The Academy of Economic Studies
The Faculty of Finance, Insurance, Banking and Stock Exchange

DOCTORAL SCHOOL OF FINANCE AND BANKING

PRE AND POST SUBPRIME CRISIS PRACTICES FOR PRICING INTEREST RATE DERIVATIVES

- Evidence from the cap – floor market -

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June 2012

Objectives & Motivation

- Empirically analyze the changes occurred during the subprime crisis in the derivatives and interest rate markets and highlight the new approach of pricing interest rate derivatives, recently adopted by the markets

- Evidence the new approach by comparing the pricing performance of three interest rate models (the Hull-White Model, the Two Factor Gaussian Model and the Libor Market Model), calibrated to cap prices for the European market

- Test which type of model, Gaussian or lognormal, provides the best pricing accuracy. Investigate if multifactor short-rate models provide more accurate pricing than one factor models.

Evolution of Interest Rate Models

- **The early days** - Black and Scholes (1973), Black (1976) and Merton (1973)

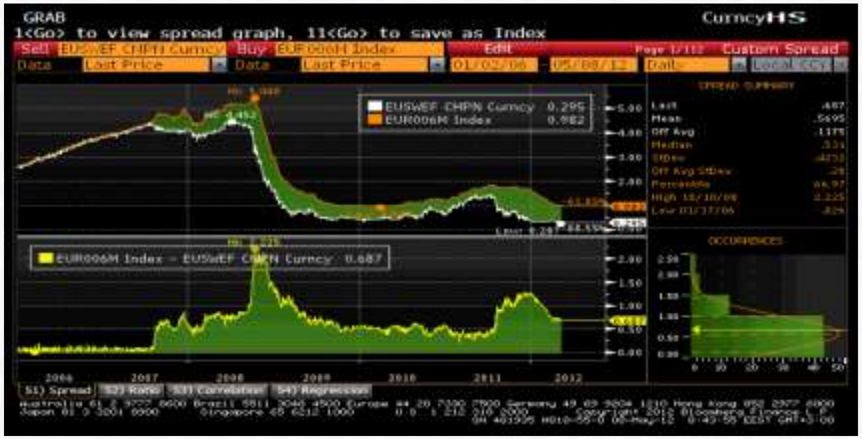
- **The one factor short-term models –** Vasicek (1977) and Cox, Ingersoll and Ross (1985), Hull and White (1990)

- **The two factor short-rate models** – The two-additive-factor Gaussian model (G2++), The two-additive-factor Extended CIR model (CIR2++), Heath, Jarrow and Morton (HJM) (1992)

- **The Market Models –** The Libor Market Model (Brace, Gatarek and Musiela) (1997)

Interest Rate Market evolution

•The subprime crisis triggered the explosion of the basis spreads



EUR 6M OIS rates vs. Euribor 6M rates (source Bloomberg)

OIS forward rates 6x12 vs. EUR 6x12 Euribor FRA (source Bloomberg)

EUR Basis Swap Euribor 3M vs. Euribor 6M, maturity 5Y (source Bloomberg)

The new pricing approach

- Such evolution of different interest rates has triggered a general reflection about the methodology used to price and hedge interest rate derivatives. The market passed from the *single – curve* approach to the *multiple – curve* framework.

- Ametrano and Bianchetti (2009) mention that the asymmetries cited above have also induced a sort of "segmentation" of the interest rate market into sub-areas, mainly corresponding to instruments with 1M, 3M, 6M, 12M underlying rate tenors, characterized by different internal dynamics, liquidity and credit risk premia, reflecting the different views and interests of the market players.

The new pricing approach

The following modified working procedure can be summarized as follows:

1)build *one discounting curve* $\boldsymbol{C_d}$ using the preferred selection of vanilla interest rate market instruments and bootstrapping procedure;

2)select *multiple separated* sets of vanilla interest rate instruments traded in real time on the market with increasing maturities, each set *homogeneous* in the underlying rate (typically with 1M, 3M, 6M, 12M tenors);

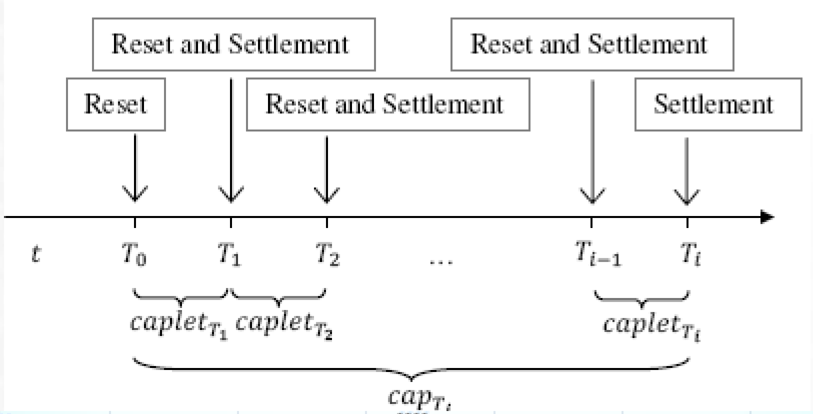
3)build *multiple separated forwarding curves* $\boldsymbol{C_{f1},\ldots, C_{fn}}$ using the selected instruments plus their bootstrapping rules;

