Models and techniques for quantifying macroeconomic risk

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Summary

- 1. Introduction
- 2. Motivation
- 3. Main Literature
- 4. Methodology
- 5. Data
- 6. Results
- 7. Concluding remarks

Introduction

- During times of financial crises, losses tend to spread across financial institutions, threatening the system as a whole;
- The spread of distress gives rise to systemic risk – the risk that the intermediation capacity of the entire financial system is impaired, with potential negative externalities to the real economy;
- A systemic risk measure should reflect the size, leverage, liquidity, interconnections, complexity and substitutability of a financial institution;

Motivation

- The recent financial crisis gave birth to extensive research in the fields of systemic risk (definition, measurement, regulation);
- Of particular interest is the identification of *SIFIs* (Systemically Important Financial Institutions) that contribute the most to the overall risk of the financial system;
- The measures used in this paper capture periods of financial disturbance;
- Related research is underlying new regulations which will be emphasized, the reorganization of financial systems, new perspectives upon financial fragility and its systemic characteristics

Main literature

- 1. Adrian, T. and M. K. Brunnermeier (2011) CoVaR. Working Paper, Princeton University and Federal Reserve Bank of New York;
- 2. Benoit, S., G. Colletaz, C. Hurlin, and C. Perignon (2013) A Theoretical and Empirical Comparison of Systemic Risk Measures. Working Paper;
- 3. Bisias, D., M. D. Flood, A. W. Lo, and S. Valavanis (2012) A Survey of Systemic Risk Analytics. Working Paper, MIT;
- 4. Brownlees, T.C., and R.F. Engle, (2012) Volatility, Correlation and Tails for Systemic Risk Measurement. Working Paper, NYU;
- 5. Koenker, R. and G. Jr. Basset (1978) Regression Quantiles. Econometrica 46 (1), 33-50;

Methodology (1)

- $\boldsymbol{\Delta CoVaR\ DCC}$:
- Consider a bivariate GARCH process for the price returns:

$$r_t = H_t^{1/2} v_t \ ,$$

where $r_t' = (r_{mt}\ r_{it})$ represents the vector of market and bank i returns, $v_t' = (\varepsilon_{mt}\ \xi_{it})$ denotes the *i.i.d.* random vector;

$$H_t = \begin{pmatrix} \sigma_{mt}^2 & \rho_{it}\sigma_{mt}\sigma_{it} \\ \rho_{it}\sigma_{mt}\sigma_{it} & \sigma_{it}^2 \end{pmatrix},$$

is the variance-covariance matrix, estimated with a *DCC GARCH* process

Methodology (2)

- Considering a Cholesky decomposition of the variance-covariance matrix H_t:

$$H_t^{1/2} = \begin{pmatrix} \sigma_{mt} & \sigma_{it}\rho_{it} \\ 0 & \sigma_{it}\sqrt{1-\rho_{it}^2} \end{pmatrix}$$

- The system's (market) returns can be written as:

$$r_{mt} = \frac{\sigma_{mt}}{\sigma_{it}\rho_{it}} r_{it} - \frac{\sigma_{mt}\sqrt{1-\rho_{it}^2}}{\rho_{it}} \xi_{it}$$

Methodology (3)

- $CoVaR$ is defined as follows:

$$\mathbb{P}\left(r_{mt} \leq CoVaR^{i}_{q,t} \middle| \, r_{it} = VaR^{i}_{q,t}\right) = q\%$$

- The system return conditional distribution is:

$$r_{mt} \mid r_{it} \sim N\left(\frac{r_{it}\sigma_{mt}\rho_{it}}{\sigma_{it}}, \left(1-\rho^{2}_{it}\right)\sigma^{2}_{mt}\right)$$

