DOFIN
Doctoral School of Finance and Banking

Extreme Events, Dependence Structures and Market Risk:

A CEE Stock Exchanges Analysis

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BUCHAREST
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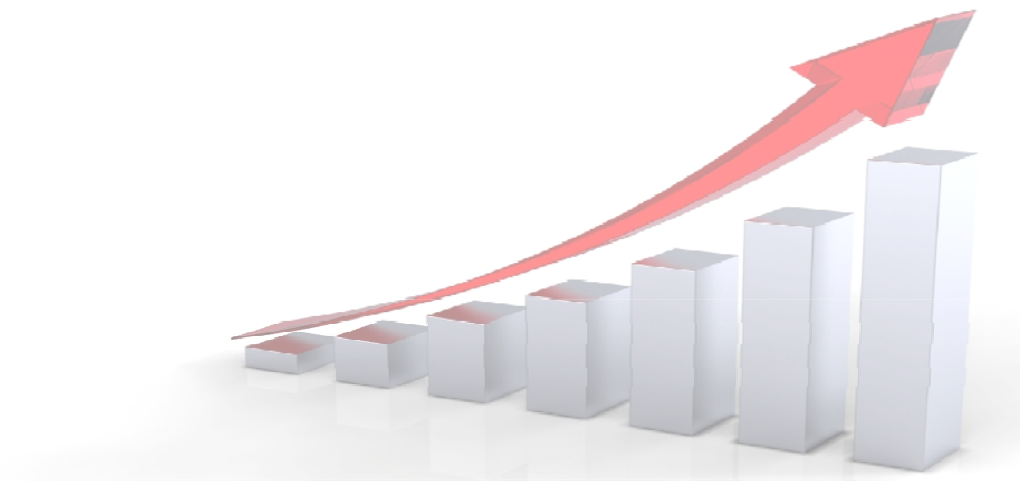


I. Motivation and Importance

- **Markets systemic view**
- **Accurate distribution for financial data**
- **Portfolios dependence structures description**
- **Portfolio and risk management**
- **Large scale utilization of Value-at-Risk models in financial and banking system**
- **Incorporating dependence structure and extreme values into market risk measurements**

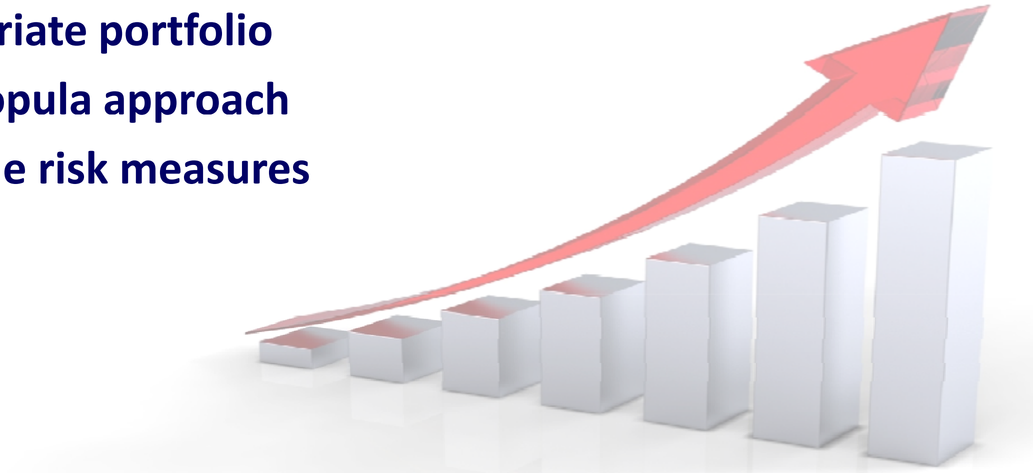
II. Target

Basic Idea

Offering a global view over the analyzed markets, incorporating EV and Dependence Structures, based on which I will construct an accurate approach for market risk

Intermediary Objectives

- Choosing the best model for autocorrelation and heteroskedasticity
- Make use of Extreme Values methodology to capture very high/ low returns in the market
- Applying Copula functions to model the dependence structures between markets
- Assess the right Copula for each bivariate portfolio
- Compute VaR based on a dynamic Copula approach
- Estimate in-sample and out-of-sample risk measures
- Backtest the results



III. Fundamentals

Extreme Value Theory

- **Peaks over Threshold Method**
- ***Literature:* Pickands (1975), Embrechts et al. (1997,1999,2005,2011), McNeil (1997,1999,2000), Longin (2001), Beirlant (2002), Poon, Rockinger and Tawn (2003), Blum and Dacorogna (2002) and others**

Copulas

- **Gaussian, T-Copula, Clayton, Rotated Gumbel, Symmetrized Joe-Clayton**
- **Time-varying Rotated Gumbel and Symmetrized Joe-Clayton**
- ***Literature:* Bouye et al. (2000), Longin (2001), McNeil et al. (2002), Embrechts (2001,2002,2003), Poon et al. (2003), Embrechts, Hoing and Juri (2003), Patton (2006) and others**

Value at Risk

- **Hybrid EVT-Copula model**
- ***Literature:* Embrechts (1999), McNeil (2000), Engle and Manganelli (2001), Alexander (2001), Kuester et at (2006), Danielsson (2000,2011) and others**

IV. Methodology

- Fitting *ARMA-GJR GARCH* to the data
- *EVT – POT* method: excesses over a high threshold *u* (i.e. tails) ~ Generalized Pareto Distribution with shape parameter - *ξ > 0* denotes fat tails

$$G_{\xi,\sigma,\beta} = \begin{cases} 1-\left(1+\xi\dfrac{E-\beta}{\sigma}\right)^{-\frac{1}{\xi}}, \text{if } \ \xi \neq 0 \\ 1-e^{-\frac{(E-\beta)}{\sigma}} \qquad\qquad\quad , \text{if } \ \xi = 0 \end{cases}$$

- C*opula* functions to d... en data series

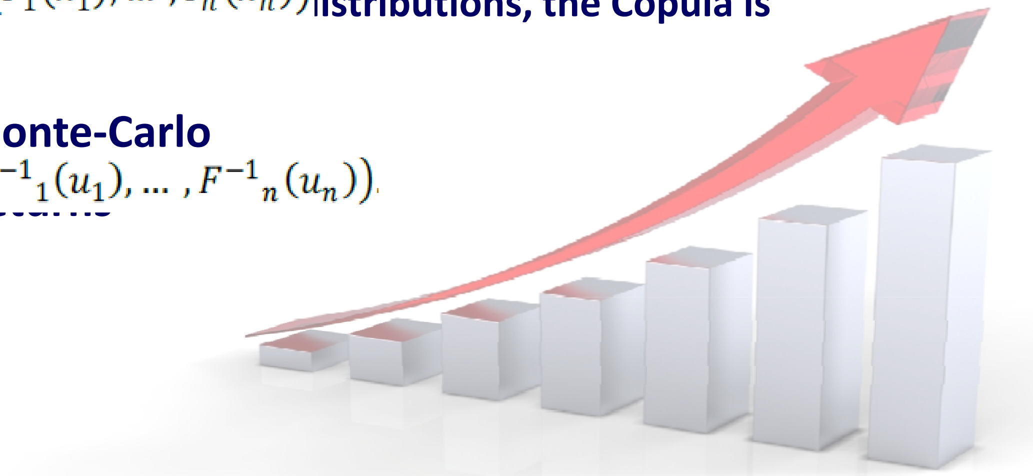
  Multidimensional distribution is linked to its univariate margins by an unique Copula

$$F(x_1, \dots, x_n) = C\big(F_1(x_1), \dots, F_n(x_n)\big)$$

  Conversely, having knov... listributions, the Copula is

$$C(u_1, \dots, u_n) = F\big(F^{-1}{}_1(u_1), \dots, F^{-1}{}_n(u_n)\big).$$

- *Value-at-Risk* is obtained from the Monte-Carlo simulated distribut...