4)compute *on each forwarding curve* the forward rates and the corresponding cash flows relevant for pricing derivatives on the *same* underlying;

5)compute the corresponding discount factors $P_d(t,T_i)$ using the discounting curve $\boldsymbol{C_d}$ and work out prices by summing up the discounted cash flows;

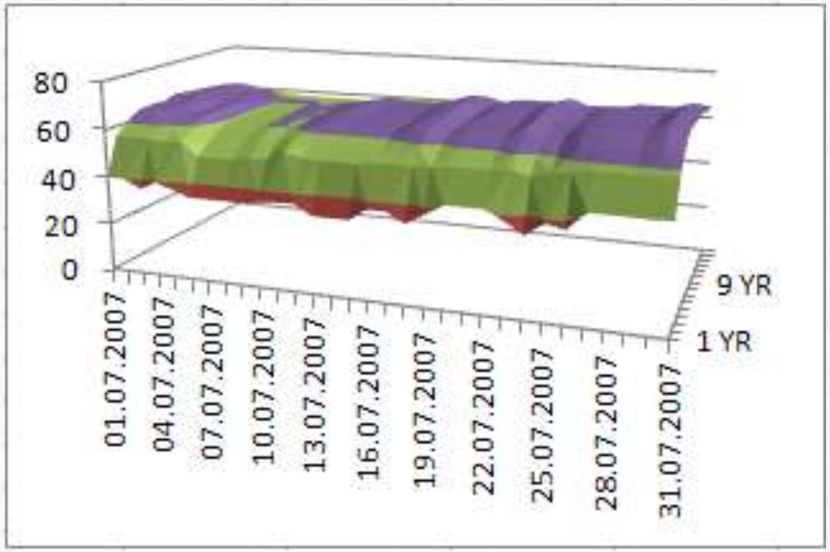
Cap – Floor Market

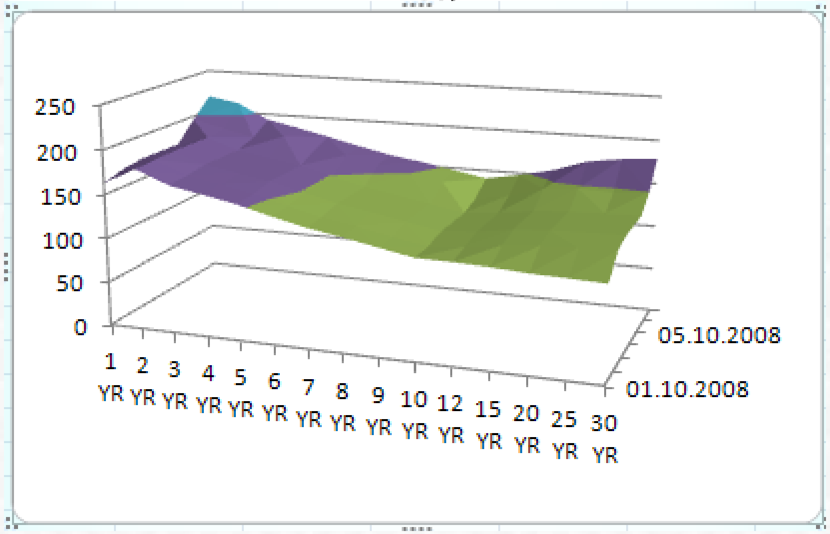
An **interest rate cap** is a derivative in which the buyer receives payments at the end of each period in which the interest rate exceeds the agreed <u>*strike price*</u>.



Cap Volatility Structure

ATM Caps on Euribor 6M





The data

The set of data consists in:

- Euribor fixing rates and Swap on Euribor 12M fixing rates, daily observations for March 2012, used to build the spot curve for the “single-curve” methodology.
- Euribor fixing rates, FRA on Euribor 6M rates and Swap on Euribor 6M rates, daily observations for March 2012, used to build the spot curve (for forwarding), in the “multiple-curves” methodology
- Lognormal (“implied”, “flat”) volatilities (measured in %) and strikes for Caps ATM and for 1,5% and 2,5% strike levels, indexed on Euribor 6M, for maturities ranging from 1 year to 10 years, daily observations for March 2012, used to calibrate the models

Data were obtained using Bloomberg and Reuters. Several applications have been applied in order to conduct the empirical study. The models and the empirical framework were implemented using MATLAB 2009 and the regression was implemented using EViews 6.

