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Resampled Mean-Variance for Romanian Stock Exchange

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Paper outline

- **Introduction**
- **Literature review**
- **Model description**
- **Data and results**
- **Conclusions**

Introduction

- This paper compares two models for asset allocation. The first one is the simple Markowitz model, while de second one is the model proposed by Michaud, the resampled efficient frontier.
- There is a lot of debate over which model is better when making investment decisions
- Relevant to understand the underlying assumptions on which these models are based, especially given nowadays financial environment

Literature review

- Markowitz (1952) proposes a model on how an investor should allocate his money between different assets in such a way that for a given level of wealth the risk is minimized.
- Kallberg and Ziemba (1994) examine the misspecification in normally distributed portfolio selection problems.
- Best and Grauer (1991) present some empirical and theoretical results on how sensitive a portfolio selection is to changes in the means of the assets.
- Chopra and Ziemba (1993) found that especially estimation errors in expected returns have a strong impact on portfolio allocation.
- Michaud (1998) proposes a new asset allocation method. Based on Markowitz model he introduces Monte Carlo simulation and suggests that the resampled frontier is better by being more efficient and by eliminating estimation error.
- Michaud and Michaud (2008) show in a new paper that the resampled efficiency leads to better outcomes when compared to the traditional approach.

Markowitz

- Diversification

- Assumptions:
  - Normal distribution
  - Quadratic utility function
  - Risk aversion and rational investors
  - Variance as a measure of risk
  - One Investment period

Markowitz

Two ways of obtaining the efficient frontier:

- Minimize the variance of the portfolios for a given range of returns specified from the very beginning;
- Maximize the expected return for a given level of variance for each portfolio.

From a mathematical perspective, in order to generate the frontier one needs to include two constraints which require that the portfolio return $w^T\mu$ equals π and that the sum of the portfolio weights equals one, the problem can be expressed as the following:

$$\begin{cases} \text{Min } w^T\Sigma w \\ w^T\mu = \pi \\ w^T I = 1 \end{cases}$$

Markowitz

- solving with Lagrangian

$$L = w^T\Sigma w + \lambda_1(\pi - w^T\mu) + \lambda_2(I - w^T I)$$

$$\begin{cases} \dfrac{dL}{dw} = 2\sum w - \lambda_1\mu - \lambda_2 I = 0 \\ \dfrac{dL}{d\lambda 1} = w^T\mu - \pi = 0 \\ \dfrac{dL}{d\lambda 2} = w^T I - 1 = 0 \end{cases}$$

- from the first equation above, we $w = \frac{1}{2}\lambda_1\Sigma^{-1}\mu + \frac{1}{2}\lambda_2\Sigma^{-1}I$ plug it in the last

two equations above, we have:

$$\begin{cases} \frac{1}{2}\lambda_1\mu^T\Sigma^{-1}\mu + \frac{1}{2}\lambda_2\mu^T\Sigma^{-1}I = \pi \\ \frac{1}{2}\lambda_1\mu^T\Sigma^{-1}I + \frac{1}{2}\lambda_2 I^T\Sigma^{-1}I = 1 \end{cases}$$

- Defining the following terms: $A = I^T\Sigma^{-1}I$, $B = \mu^T\Sigma^{-1}I$, $C = \mu^T\Sigma^{-1}\mu$ here A, B,

C are constants, and rewrite the above formula:

Markowitz

$$\begin{cases} \frac{1}{2}C\lambda_1 + \frac{1}{2}B\lambda_2 = \pi \\ \frac{1}{2}B\lambda_1 + \frac{1}{2}A\lambda_2 = I \end{cases}$$

- solve the equations above we have the values of two multipliers:

$$\lambda_1 = \frac{2(A\pi - B)}{AC - B^2} \qquad \lambda_2 = \frac{2(C - B\pi)}{AC - B^2}$$

- Plugging the two multipliers back to the expression of w,we have:

$$w(\pi) = \frac{\left(A\Sigma^{-1}\mu - B\Sigma^{-1}I\right)\pi + \left(C\Sigma^{-1}I - B\Sigma^{-1}\right)\mu}{AC - B^2}$$

Notice that the optimal portfolio weight vector is only a function of the absolute expected return π

The portfolio variance is thus:

$$w^T \Sigma w = \frac{A}{AC - B^2}\pi^2 - \frac{2B}{AC - B^2}\pi + \frac{C}{AC - B^2}$$

Markowitz

Practical criticism

- One important flaw that comes more from the practical implementation of the model is the instability of the input parameters in the mean variance model.
- In other words, in theory, mean variance model can work very well, however in practice we find that the portfolios that are mean-variance efficient are very unstable.
- The instability in the parameters has the biggest impact on the performance of the method in day to day life; By this instability, it is meant that very small changes in the input parameters mentioned above can lead to completely different structures of the portfolios that are on the efficient frontier and also different efficient frontiers.
- This is the main reason why Michaud proposed an alternative method, which is supposed to eliminate the effect of the instability of the initial parameters.