IV. Methodology

Steps

- Analyzing the data and asses the stylized facts

- Filtering the data series by the ARMA-GJR GARCH model, in order to take care of autocorrelation, heteroskedasticity, leverage and obtain the i.i.d. standardized residuals

- Choosing the right threshold, depending on the data series, based on ME and Hill Plot

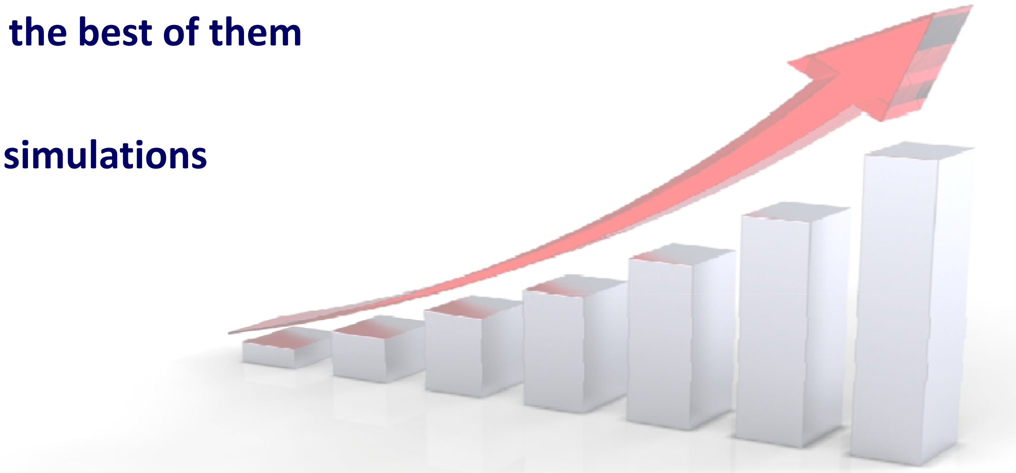
- Fitting the best semi-parametric distribution: GPD in tails and Kernel in center

- Obtaining uniform data for the Copula modeling by two methods: CML and IFM

- Fitting the Copulas to the data and choose the best of them

- Computing VaR Measures by Monte-Carlo simulations

- Backtesting



V. Data Description

Data

Four main stock indices from Romania, Hungary, Czech Republic, Poland, respectively *BET,BUX, PX, WIG20*

The analysis is performed mainly in *Matlab*

Data is provided by *Bloomberg*

Descriptive Statistics

Series	BET	BUX	PX	WIG20
Mean	0.00017	0.00012	0.00007	0.00005
Maximum	0.04588	0.05723	0.05370	0.03542
Minimum	-0.05697	-0.05493	-0.07029	-0.03612
Std. Dev.	0.00841	0.00783	0.00723	0.00713
Skewness	-0.43910	-0.12800	-0.52320	-0.17390
Kurtosis	9.27627	8.93445	15.99637	5.53199
Jarque-Bera	3,365.14*	2,929.98*	14,162.20*	556.99*
Probability	0.0000	0.00000	0.00000	0.00000
Observations	2,000	2,000	2,000	2,000

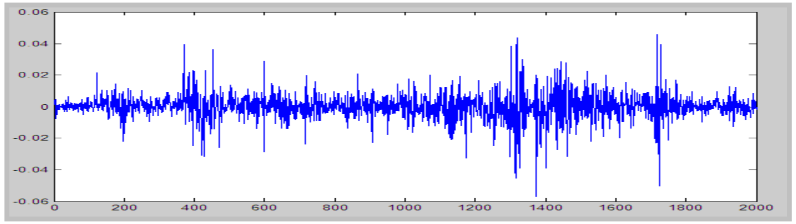
*Denotes rejection of null hypothesis at 1% significance level

Critical value 1%: 9.6981

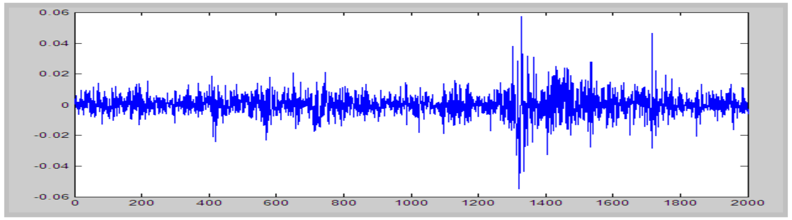
VI. Results

Facts: skewed, leptokurtic, volatility clusters, heteroskedasticity, fat tails, contagion, weakly AC, strong AC on squared raw data

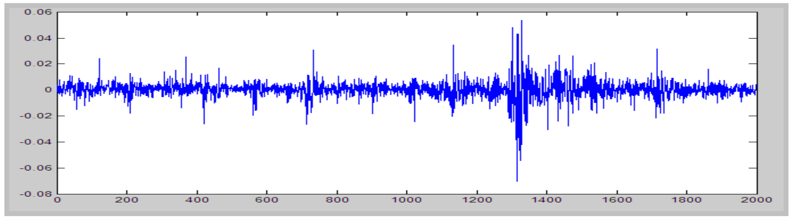
BET



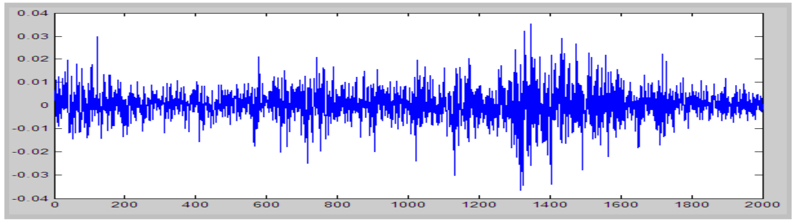
BUX

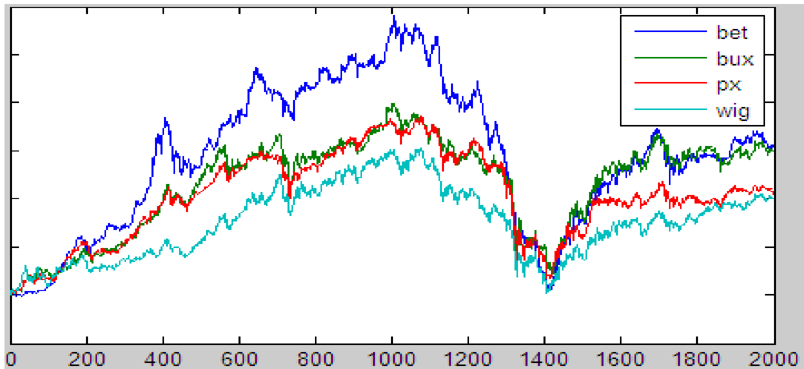


PX



WIG





Autocorrelation and Heteroskedasticity

Test	Ho – No serial correlation	Q-Statistic			
		BET	BUX	PX	WIG20
Ljung-Box	Returns*	59.8785	55.4795	89.6121	51.8417
Ljung-Box	Squared returns*	1096.70	1829.50	2638.30	1199.75