Comparison Classical vs. Modern Approach

Classical Methodology (Single-Curve)	Modern Methodology (Multiple-Curves)

$$CF^{Black}(t;T,K,w)=\sum_{k=2}^{m} cf(t;T_{k-1},T_k,K,w)$$

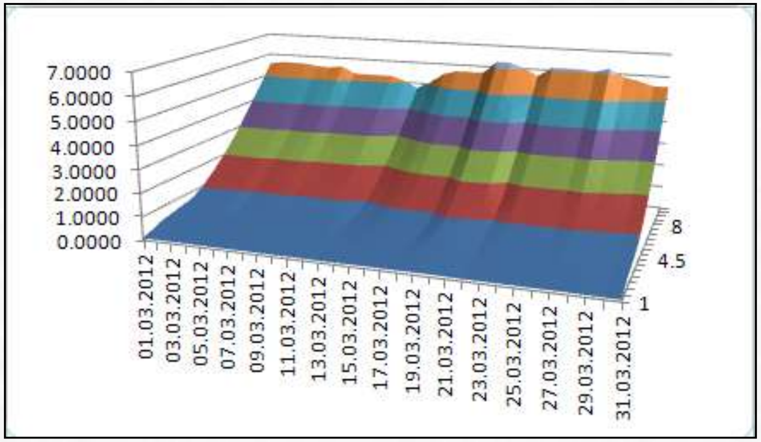
$$CF^{Black}(t;T,K,w)=\sum_{k=2}^{m} cf(t;T_{k-1},T_k,K,w)$$

$$cf(t;T_{k-1},T_k,K,w)=NwP(t,T_k)[F_k(t)\phi(wd_k^+)-K\phi(wd_k^-)]\tau_k,$$

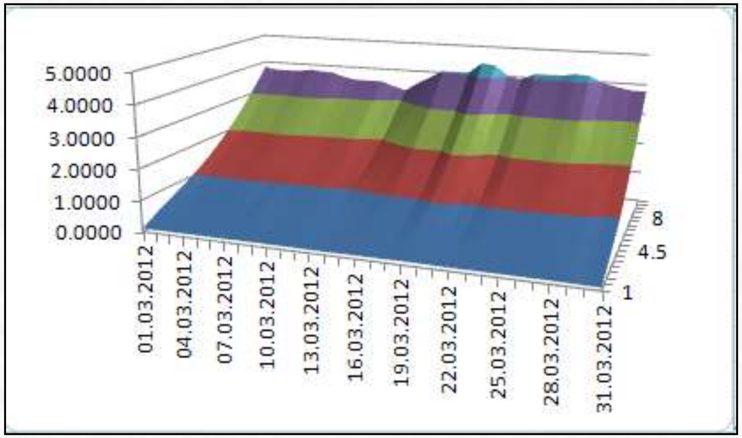
$$cf(t;T_{k-1},T_k,K,w)=NwP_d(t,T_k)\left[\tilde{F}_{x,k}(t)\phi(wd_k^+)-K\phi(wd_k^-)\right]\tau_{x,k}$$

$$d_k^{\pm}=\frac{\ln\left(\frac{F_k(t)}{K}\right)\pm\frac{1}{2}\sigma_k(t)^2\tau_{k-1}(t)}{\sigma_k(t)\sqrt{\tau_{k-1}(t)}}$$

$$d_k^{\pm}=\frac{\ln\left(\frac{\tilde{F}_{x,k}(t)}{K}\right)\pm\frac{1}{2}\tilde{\sigma}_{x,k}(t)^2\tau_{x,k-1}(t)}{\tilde{\sigma}_{x,k}(t)\sqrt{\tau_{x,k-1}(t)}}$$



Cap ATM price march 2012 – One Curve



Cap ATM price march 2012 – Two Curves

Pricing interest rate caps with Hull - White

The evolution of the short rate: $dr(t) = [\vartheta(t) - ar(t)]dt + \sigma dW(t)$