Resampled Efficient Frontier

- Motivated by the fact that the biggest impact on the model outcome is determined by the instability in the input parameters, Michaud (1998) proposed a method that would solve this issue. He came up with the *Resampled Efficient Frontier*.
- If it is supposed that the variance and the excess return are estimated using n number of observations, then it is possible to note that the point estimates are random variables which results from obtaining different estimates even if a sample of the same size and from the same distribution is used.
- In order to eliminate the instability or uncertainty in the investment information, the method introduced by Michaud employs Monte Carlo simulation into the optimization process.

Resampled Efficient Frontier

- ***Steps used to generate the Resampled Efficient Frontier.***

1. First of all one needs to estimate the three input parameters: the expected returns on each asset, the standard deviation for each asset and the variance-covariance matrix.
2. The second step would be to generate random numbers drawn from a standardized normal distribution for each asset we have. The length of the simulated path should be the same with the number of observations we have in case of real data. In our case we make 500 generated observations for each asset.
3. At this step we should make sure that the generated numbers through Monte Carlo simulation have the same statistical properties as the real ones by using the expected returns and standard deviation estimated on the real data. To do that we are using the following formula:

$$r_t = \mu + \sqrt{\sigma_t^2}\,\varepsilon_t$$

Resampled Efficient Frontier

4. At this step one has a simulated set of data similar to the real one. As a result it is possible calculate the expected mean, standard deviation and variance covariance matrix.
5. Having obtained the three inputs we can simply estimate the efficient frontier using Markowitz model. In this sense, we use the three estimated inputs in step 4 and obtain our weights to obtain the portfolios from the efficient frontier.
6. At this step one has to repeat everything between step 2 and 5 as many times as possible (in our case 500 simulations were done)
7. Having obtained 500 efficient frontiers from the simulated data, we take each portfolio and average the weights.
8. The obtained average weights, are applied on the input parameters that were estimated on the real data (step 1). In this way, it is possible to draw the Resampled Efficient Frontier. This then might be compared to the efficient frontier.

Resampled Efficient Frontier

- ***Advantages and disadvantages of using resampled efficient frontier***

- The main benefit of using the resampled efficient frontier should be the use of data in such a way that the allocations made to certain assets in a portfolio should be less sensitive to small modifications in the input parameters. This is because a very important aspect that Michaud is using is by “looking into the future”.
- If one has to talk about the drawbacks of the model, then the first one that comes in mind is that there is no theoretical framework on which it should be based.
- Another major drawback of the resampled efficient it that it might suffer from the problem it actually is intended to solve. If the initial estimated parameters are not the same as the real ones, than the estimation errors are increased even much more when we do estimate the resampled efficient frontier.