*Denotes rejection at 1% semnificance level

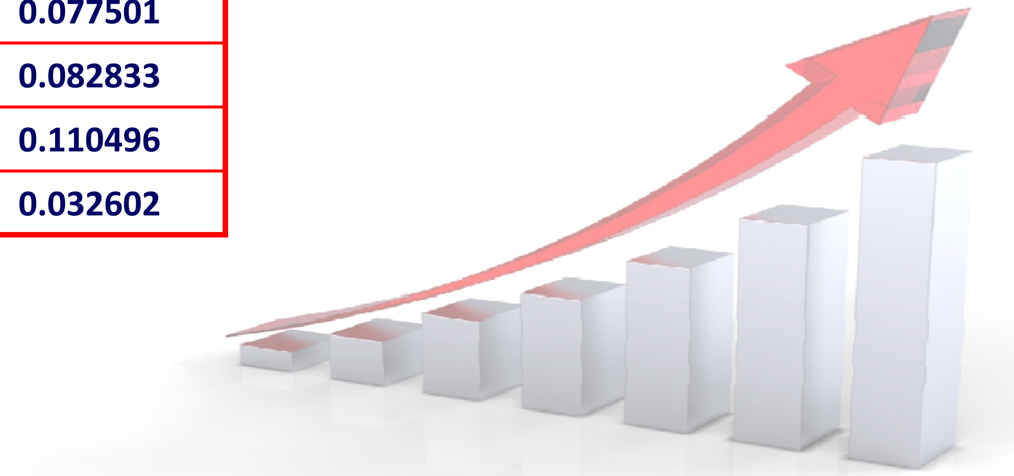
Ljung-Box critical value: 44.3141

Mean and Volatility Modeling

Series	Model	Asymmetric Term
BET	AR (1) - GJR GARCH (1,1)	0.077501
BUX	ARMA (1,1) - GJR GARCH (1,1)	0.082833
PX	ARMA (2,2) - GJR GARCH (1,1)	0.110496
WIG	ARMA (1,1) - GJR GARCH (1,1)	0.032602

Standardized residuals ~ i.i.d. returns

$$r(t) = \frac{\varepsilon(t)}{\sigma(t)}$$



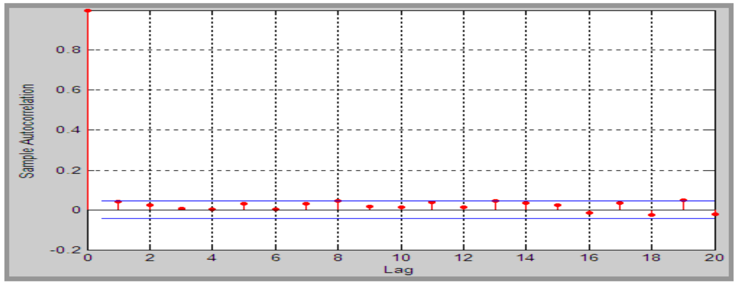
Result

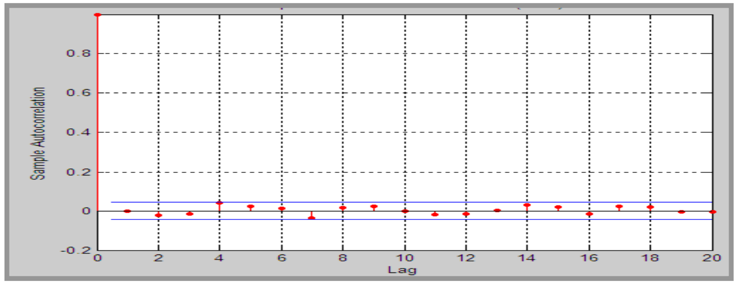
Successfully applied models

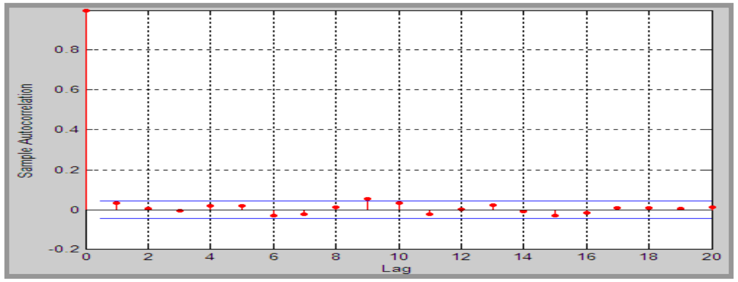
Test	Ho – No serial correlation	Q-Statistic			
		BET	BUX	PX	WIG20
Ljung-Box	Returns*	33.4744	21.6250	25.2338	22.6438
Ljung-Box	Squared returns*	22.5175	25.3037	20.8985	27.8767

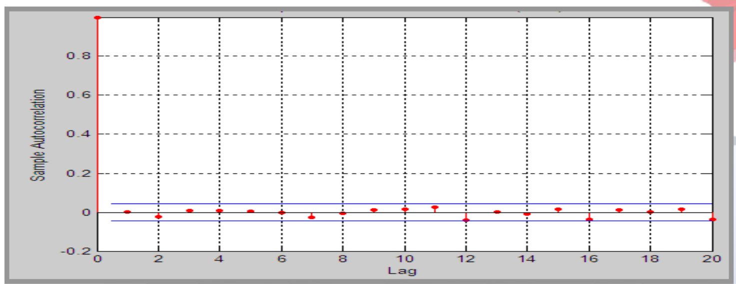
*Denotes acceptance at 1% semnificance level