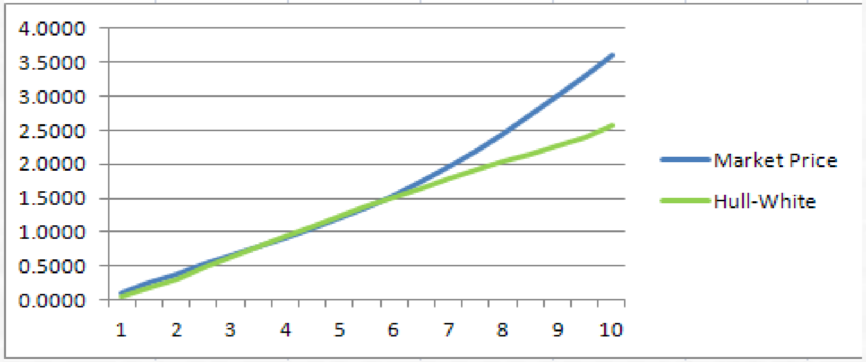
Cap pricing formula:
$$\boldsymbol{Cap}(t,T,N,X) = N\sum_{i=1}^{n}(1+X\tau_i)\boldsymbol{ZBP}\left(t,t_{i-1},t_i,\frac{1}{1+X\tau_i}\right)$$
$$\boldsymbol{Cap}(t,T,N,X) = N\sum_{i=1}^{n}\left[P_d(t,t_{i-1})\Phi(-h_i+\sigma_p^i) - (1+X\tau_i)P_d(t,t_i)\Phi(-h_i)\right]$$

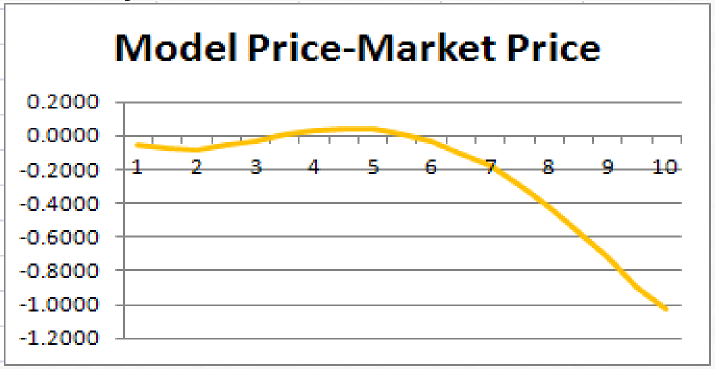
Where

$$\sigma_p^i = \sigma\cdot\sqrt{\frac{1-exp\left(-2a(t_{i-1}-t)\right)}{2a}}\cdot B(t_{i-1},t_i)$$

$$h_i = \frac{1}{\sigma_p^i}\ln\frac{P_d(t,t_i)(1+X\tau_i)}{P_d(t,t_{i-1})} + \frac{\sigma_p^i}{2}$$

Calibration of the Hull-White model to market prices for one day





Pricing interest rate caps with G2++

The evolution of the short rate: $r(t) = x(t) + y(t) + \varphi(t)$

$$dx(t) = -ax(t)dt + \sigma dW_1(t)$$
$$dy(t) = -by(t)dt + \eta dW_2(t)$$

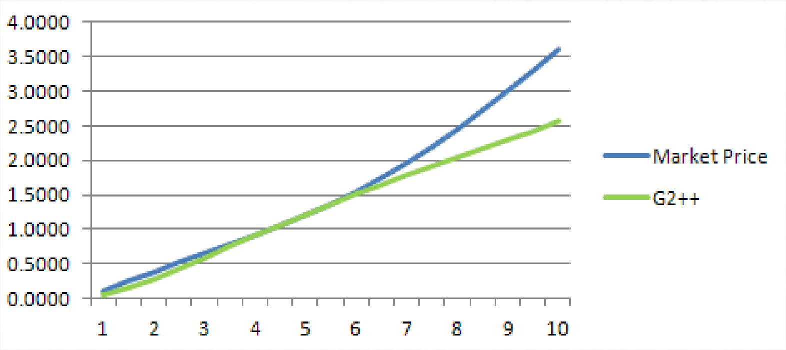
Cap pricing formula:

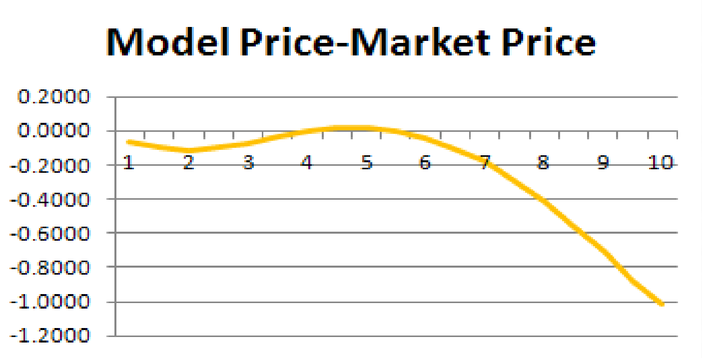
$$\boldsymbol{Cap}(t,T,\tau,N,X) = \sum_{i-1}^{n} \boldsymbol{CPL}(t,T_{i-1},T_i,N,X)$$