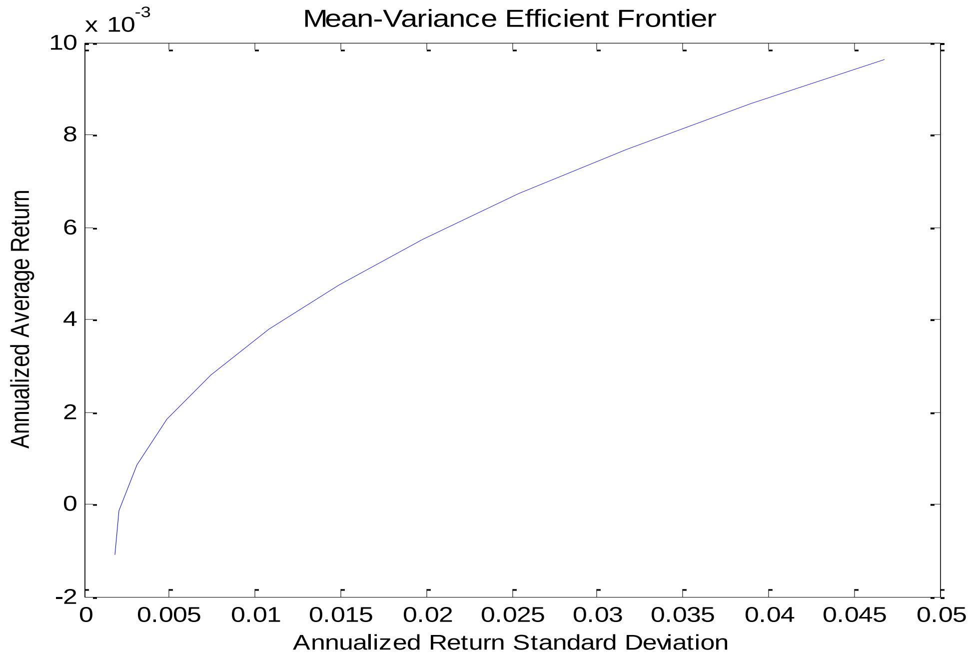
Data

- This paper uses data with a daily frequency on 45 stocks from the Romanian stock exchange. The period between which these data is extracted is from January 1st,2008 till December 31 2010. It was provided from the ING Investment Management and represents real investments of the asset manager in the above specified period.
- In order to conduct our study, 10 random portfolio were formed using the above stocks, each portfolio consisting of 15 assets. It was also divided into in-sample and out-of-sample. For the in-sample a period of 2 years was used while for the out-of-sample a year of data.

Results

1. The mean-variance model

One of the key elements from Markowitz's work is that the risk of a portfolio is usually less than the weighted average risk of the individual assets, and is the key to diversification. In the following figure a mean-variance efficient frontier is represented, and it is for one of the portfolios formed on our data. What I did, was to form portfolios of 10 assets and run the model on them.



Results

- One of the most important drawbacks of the mean variance model is represented by the uncertainty in the input parameters. There is extensive literature on the impact of small changes in the input parameters on the efficient frontier when data for the same asset is used. In effect, these small changes have a big impact on the estimated weights, and thus on the frontiers. The following figure is an exemplification of this.