Ljung-Box critical value: 44.3141







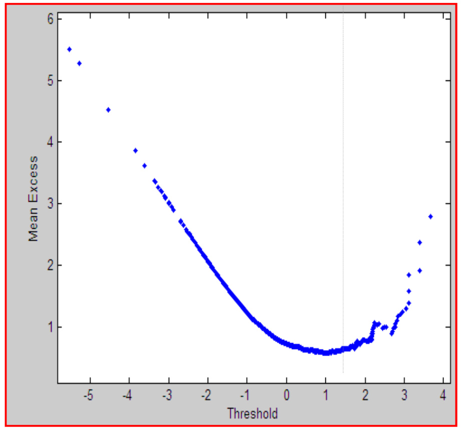


EVT: Choosing the Threshold – ME and Hill Plots

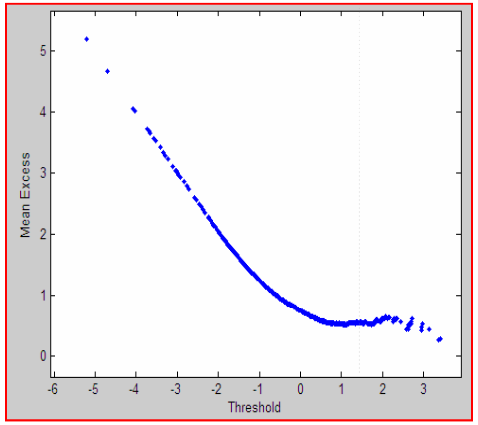
BET example

Mean Excess Plots

L



R



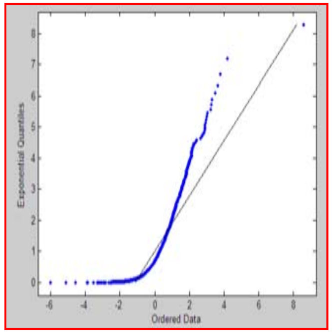
Average distance from the increasing threshold

$$e_n(u) = \frac{\sum_{i=1}^{n} (X_i - u)}{\sum_{i=1}^{n} I_{\{X_i > u\}}}$$

$$\left\{ (X_{k,n}, e_n(X_{k,n})) \mid k = 1, \dots, n \right\}$$

Q-Q Plots

R



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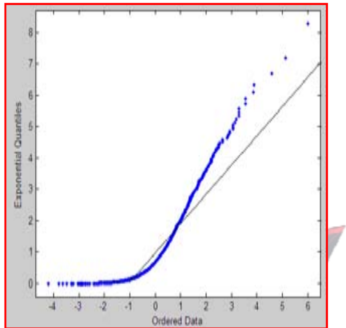
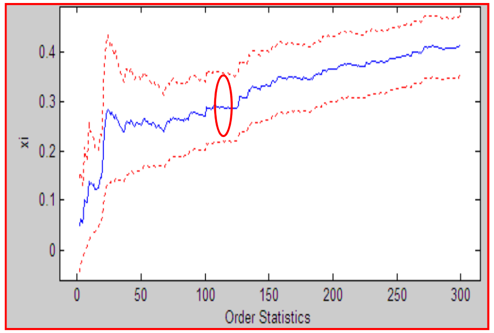


Hill Plots

L



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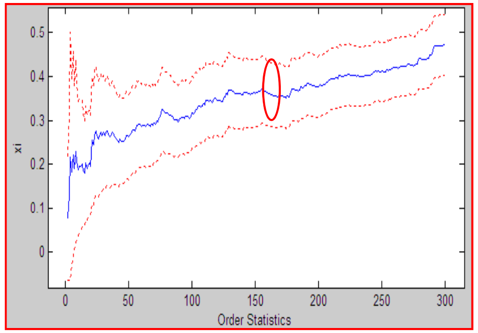


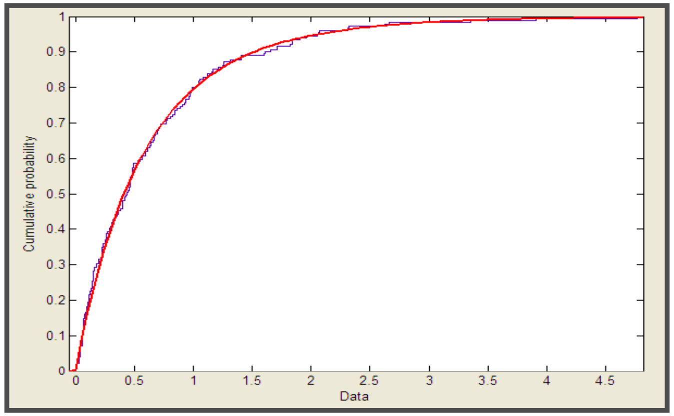
$$\xi_{k,n}^{Hill} = \frac{1}{k} \sum_{i=1}^{k} \log X_{n+1-i,n} - \log X_{n-k,n}$$

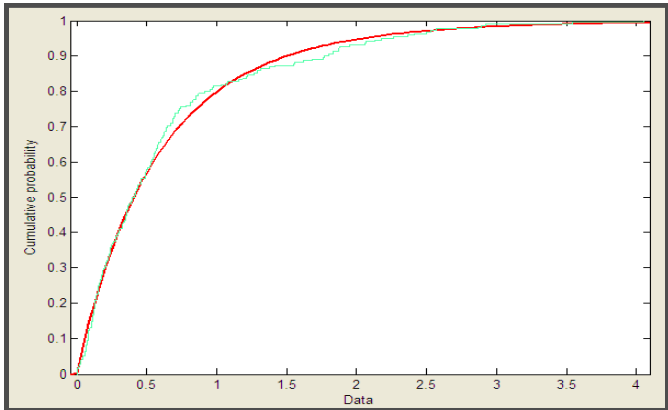
$$\left\{ (k, \hat{\xi}_{k,n}), 1 \le k \le n \right\}$$

EVT: Threshold, Tail Index and GPD Fit

Parameter		BET	BUX	PX	WIG20
Threshold	Left	1.2280~9%	1.3810~8%	1.1563~10%	1.1990~9%
	Right	1.1860~6.5%	-	1.0800~10%	-
Shape - ξ *(ML estimates)*	Left	0.1543	0.0787	0.1692	0.1271
	Right	0.0412	-	0.1310	-
Scale - σ *(ML estimates)*	Left	0.5772	0.4879	0.5213	0.4813
	Right	0.5625	-	0.5221	-

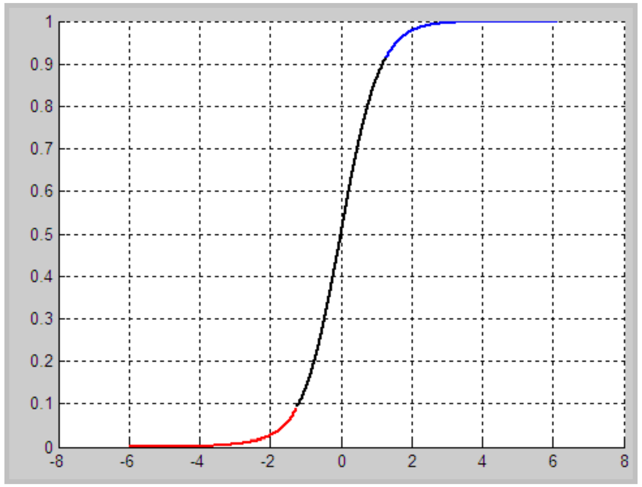
GPD Fit Assessment (BET)