$$\Sigma(t,T_{i-1},T_i) = \frac{\sigma^2}{2a^3}\left[1-e^{-a(T_i-T_{i-1})}\right]^2\left[1-e^{-2a(T_i-t)}\right] + \frac{\eta^2}{2b^3}\left[1-e^{-b(T_i-T_{i-1})}\right]^2\left[1-e^{-2b(T_i-t)}\right] + 2\rho\frac{\sigma\eta}{ab(a+b)}\left[1-e^{-a(T_i-t)}\right]\left[1-e^{-b(T_i-t)}\right]\left[1-e^{-(a+b)(T_i-t)}\right]$$

$$\boldsymbol{Cpl}(t,T_1,T_2,N,X) = -N\left(1+X\alpha(T_1,T_2)\right)P_d(t,T_2) \cdot \Phi\left(\frac{\ln\frac{P_d(t,T_1)}{\left(1+X\alpha(T_1,T_2)\right)P_d(t,T_2)}}{\Sigma(t,T_1,T_2)} - \frac{1}{2}\Sigma(t,T_1,T_2)\right) + P_d(t,T_1)N\Phi\left(\frac{\ln\frac{P_d(t,T_1)}{\left(1+X\alpha(T_1,T_2)\right)P_d(t,T_2)}}{\Sigma(t,T_1,T_2)} + \frac{1}{2}\Sigma(t,T_1,T_2)\right)$$

Calibration of the G2++ model to market prices for one day





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Pricing interest rate caps with Libor Market Model

Exponential Volatility Parameterization:

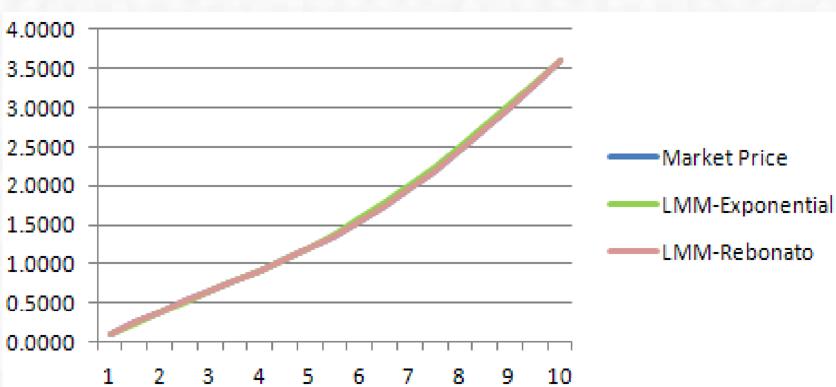
$$\sigma_i(t) = a \cdot e^{-c(T_{i-1}-t)}$$

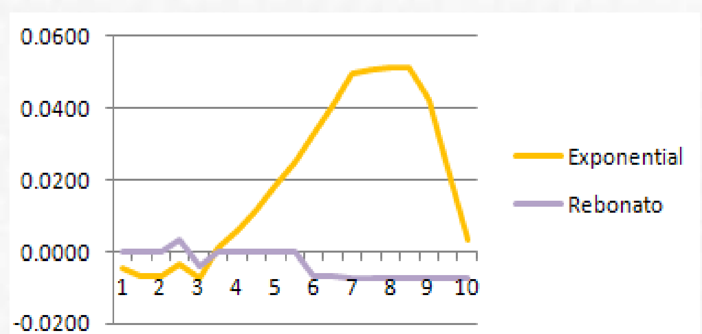
Rebonato Volatility Parameterization:

$$\sigma_i(t) = [a(T_{i-1}-t)+b] \cdot e^{-c(T_{i-1}-t)}$$

In order to obtain the prices of caps for LIBOR Market Model, we used the following procedure: having quoted by the market the caps volatilities, we computed caps market prices, and then caplet market prices. By using the Black formula, we calculated the market caplets volatilities. We performed the calibration of the LIBOR Market Model and thus we had the model caplets volatilities and going on we computed the caplets and then caps prices of the model.

Calibration of the LMM model to market prices for one day





Out of sample estimation

- In order to calibrate the three models to market prices, an out-of-sample estimation using the Least Squares Method was performed.

  - The method of least squares assumes that the best-fit curve of a given type is the curve that has the minimal sum of the deviations squared from a given set of data.

  - Suppose that the data points are $(x_1,y_1), (x_2,y_2), \ldots, (x_n,y_n)$ where x is the independent variable and y is the dependent variable. The fitting curve $f(x)$ has the deviation d from each point, i.e. $d_i = y_i - f(x_i)$.