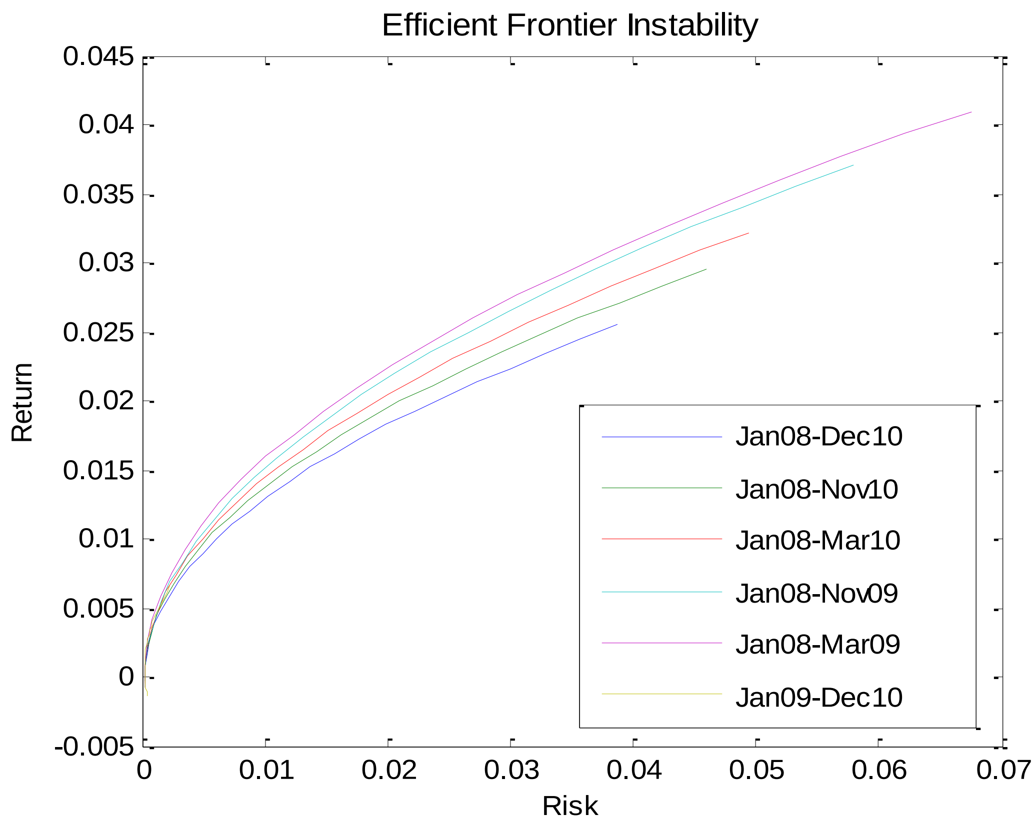


Figure 2. Instability of efficient frontier

Results

2. *The Resampled model*

In order to address the before mentioned flaw of the Markowitz model, Michaud comes with a new solution. His model suggests that by introducing Monte Carlo simulation one can eliminate the uncertainty in the investor's information. The following graph provides a practical explanation on how we do form a Resampled Efficient Frontier.

Results

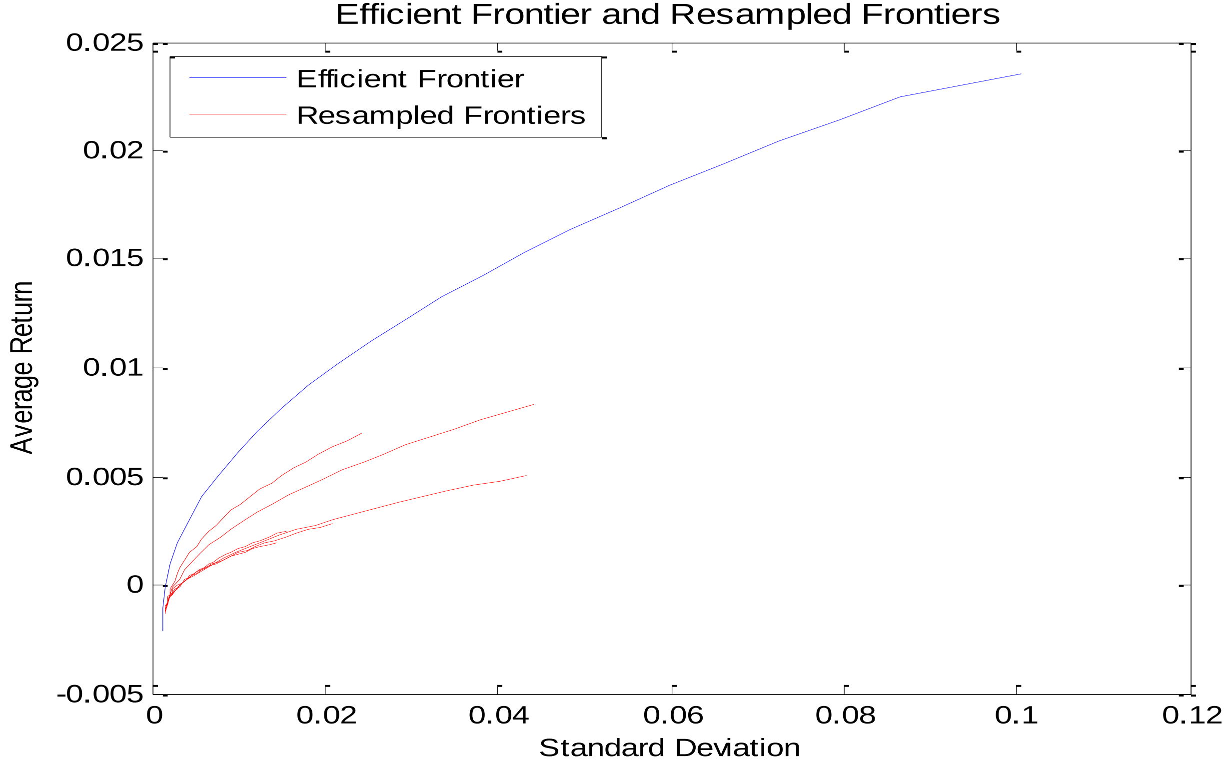
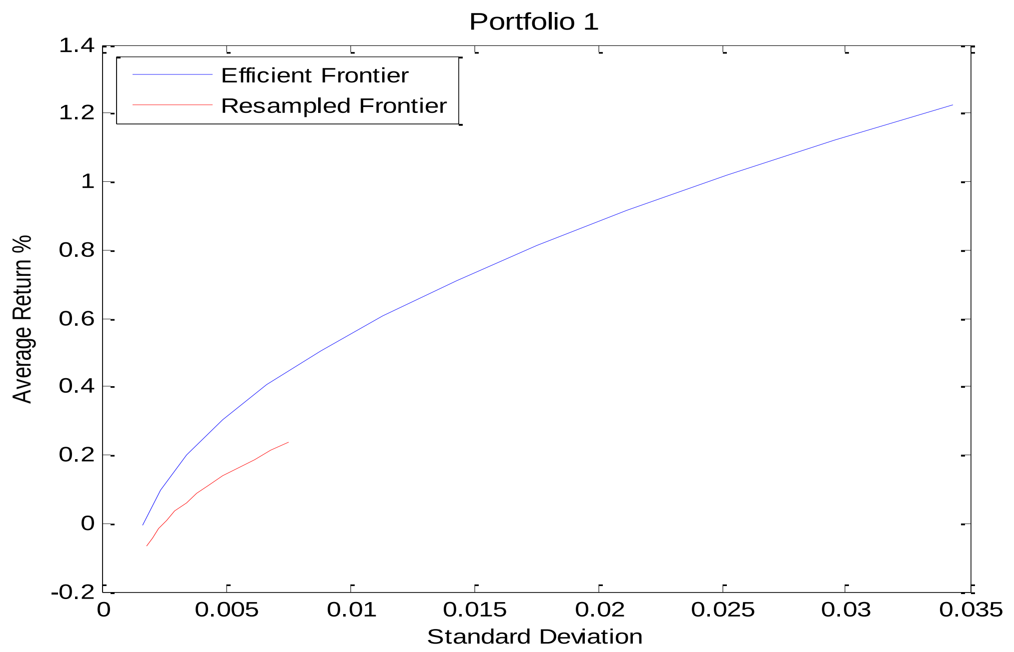