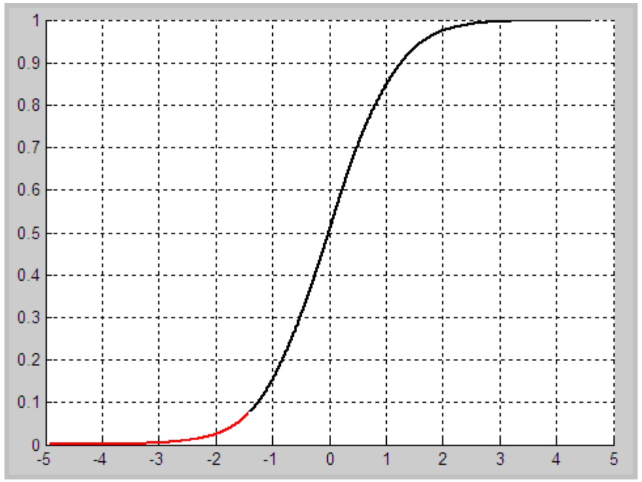


Semi-parametric Fit: GPD & Kernel Smooth

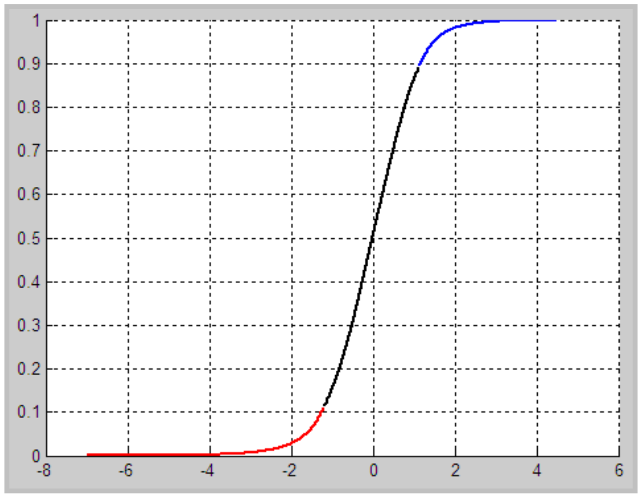
BET



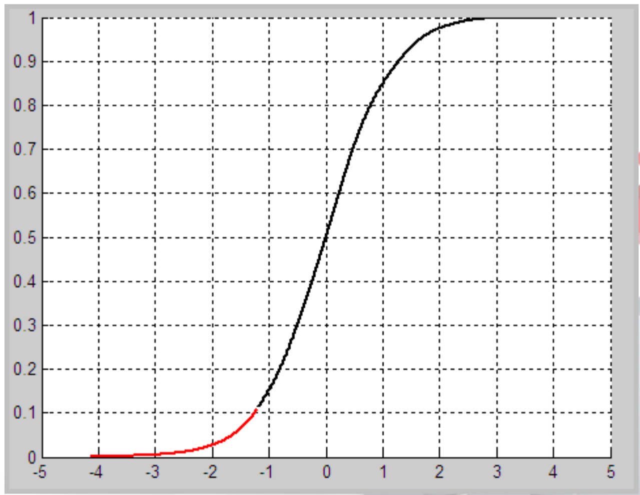
BUX



PX

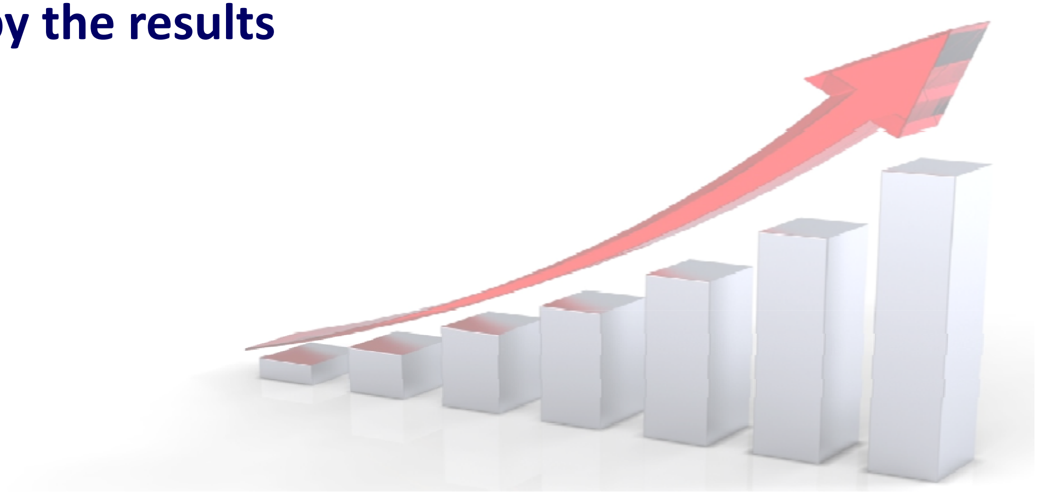


WIG20



Dependence structures

- Two approaches: Copula CML and IFM
- Assess the benefit of using EVT
- Strong positive dependence among the four indices
- Asymmetry is further sustained by the results



Dependence measures

Financial Center for CEE

Stock Market Dimension

Small Economy

Vulnerability

EU Accession

Rankings:

1. WIG20-BUX
2. WIG20-PX
3. PX-BUX
4. PX-BET
5. WIG20-BET
6. BUX-BET

Economic Development

Economic Stability

The Second Stock Exchange

Recession Impact

Driving Market

Wig20-Bux: CML or IFM?

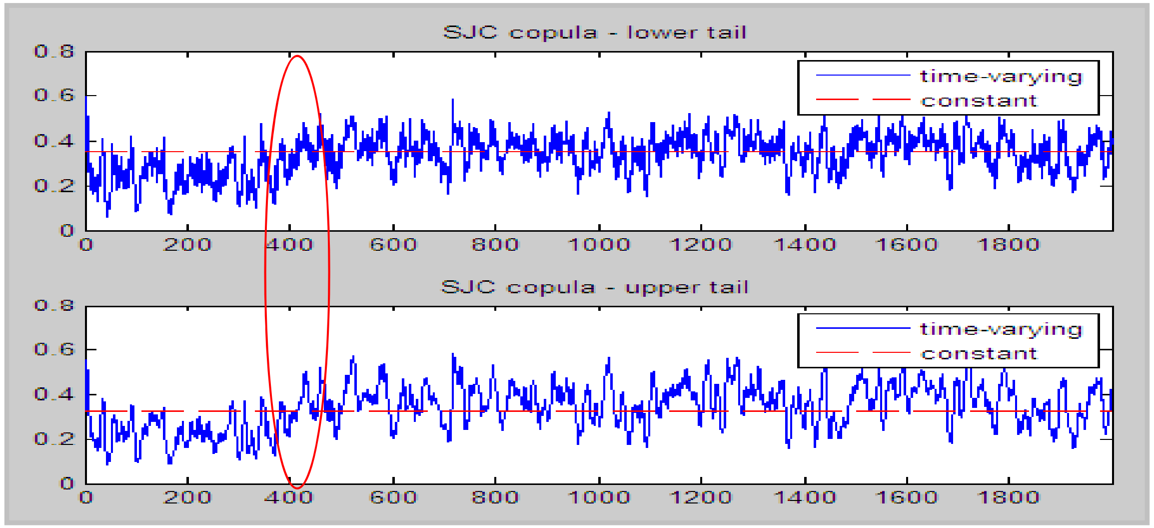
Copula	Canonical Maximum Likelihood					
	Parameters	Low - TD	Up - TD	LL	AIC	Rankings
Gaussian	0.5241	0	0	-321.581	-643.16	3
Clayton	0.7974	0.4192	0	-266.052	-532.104	5
R.Gumbel	1.5166	0.4182	0	-316.811	-633.621	4
T	0.5319/ 6.5449	0.1696	0.1696	-346.040	-693.479	1
SJC	0.3537/ 0.3106	0.3537	0.3106	-331.400	-662.798	2

