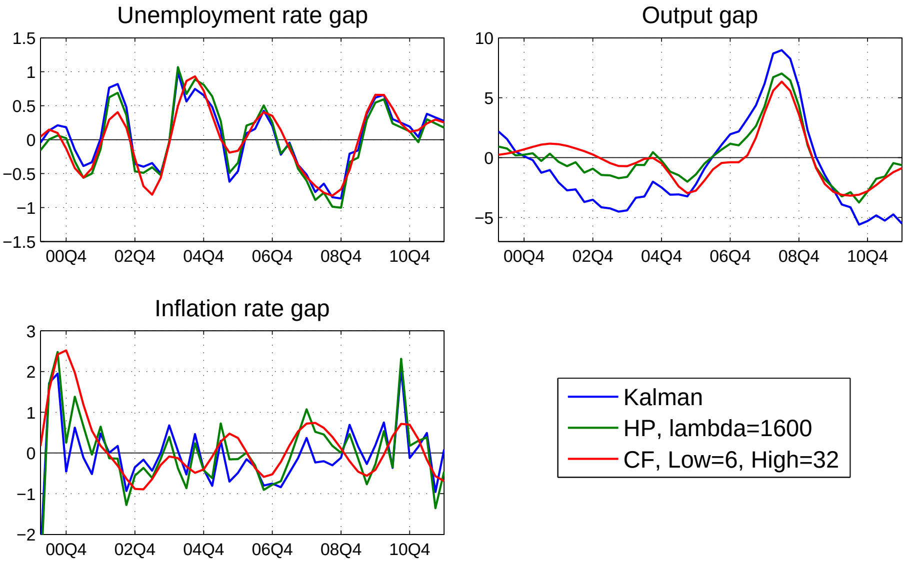
Unobserved components (1)



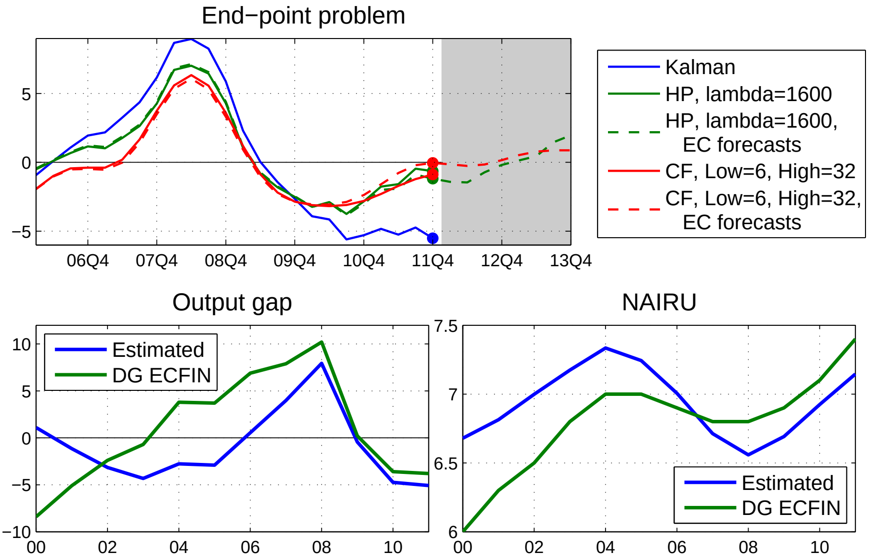
Unobserved components (2)



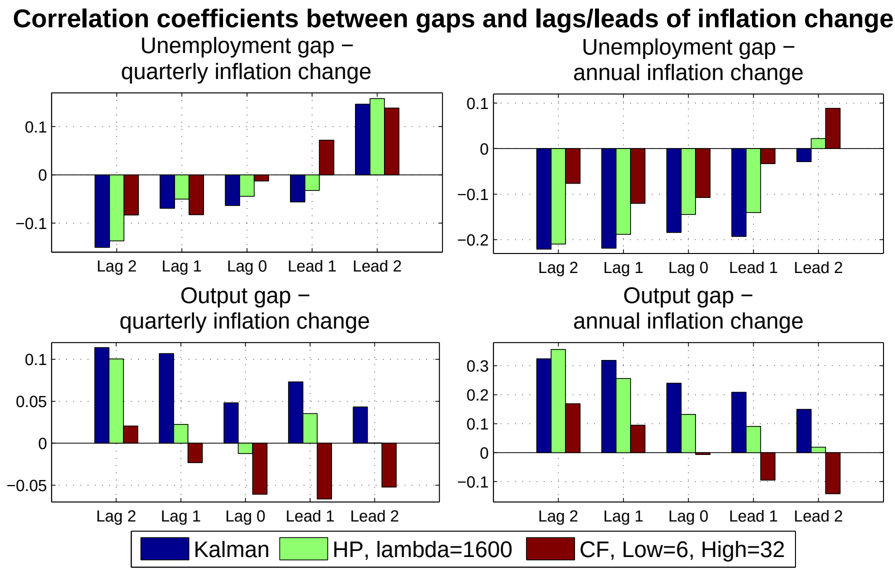
Reliability of estimated states (1)