  - According to the method of least squares, the best fitting curve has the property that:

  - $\prod = d_1^2 + d_2^2 + \ldots + d_n^2 = \sum_{i=1}^{n} d_i^2 = \sum_{i=1}^{n} [y_i - f(x_i)]^2$ = a minimum

  - **SSR** = min $\sum_{i=1}^{n} (Cap_{i,model} - Cap_{i,market})^2$

Out of sample estimation

- The pricing performance of different interest rate models is examined by using out-of-sample pricing errors. The models are compared on both **time-dimensional** (lagged model parameters) out-of-sample pricing and **cross-sectional** (across moneyness) out-of-sample pricing.

- **Time dimensional**

1) Back out the implied model parameters on date *T* by calibrating the model to market prices of caps at date *T*.
2) On the future date (*T*+*h*) apply the same model together with the parameters obtained in step 1 and the interest rate structure from the date (*T*+*h*) to calculate the predicted model prices of the model on that date
3) Compute the out of sample pricing errors by subtracting the market prices on date (*T*+*h*) from the predicted model prices estimated in step 2.
4) Repeat step 1 to 3 for each option on each date in the dataset to compute the average out of sample pricing errors.

Out of sample estimation

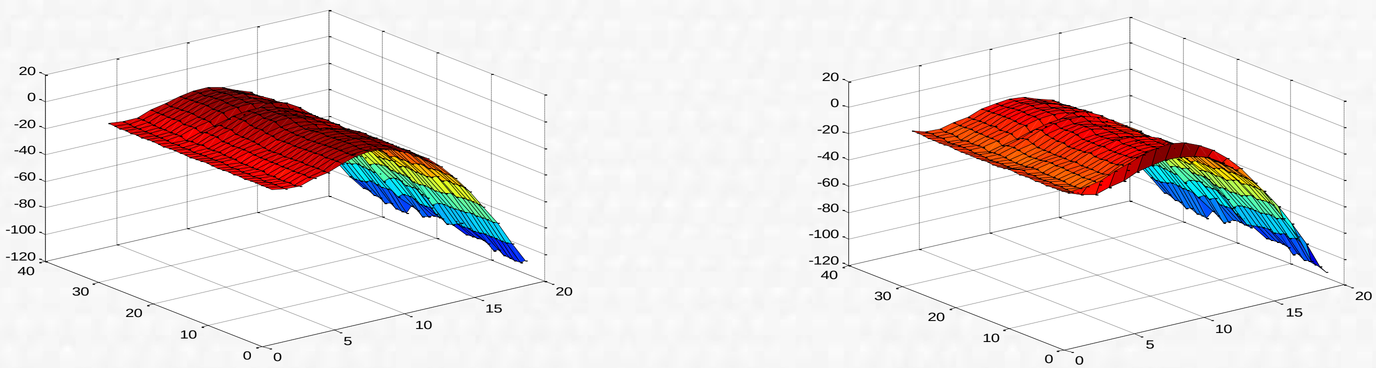
- **Cross-sectional**

1) Back out the implied model parameters on date *T* by calibrating the model to market prices of at-the-money caps at date *T*.
2) Apply the same model on the same date *T* to price away-from-the-money interest rate caps of all maturities.
3) Compute the cross-sectional pricing errors by subtracting the market prices on date *T* from the model prices estimated in step 2.
4) Repeat step 1 to 3 for each option on each date in the dataset to compute the average cross-sectional pricing errors.

Out of sample estimation – Time dimensional results

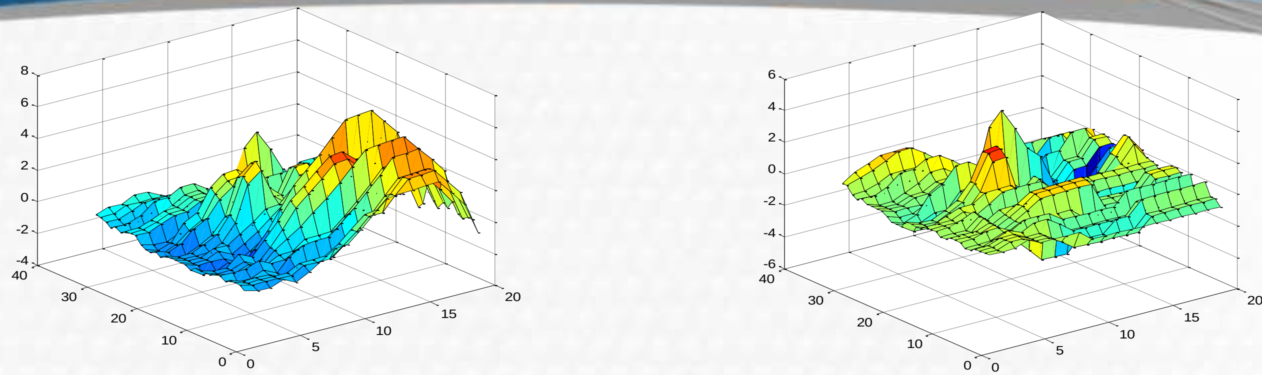
Monthly average errors in basis points, for ATM caps with maturities between 1 year and 10 years.