Copula	Inference Functions for Margins					
	Parameters	Low - TD	Up - TD	LL	AIC	Rankings
Gaussian	0.5246	0	0	-321.894	-643.787	3
Clayton	0.7966	0.4289	0	-269.972	-539.943	5
R.Gumbel	1.5193	0.4219	0	-319.636	-639.271	4
T	0.5502/ 6.7800	0.1864	0.1864	-348.744	-698.426	1
SJC	0.3701/ 0.3242	0.3701	0.3242	-334.535	-669.068	2

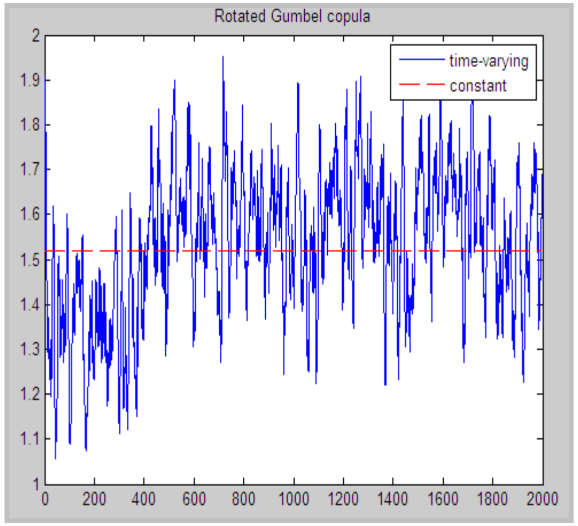
Incorporating the semi-parametric distribution (GPD and Kernel Smooth) provides better and higher results

Time Varying Copulas

WIG-BUX



L.tail: 2.075,-8.742,-2.718; U.tail: -0.337, -4.458; 1.734: LL: -375.89



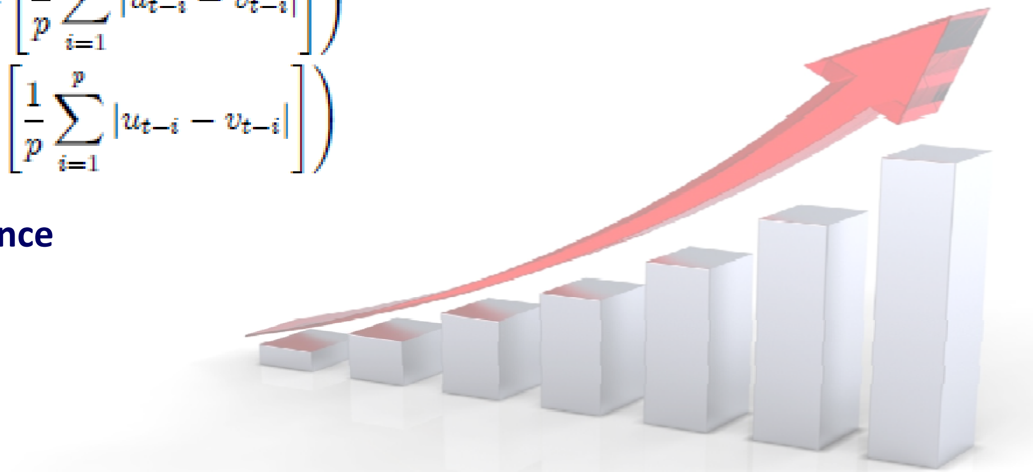
L.tail: 0.916, 0.079, -1.481; LL:-338.21

- **Copula-GARCH (Patton 2006)**

$$\tau_t^U = \Lambda\left(\omega_U + \alpha_U \tau_{t-1}^U + \beta_U \left[\frac{1}{p}\sum_{i=1}^{p} |u_{t-i} - v_{t-i}|\right]\right)$$

$$\tau_t^L = \Lambda\left(\omega_L + \alpha_L \tau_{t-1}^L + \beta_L \left[\frac{1}{p}\sum_{i=1}^{p} |u_{t-i} - v_{t-i}|\right]\right)$$

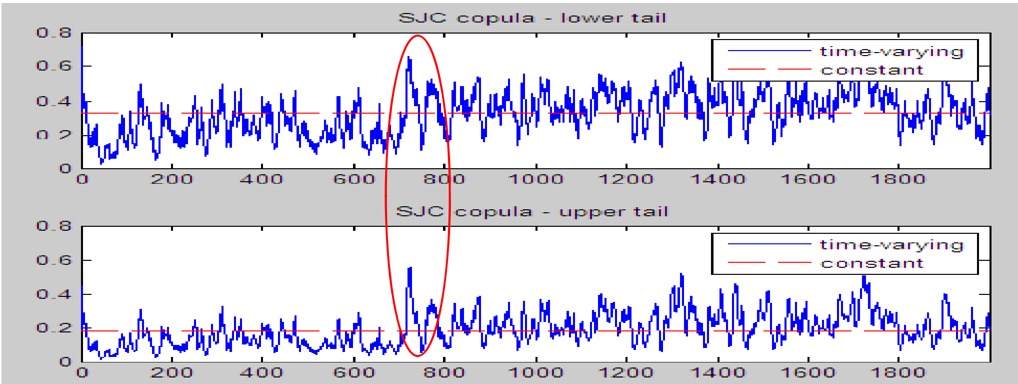
- **EU Accession – 2004 => increase in dependence**



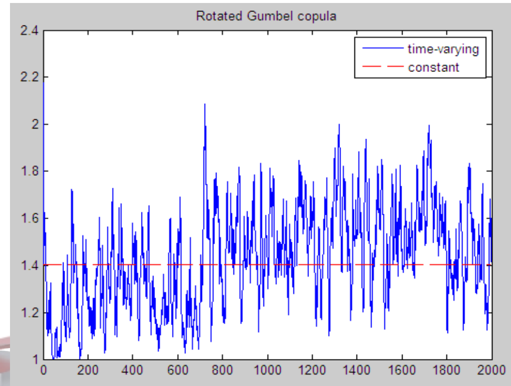
Wig20-Px: CML or IFM?