Reliability of estimated states (2)



Reliability of estimated states (3)



Conclusions

- Estimated current cyclical position (positive unemployment gap and strong negative output gap) indicates need for deeper structural reforms in order to:
  - boost economic activity;
  - equilibrate labour market.
- Current inflation rate seems to be at equilibrium as NBR met inflation target for 2011
- Model captures effects of economic crisis:
  - growth rate of potential output halves, from 5.0% in 2001-2008 to 2.4% in 2009-2011;
  - NRU is still on an upward shape.
- Persistence effects are important for unemployment rate dynamics
- Gaps are (potentially) useful in forecasting inflation

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Annex: Testing assumptions

OLS relation						
Net wages' inflation	=	1.86	+	0.94	·	CPI inflation
Prob.		[0.01]		[0.00]		
Wald test						
Net wages' inflation	=	const.	+	1	·	CPI inflation
χ^2 Prob.	[**0.73**]					

Tab. A1. Wald test for coefficients restriction (1)

OLS relation						
CPI inflation at *t*	=	0.13	+	0.91	·	CPI inflation at *t-1*
Prob.		[0.54]		[0.00]		
Wald test						
CPI inflation at *t*	=	0	+	1	·	CPI inflation at *t-1*
χ^2 Prob.	[**0.09**]					

Tab. A2. Wald test for coefficients restriction (2)

t-Statistic	-2.42
Prob.	[**0.14**]

Tab. A3. Augmented Dickey-Fuller test for quarterly inflation ratec

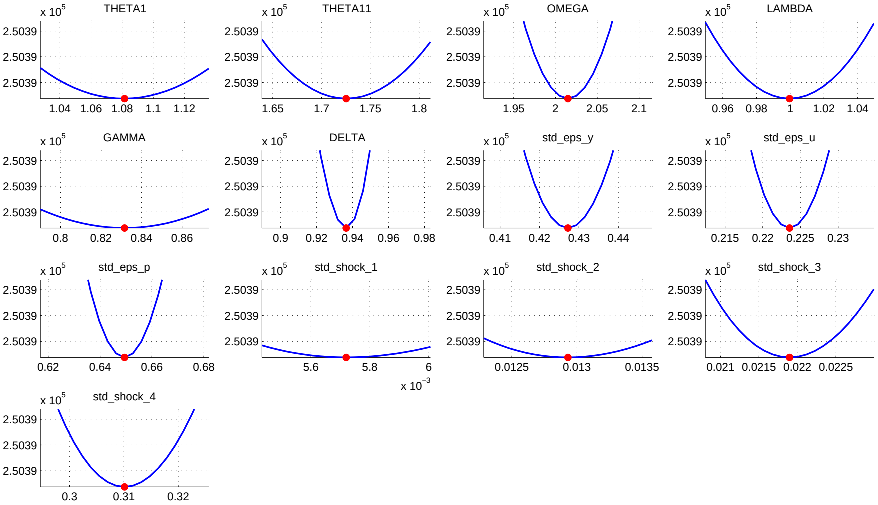
Annex: Reduced-form measurement equations

$$\begin{pmatrix} y_t \\ u_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} \bar{y}_t \\ \bar{u}_t \\ \bar{\pi}_t \end{pmatrix} + \begin{pmatrix} \frac{\omega\lambda}{\alpha} & -\frac{\omega\theta_{11}}{\alpha} & -\frac{\omega}{\alpha} & \frac{\omega\lambda\gamma}{\alpha} \\ -\frac{\lambda}{\alpha} & \frac{\theta_{11}}{\alpha} & \frac{1}{\alpha} & -\frac{\lambda\gamma}{\alpha} \\ \frac{(\theta_1+\theta_{11})\lambda}{\alpha} & \frac{\omega\theta_{11}\lambda}{\alpha} & \frac{\omega\lambda}{\alpha} & -\frac{(\theta_1+\theta_{11})\lambda\gamma}{\alpha} \end{pmatrix} \times$$