- It follows that:

$$\mathbb{P}\left(\left(\frac{r_{mt} - r_{it}\rho_{it}\sigma_{mt}/\sigma_{it}}{\sigma_{mt}\sqrt{1-\rho^{2}_{it}}} \middle| r_{it} = VaR^{i}_{q,t}\right) \leq \frac{CoVaR^{i}_{q,t} - r_{it}\rho_{it}\sigma_{mt}/\sigma_{it}}{\sigma_{mt}\sqrt{1-\rho^{2}_{it}}}\right) = q\%$$

Methodology (4)

- Combining the formulas and holding on the Gaussian distribution assumption:

$$CoVaR^{i}_{q,t} = -\phi^{-1}(q\%)\sigma_{mt}\sqrt{1-\rho^{2}_{it}} + \phi^{-1}(q\%)\frac{\rho_{it}\sigma_{mt}}{\sigma_{it}}$$

$$CoVaR^{i}_{50\%,t} = -\phi^{-1}(q\%)\sigma_{mt}\sqrt{1-\rho^{2}_{it}} + \phi^{-1}(50\%)\frac{\rho_{it}\sigma_{mt}}{\sigma_{it}}$$

- $\Delta CoVaR$ captures the increase in $CoVaR$ when institution i moves from the median state to its VaR state;

Methodology (5)

- $\Delta CoVaR$ becomes:

$$\Delta CoVaR^{i}_{q,t} = CoVaR^{m|r_{it}=VaR^{i}_{q,t}}_{t} - CoVaR^{m|r_{it}=VaR^{i}_{50\%,t}}_{t}$$

$$= \frac{\rho_{it}\sigma_{mt}}{\sigma_{it}}(VaR^{i}_{q,t} - VaR^{i}_{50\%,t})$$

where σ_{it} and σ_{mt} are bank i's and system's volatilities at time t and ρ_{it} is the conditional correlation;

- The above variables are estimated by the *DCC GARCH* process using standard QML techniques;

Methodology (6)

- $\boldsymbol{\Delta CoVaR\ Quant}$:

- Another way to estimate $\Delta CoVaR$ is through quantile regressions;

$$CoVaR_q^{m|r_i=VaR_q^i} = VaR_q^m|VaR_q^i = \hat{\alpha}_q^i + \hat{\beta}_q^i VaR_q^i$$

- Then, $\Delta CoVaR$ is given by:

$$\begin{aligned}\Delta CoVaR_{q,t}^i &= CoVaR_{q,t}^i - CoVaR_{50\%,t}^i \\ &= \hat{\beta}\left(\widehat{VaR}_{q,t}^i - \widehat{VaR}_{50\%,t}^i\right)\end{aligned}$$

where $\hat{\beta}$ is the coefficient estimated by the quantile regression;

Methodology (7)

- ***MES***:

- MES is the marginal contribution of an institution i to systemic risk, as measured by the Expected Shortfall (ES) of the system;

$$ES_{mt}(C) = \mathbb{E}_{t-1}(r_{mt}|r_{mt} < C) = \sum_{i=1}^{N} w_{it}\, \mathbb{E}_{t-1}(r_{it}\,|r_{mt} < C)$$

- MES corresponds to the partial derivative of the system ES with respect to the weight of bank i in the economy:

$$MES_{it}(C) = \frac{\partial ES_{mt}(C)}{\partial w_{it}} = \mathbb{E}_{t-1}(r_{it}\,|r_{mt} < C)$$

Methodology (8)

- Choosing the transposed matrix from the Cholesky factorization of the variance-covariance matrix, the following equations are obtained:

$$r_{mt} = \sigma_{mt}\varepsilon_{mt}$$

$$r_{it} = \sigma_{it}\rho_{it}\varepsilon_{mt} + \sigma_{it}\sqrt{1-\rho_{it}^2}\xi_{it}$$