Copula	Canonical Maximum Likelihood					
	Parameters	Low - TD	Up - TD	LL	AIC	Rankings
Gaussian	0.4458	0	0	-221.338	-442.675	4
Clayton	0.6680	0.3542	0	-206.203	-412.404	5
R.Gumbel	1.4020	0.3604	0	-235.356	-470.711	3
T	0.4497/ 7.8904	0.0993	0.0993	-236.637	-473.272	2
SJC	0.3280/ 0.1778	0.3280	0.1778	-238.255	-476.507	1

Copula	Inference Functions for Margins					
	Parameters	Lo - TD	Up - TD	LL	AIC	Rankings
Gaussian	0.4470	0	0	-222.657	-445.314	4
Clayton	0.6752	0.3632	0	-207.289	-414.576	5
R.Gumbel	1.4040	0.3616	0	-236.222	-472.443	3
T	0.4628/ 7.0148	0.1104	0.1104	-237.999	-475.996	2
SJC	0.3365/ 0.1824	0.3365	0.1824	-239.789	-479.577	1



L.tail: 1.134,-8. 003,-0.119 U.tail: -0.545, -6.142; 2.057: LL: -278.59

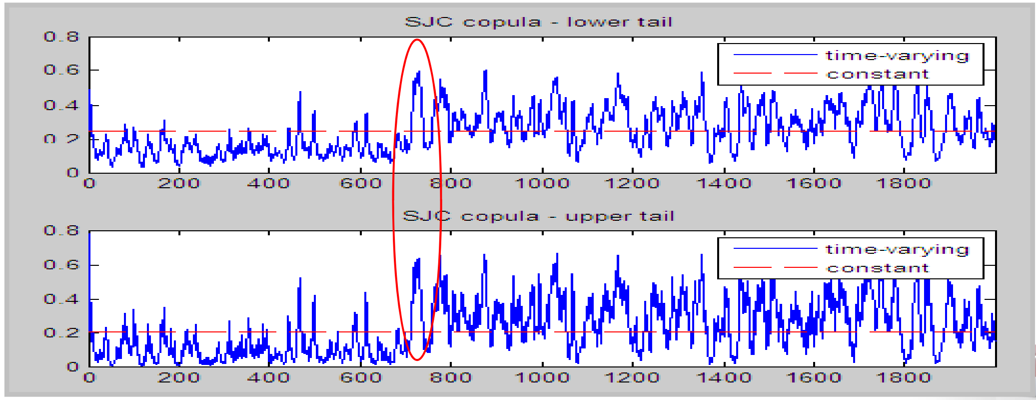


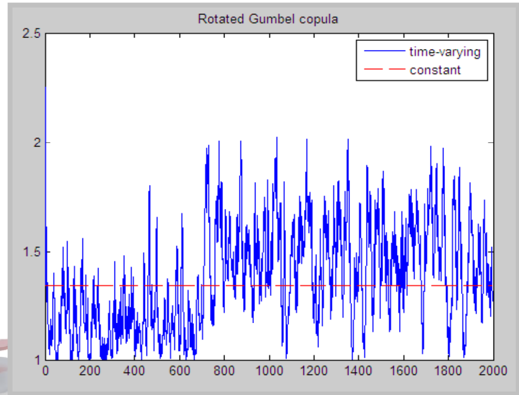
L.tail: 1.091, 0.024, -2.091; LL:-268.92

Bux-Px: CML or IFM?

Copula	Canonical Maximum Likelihood					
	Parameters	Low - TD	Up - TD	LL	AIC	Rankings
Gaussian	0.4132	0	0	-186.996	-373.992	3
Clayton	0.5525	0.2852	0	-157.781	-315.560	5
R.Gumbel	1.3406	0.3229	0	-183.194	-366.387	4
T	0.4156/ 9.4281	0.0626	0.0626	-195.957	-391.913	2
SJC	0.2497/ 0.2028	0.2497	0.2028	-196.361	-392.719	1

Copula	Inference Functions for Margins					
	Parameters	Lo - TD	Up - TD	LL	AIC	Rankings
Gaussian	0.4244	0	0	-188.156	-376.310	3
Clayton	0.5624	0.2889	0	-158.605	-317.209	5
R.Gumbel	1.3441	0.3245	0	-183.479	-366.956	4
T	0.4261/ 9.8231	0.0684	0.0684	-196.721	-393.439	2
SJC	0.2596/ 0.2048	0.2596	0.2048	-198.317	-396.947	1





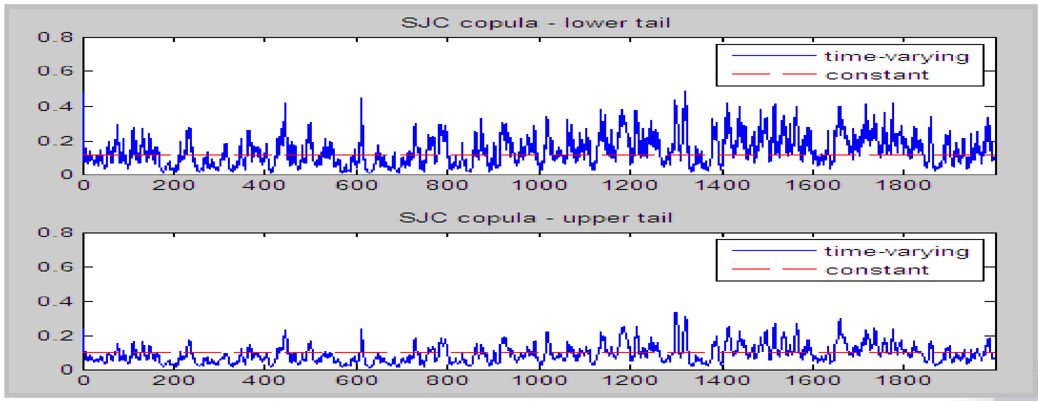
L.tail: 0.010,-6.648,1.520; U.tail: -2.579, -12.338; 1.451: LL: -248.79

L.tail: 1.569, -0.185, -3.012; LL:-231.29

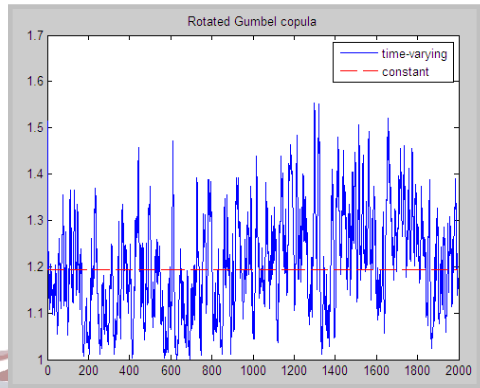
Px-Bet: CML or IFM?