	1	2	3	4	5	6	7	8	9	10
Hull-White	-5.463	-7.284	-1.589	4.575	3.902	-3.828	-19.840	-41.855	-70.654	-101.036
G2++	-6.514	-11.086	-6.133	0.761	1.443	-4.832	-19.543	-40.499	-68.488	-100.963
LMM-Exp	-0.303	-0.419	-0.127	0.913	0.962	1.505	1.617	2.037	2.000	0.243
LMM-Reb	-0.139	-0.106	0.061	0.591	-0.200	-0.604	-1.288	-0.967	-0.372	-0.054



Monthly average calibration errors in basis points for Hull-White (left) and G2++ (right) models.

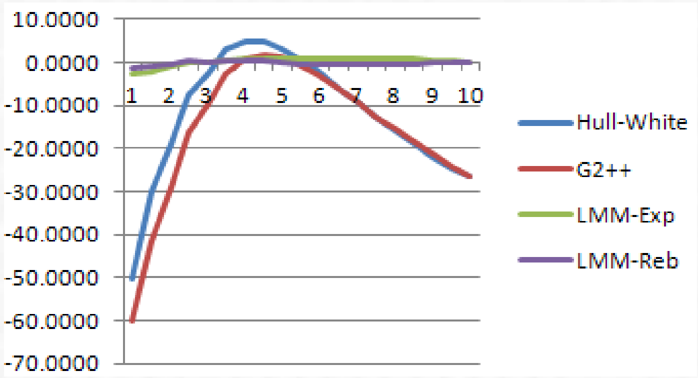
Out of sample estimation – Time dimensional results



Monthly average calibration errors in basis points for the LMM Model – Exponential (left) and Rebonato (right) parameterization

	1	2	3	4	5	6	7	8	9	10
Hull-White	-50.0755	-19.1777	-2.4414	4.9356	3.0953	-2.3124	-9.3835	-15.8983	-22.0029	-26.2976
G2++	-59.8062	-29.3299	-9.5258	0.7780	1.1349	-2.9243	-9.2478	-15.3896	-21.3338	-26.2727
LMM-Exp	-2.7074	-1.0806	-0.1916	0.9682	0.7795	0.9614	0.8299	0.8191	0.6431	0.0598
LMM-Reb	-1.2026	-0.2516	0.1154	0.6411	-0.1380	-0.3377	-0.5767	-0.3460	-0.1112	-0.0184

Monthly average percentage errors, for caps with maturities between 1 year and 10 years.

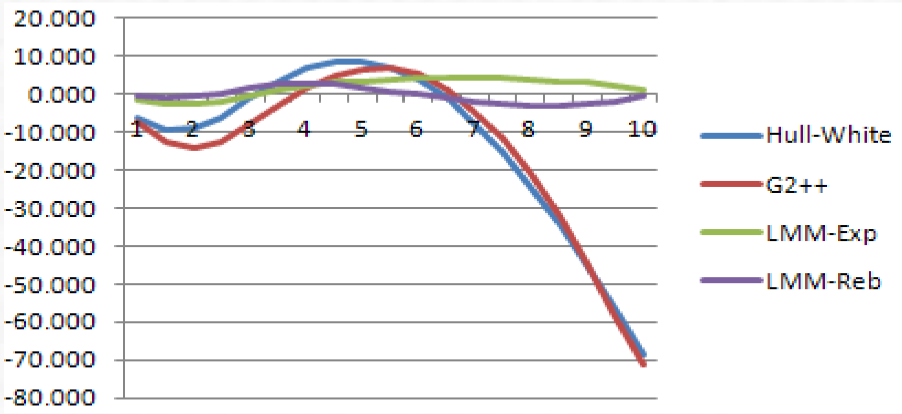


Out of sample estimation – Cross sectional results

Below are listed the cross-sectional pricing errors. The estimation was performed for 1,5% and 2,5% strike levels for one day.