Figure 3. Efficient Frontier and Resampled Frontiers

What this graph displays are the alternative efficient frontiers that derive from the original data. So what I did was to compute the input parameters on the real data. Then using these estimated parameters, the formula in the methodology part and the Monte Carlo simulation, samples of future possible returns were generated.

Results

- Now following the previous explanation, it is possible to indicate how exactly the real resampled efficient frontier is obtained. The difference with the above example lies in the fact that instead of applying the obtained weights from one simulation straight on the real data, one has to take all weights obtained on different the different simulations and average them. Then, the obtained weights are applied on the real data. In our case, there have been used 500 simulations for the resampled efficient frontier.



- *Figure 4*: Markowitz vs. Michaud: in sample

Results

- As one can observe that the resampled frontier is to the left of the one generated using Markowitz model. As explained in the previous example on how Michaud's frontier is generated, the averaged weights, which were estimated on the simulations, are suboptimal for the real return compared the weights estimated on these real data.

- For the ten portfolios, the table below shows the results:

Portfolio	1	2	3	4	5	6	7	8	9	10
Best model	M	M	M	M	M	M	M	M	M	M

Table 1. Model performance in-sample

As it was expected, in each case, Markowitz model performed better than the resampled efficient frontier. According to Michaud, these results are very easy to interpret. At first glance, the resampled model looks inferior to the mean variance one. However, an investor should use the latter one only when he is very sure that the input parameters he has calculated are the true ones. In case he is not sure about that, then he should use the resampled technique. Nonetheless, the easiest way to see whether Michaud is right is to test his model against the mean variance one on the out-of-sample.

Results

- Out of sample results – three cases
- First, the Michaud model outperformed the Markowitz model and the results are illustrated in the figure below:

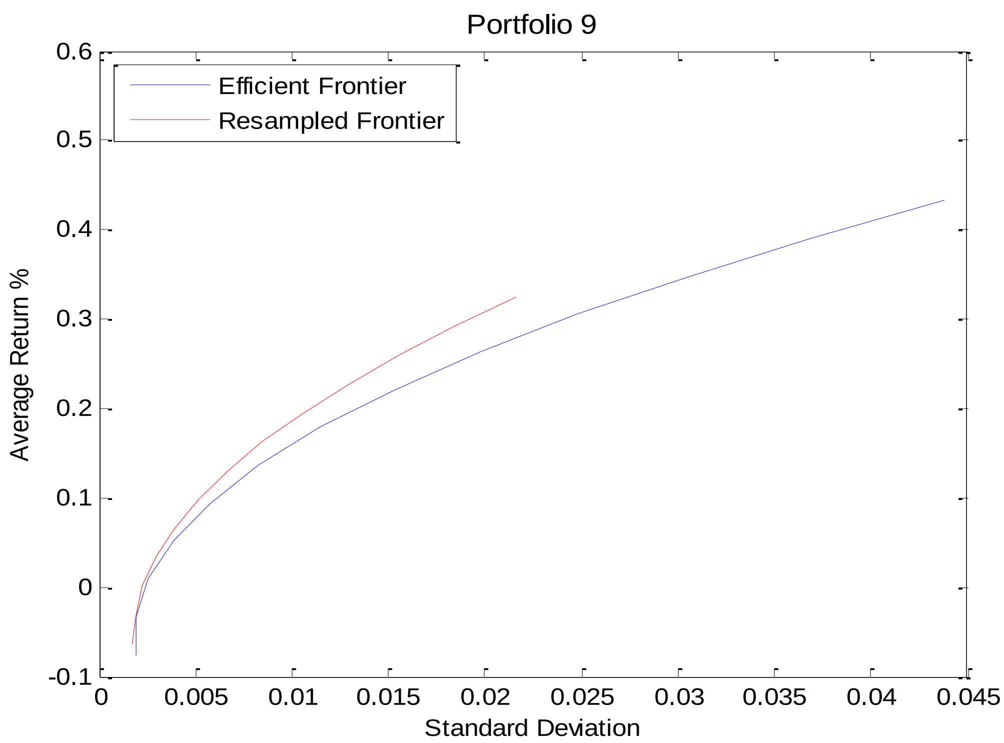


Figure 5:Markowitz vs. Michaud: out of sample

Results

- One can notice that the resampled frontier is to the left compared to the mean variance one. It means that for a given level of return, an investor that uses Michaud`s model would achieve less volatility than by using the other model. It is also a good opportunity to talk about statistical equivalence between portfolios. It is not necessarily that two portfolios that apply to this rule have the same expected return or standard deviation. Usually, these portfolios have returns and risk close to each other. However, as investor moves to higher returns, this divergence between these parameters appear to be more pronounced.
- The above result was as one might have expected. However, there are also cases when it is not possible to say which of the two models performs better:

Results

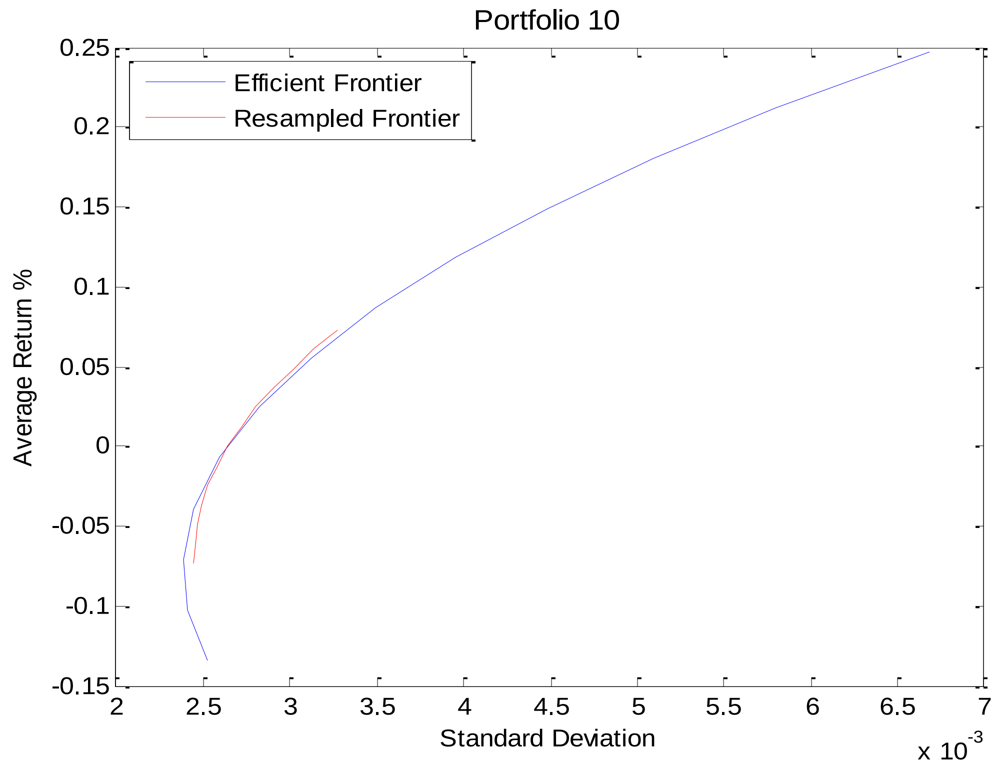


Figure 6:Markowitz vs. Michaud: out of sample

- The reader can see that frontiers are almost the same and that an investor would be indifferent between choosing one of the models. There is also a case where the resampled model underperforms for low returns and over performs for high returns compared to the other model.