Copupla	Canonical Maximum Likelihood					
	Parameters	Low - TD	Up - TD	LL	AIC	Rankings
Gaussian	0.2856	0	0	-85.028	-170.057	4
Clayton	0.3525	0.1399	0	-72.858	-145.719	5
R.Gumbel	1.1211	0.2276	0	-86.759	-173.518	3
T	0.2891/ 9.4136	0.0367	0.0367	-95.483	-190.965	1
SJC	0.1446/ 0.0994	0.1446	0.0994	-94.603	-189.204	2

Copula	Inference Functions for Margins					
	Parameters	Lo - TD	Up - TD	LL	AIC	Rankings
Gaussian	0.2864	0	0	-85.514	-171.029	4
Clayton	0.3594	0.1453	0	-75.401	-145.921	5
R.Gumbel	1.2182	0.2334	0	-86.445	-172.891	-3
T	0.2987/ 9.1097	0.0413	0.0413	-96.742	-193.483	1
SJC	0.1473/ 0.1109	0.1473	0.1109	-95.049	-190.098	2



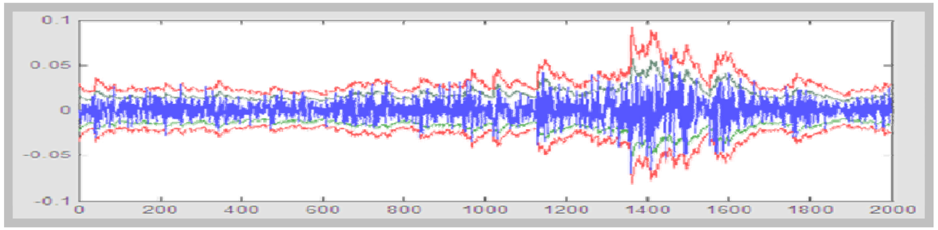
L.tail: 2.515,-14.352,-3.092; U.tail: -1.227, -6.008; 4.177; LL:-187.56



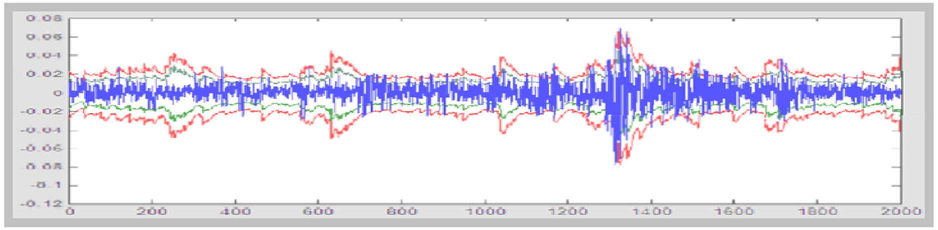
L.tail: 0.077, 0.525, -0.970; LL:-114.25

Value-at-Risk

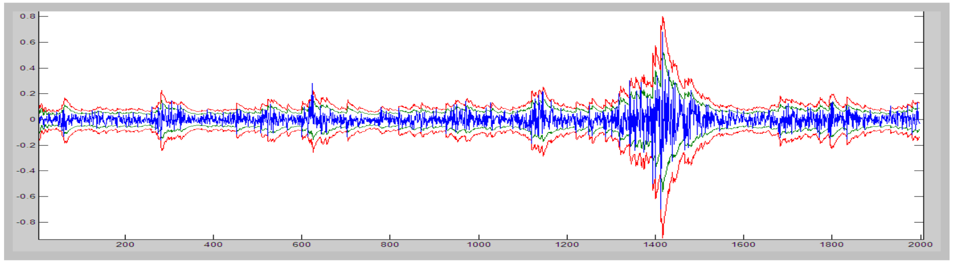
WIG20-BUX



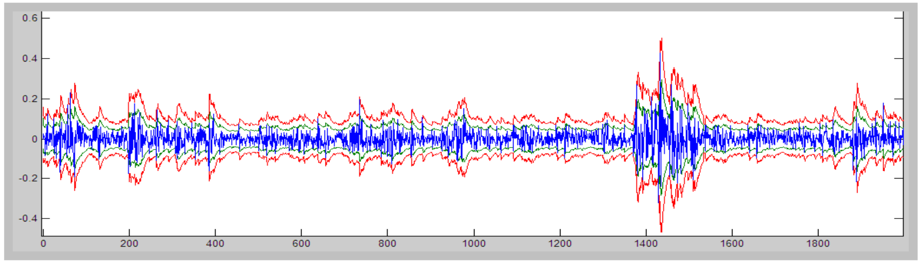
WIG20-PX



PX-BUX



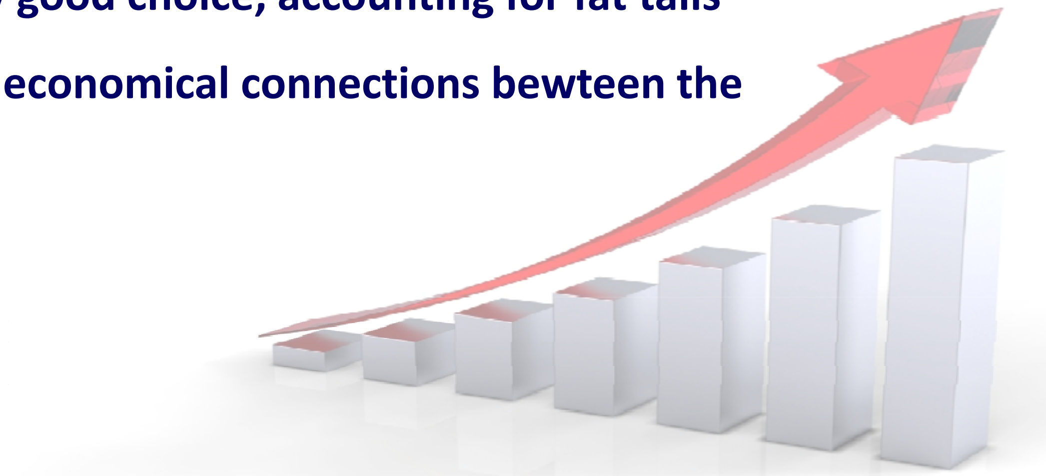
PX-BET



Portfolio	Bernoulli *Traffic Light* Backtest Time Varying SJC Copula			
	In Sample		Out of Sample	
	1%	5%	1%	5%
WIG-BUX	1.05%	5.25%	0.45%	4.84%
WIG-PX	1.16%	6.75%	1.26%	6.31%
PX-BUX	1.01%	5.96%	0.81%	3.96%
PX-BET	0.92%	4.71%	0.44%	3.52%

VII. Conclusions

- EV framework provides a very good fit for my data
- Depending on the macroeconomic framework of each country, each index has different characteristics captured by the semiparametric fit
- Strong positive dependence among the four indices
- WIG20 – the Polish Index – biggest market, greatest influence
- BET presents the weakest dependence structure because of the financial instabilities
- Asymmetry is sustained by all results
- T-Copula and SJC prove to be a very good choice, accounting for fat tails
- TV Copulas offer a reflection of the economical connections bewteen the analyzed markets



Thank you very much
for your attention!

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