Result for 1,5% strike level – measured in basis points

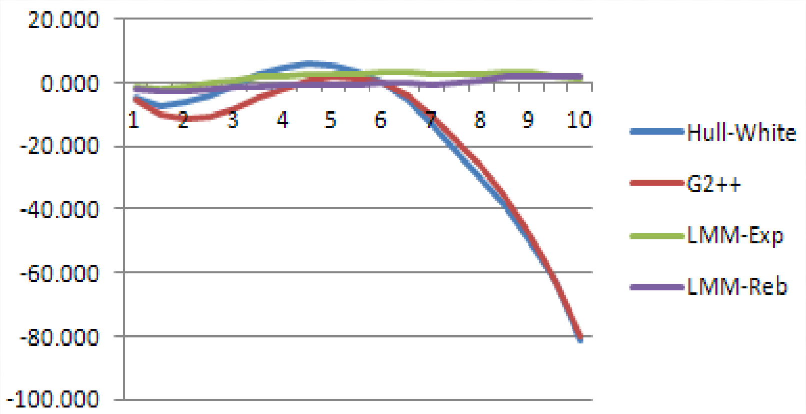
	1	2	3	4	5	6	7	8	9	10
Hull-White	-6.033	-8.971	-1.172	6.767	8.730	4.030	-7.674	-24.483	-44.924	-68.648
G2++	-7.076	-14.022	-7.829	1.527	6.594	5.134	-4.479	-21.263	-44.320	-71.242
LMM-Exp	-1.373	-2.770	-0.345	2.252	3.355	4.234	4.211	3.831	3.078	1.343
LMM-Reb	-0.497	-0.692	1.701	2.940	1.740	0.097	-1.861	-2.870	-2.367	-0.588



Out of sample estimation – Cross sectional results

Result for 2,5% strike level level – measured in basis points

	1	2	3	4	5	6	7	8	9	10
Hull-White	-4.613	-6.303	-0.789	4.816	5.402	-0.217	-13.244	-29.925	-50.146	-81.199
G2++	-5.303	-11.299	-8.279	-1.753	1.955	-0.044	-10.406	-26.401	-48.528	-79.739
LMM-Exp	-1.389	-1.023	0.720	2.310	2.938	3.518	2.700	3.014	3.186	1.325
LMM-Reb	-1.818	-2.451	-1.640	-0.765	-0.530	0.033	-0.379	0.790	2.291	2.260



For the cross sectional out of sample estimation, based on the above results, we cannot formulate a general rule about the evolution of the errors when increasing the strike level. For the 1,5% strike level, all the three models present slightly increased errors, in absolute value, than for the ATM caps, while for 2,5% strike level, the performance of the models improves, the errors being equal or even smaller, in absolute value, than for the ATM caps.

Test of prediction ability

- In order to test the prediction ability of the models and to investigate possible systematic biases of the different models when pricing interest rate caps out-of-sample, we run the following regression:

$$Cap_{market,i} = \beta_0 + \beta_1 * Cap_{model,i} + \varepsilon_i$$

- The results from the regressions, based on data of ATM interest rate caps, indicate that the smallest out-of-sample pricing errors and hence the least model misspecification appears in the lognormal model.

1 YEAR CAP	β_0	β_1	R^2	t-stat
Hull-White	0.0178	0.9995	0.6128	1.7668
G2++	-0.0373	0.9882	0.8863	-1.1835
LIBOR -Rebonato	0.0406	0.9989	0.9690	3.9551
5 YEARS CAP	β_0	β_1	R^2	t-stat
Hull-White	-0.0313	1.0559	0.9925	-1.4607
G2++	-0.0422	1.0451	0.9155	-0.5685
LIBOR -Rebonato	0.0780	0.9364	0.9800	2.4918
10 YEARS CAP	β_0	β_1	R^2	t-stat
Hull-White	-0.1085	1.0356	0.9831	-5.6909
G2++	-0.9638	1.0071	0.9803	-4.5716
LIBOR -Rebonato	-0.0034	0.9854	0.9995	-2.0353

Conclusions

- In this paper we empirically analyzed the practices for pricing interest rate derivatives used before and after the subprime crisis. The results show us that banks and financial brokers should pay more attention to the changes occurred in the markets after the crisis, because the difference between the two methodologies is no longer negligible.
- For the time dimensional out of sample estimation, we can conclude that the results for one day anticipate those for one month, so increasing the timescale does not alter the results. The pricing errors show that Hull-White and G2++ models undervalue the prices for longer maturities, while LIBOR Market Model provides the best pricing estimation in this case. Also this estimation proves that in same cases multifactor models cannot provide more accurate pricing than their single factor equivalents.
- For the cross sectional out of sample estimation, we cannot formulate a general rule about the evolution of the errors when increasing the strike level. For the 1,5% strike level, all the three models present slightly increased errors, in absolute value, than for the ATM caps, while for 2,5% strike level, the performance of the models improves, the errors being equal or even smaller, in absolute value, than for the ATM caps.
- The results from the regressions based on data of ATM interest rate caps, indicate that the smallest out-of-sample pricing errors appear in the LIBOR Market Model.

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Thank you !