Results

- The third case is when Michaud`s model actually is clearly inferior to the mean variance one. The below graph is similar to the one obtained in the in-sample:

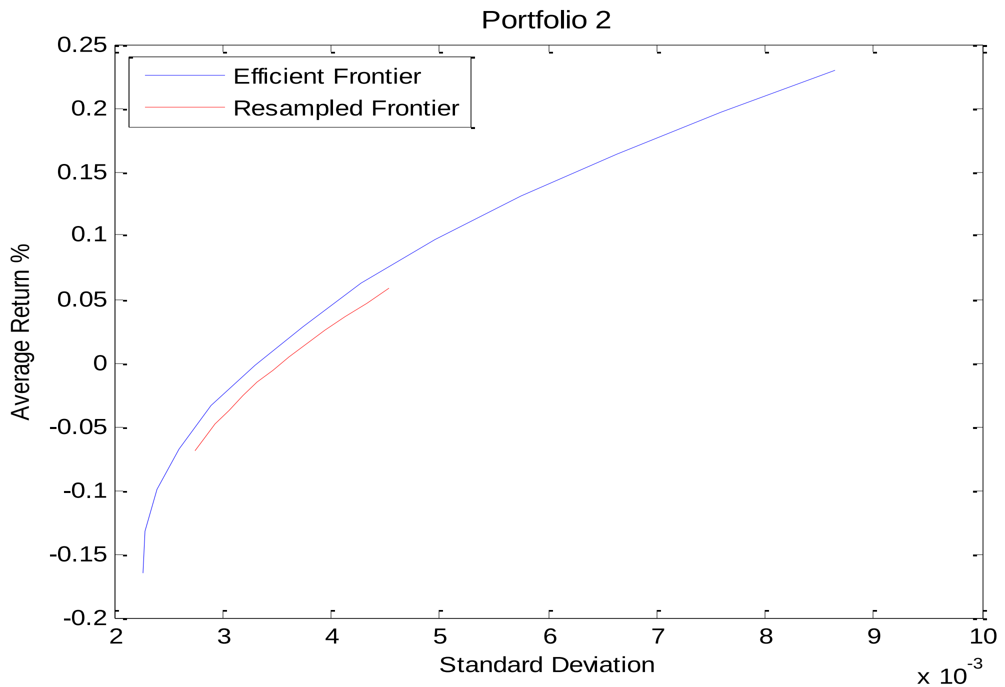


Figure 7. Markowitz vs. Michaud: in sample

Results

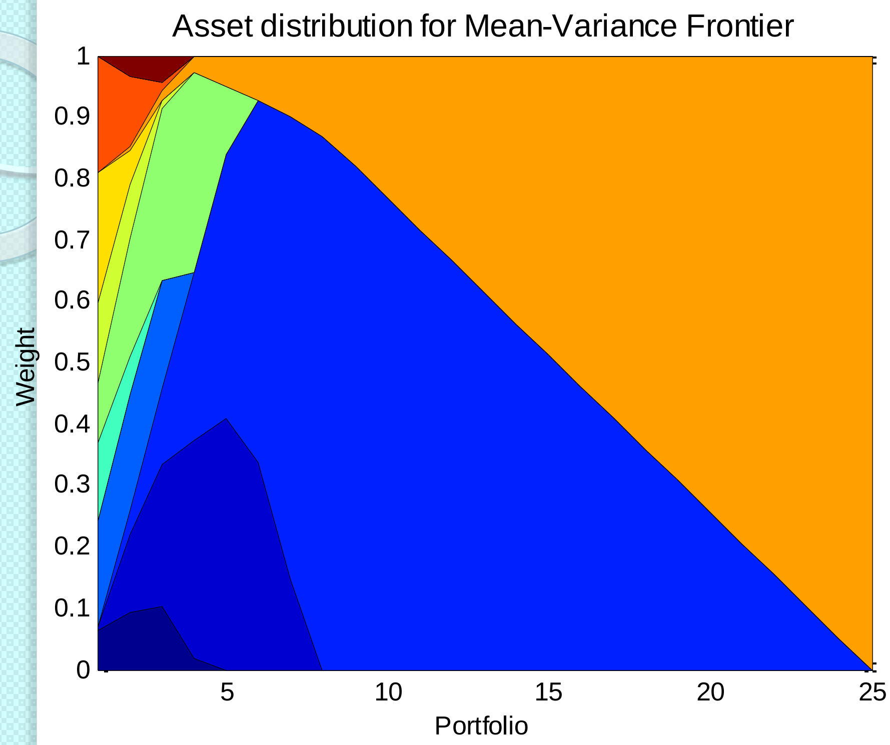
- We can notice that the model using Monte Carlo simulation does not necessarily perform better than the simple one. If we would look over all on which model is better for each portfolio, than these are the results:

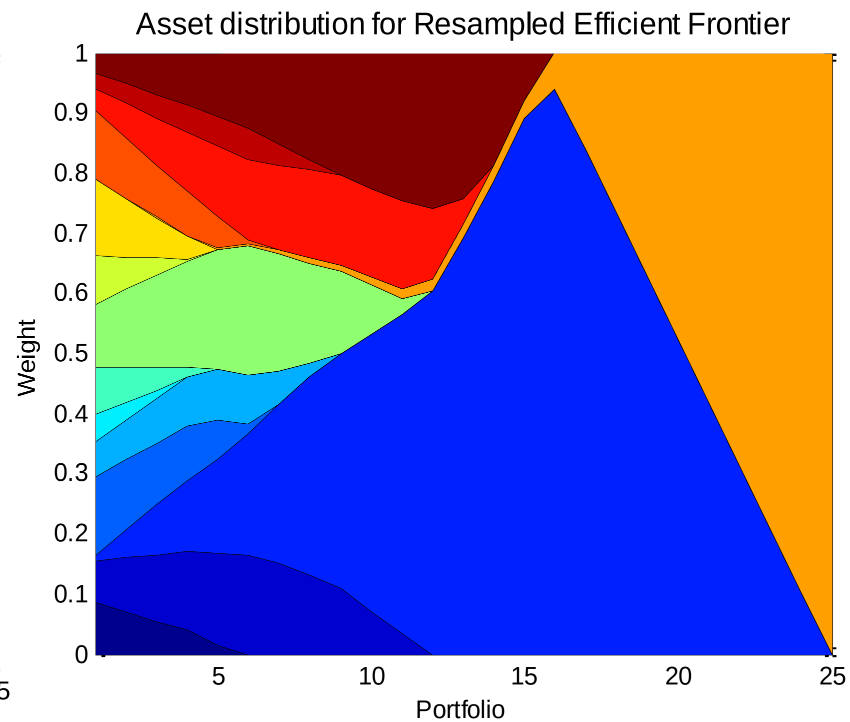
Portfolio	1	2	3	4	5	6	7	8	9	10
Best result	M	M	M	M	M	M	R	R	R	R

Table 2. Model performance out of sample

- It is important to mention that when computing resampled efficient frontier, simulations that were lying far away from the usual ones (outliers) were not eliminated as it is suggested by Michaud. The main thought behind this decision is because I do not find any reason why one should eliminate the outliers especially when we live in a volatile world in which sometimes variance, and not expected return, is the thing that counts.

Results





- This figures illustrate the portfolio composition differences. In case of MV model for the portfolio with lowest risk, only 9 stocks are included, while for the REF 14. As we move to the right, the number of stocks used in the second model decreases more smoothly than in the case of the efficient frontier. However, for the riskiest portfolios, REF is still allocating most resources to just a few stocks.

Conclusions

- This study analyzed two models for portfolio allocation. The first one is the Mean Variance Efficient Frontier model and the second one is the Resampled Efficient Frontier. Contrary to our expectations, the second model did not perform better than Markowitz model. It did not outperform it even on average, being inferior in out-of-sample 6 times out of 10. One of the reasons can be the fact that there is no theoretical reason on why Michaud model`s should actually be better than the mean variance one. Moreover, since it is actually build upon the mean variance one, one should not expect that it will outperform by far. Moreover, one of its pretended advantages, the model stability might actually suffer from the very initial estimated parameters.
- Suggestions for further research would be to eliminate the restrictive assumptions of the two models such as normality of returns and rationality of the investors. There should be done much work into developing a model that incorporates behavioral finance concepts.

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