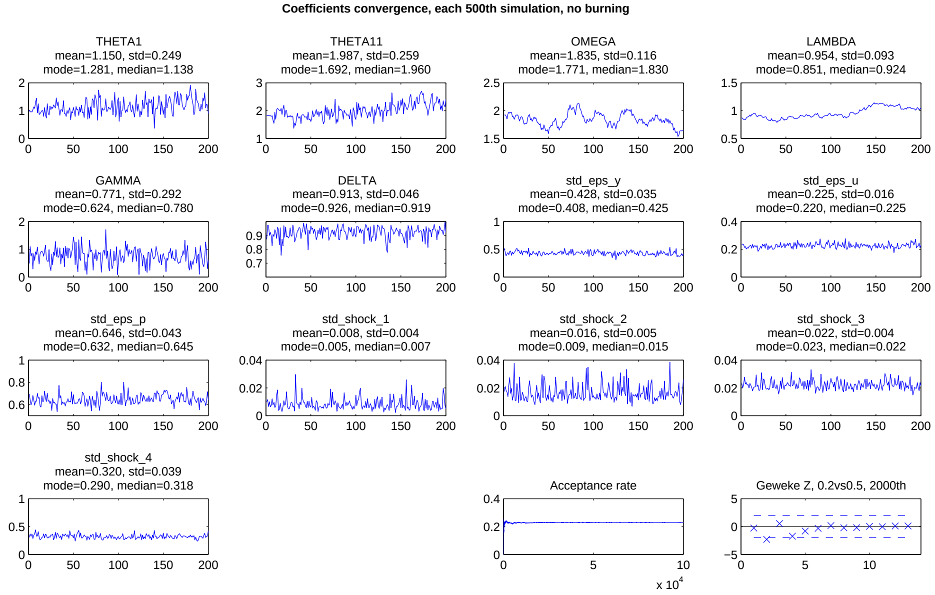
$$\times \begin{pmatrix} y_{t-1} - \bar{y}_{t-1} \\ u_{t-1} - \bar{u}_t \\ \pi_{t-1} - \bar{\pi}_t \\ \Delta s_t \end{pmatrix} + \begin{pmatrix} \epsilon_t^y \\ \epsilon_t^u \\ \epsilon_t^\pi \end{pmatrix}$$

where $\alpha = \theta_1 + \theta_{11} + \lambda\omega > 0$

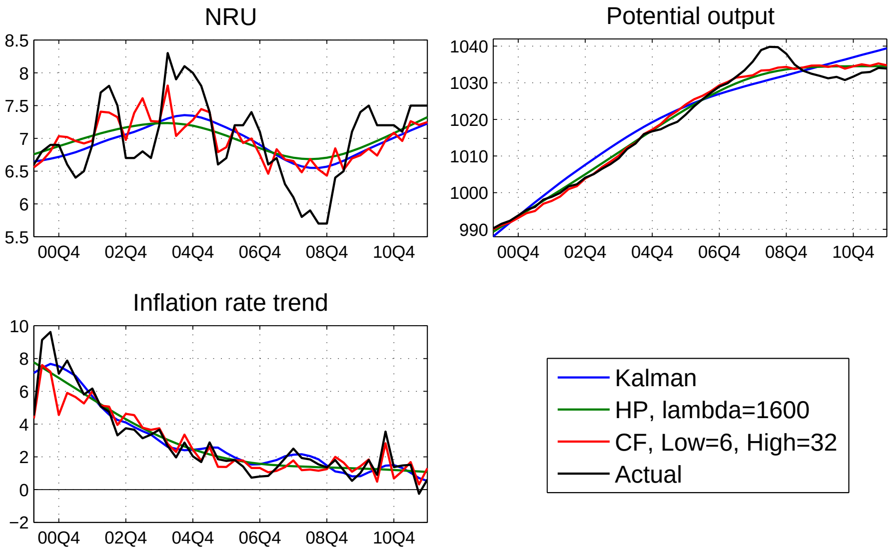
Annex: Neighbourhoods around the optimum



Annex: Convergence of the chains



Annex: Trend components



Annex: Alternative measures for equilibrium unemployment rate