- Considering a systemic event C, MES is defined as:

$$MES_{it}(C) = \mathbb{E}_{t-1}(r_{it}|r_{mt} < C)$$

$$MES_{it}(C) = \sigma_{it}\rho_{it}\mathbb{E}_{t-1}\left(\varepsilon_{mt}|\varepsilon_{mt} < \frac{C}{\sigma_{mt}}\right) + \sigma_{it}\sqrt{1-\rho_{it}^2}\,\mathbb{E}_{t-1}\left(\xi_{it}|\varepsilon_{mt} < \frac{C}{\sigma_{mt}}\right)$$

Methodology (9)

- ***SRISK***:

- The $SRISK$ measure extends the MES in order to take into account both the liabilities and the size of the financial institution;
- The $SRISK$ corresponds to the expected capital shortfall of a given financial institution, conditional on a crisis affecting the whole financial system:

$$SRISK_{it} = max\left[0\ ;\ \overbrace{k\ (D_{it} + (1 - LRMES_{it})W_{it})}^{Required\ Capital} - \overbrace{(1 - LRMES_{it})W_{it}}^{Available\ Capital}\right]$$

- The banks with the largest capital shortfall are considered systemically risky

Methodology (10)

- $SRISK$ can be rewritten as:

$$SRISK_{it} = max[0\ ;\ [k\ L_{it} - 1 + (1-k)\ LRMES_{it}]W_{it}]$$

where k is the prudential capital ratio (8%), D_{it} is the book value of total liabilities and W_{it} is the market valued equity;

- The $SRISK$ also considers the interconnection of a bank with the rest of the system through the long-run marginal expected shortfall ($LRMES$):

$$LRMES_{it} \cong 1 - \exp\ (18 \times MES_{it})$$

- This approximation represents the bank expected loss over a six-month horizon, obtained conditionally on the market falling by more than 40% within the next six months;

Data

- The data used in the empirical analysis are from 40 publicly traded commercial banks, components of STOXX Europe 600 Banks as a proxy for the European financial system. The main datasets are time series of stock price with daily frequency, total debt and number of shares outstanding, extracted from quarterly balance sheets;
- The system's serie (daily returns) is calculated by summing the market value equity weighted stock return of each bank as:

$$r_{mt} = \sum_{i=1}^{N} \frac{MV_{t-1}^{i} \times r_{it}}{\sum_{j=1}^{N} MV_{t-1}^{j}}$$

- Period analysed: 25.apr.2005 – 30.sep.2014;
- Source: Thomson Reuters Eikon Datastream

Results (1)

- An important remark is that different systemic risk measures return different rankings:

Table 5.1: Systemic risk rankings (30.09.2014)

Ranks	ΔCoVaR DCC	MES	SRISK
1	HSBC	BCA dei Paschi	Deutschebank
2	RBS	BCA Popolare	BNP
3	Lloyds	UBI BCA	Credit Agricole
4	Deutschebank	BCA Milano	Societe Generale
5	Mediobanca	RBS	ING

Results (2)

- MES and equity $Beta$ rankings are similar because of the existance of a strong correlation;
- If ξ_{it} and ε_{mt} are independent, then:

$$MES_{it}(C) = \sigma_{it}\rho_{it}\mathbb{E}_{t-1}\left(\varepsilon_{mt}|\varepsilon_{mt} < \frac{C}{\sigma_{mt}}\right)$$

$$MES_{it}(C) = \sigma_{it}\rho_{it}\mathbb{E}_{t-1}(\varepsilon_{mt}|r_{mt} < C)$$

$$\beta_{it} = cov(r_{it}, r_{mt})/var(r_{mt}) = \rho_{it}\sigma_{it}/\sigma_{mt}$$

- Rewriting:

$$MES_{it}(C) = \beta_{it}\sigma_{mt}\mathbb{E}_{t-1}(\varepsilon_{mt}|r_{mt} < C)$$
$$= \beta_{it}\mathbb{E}_{t-1}(r_{mt}|r_{mt} < C)$$

Results (3)

- MES of financial institution i is proportional with its systematic risk measured by time-varying $Beta$. The proportionality coefficient is the ES of the system:

$$MES_{it}(q) = \beta_{it} ES_{mt}(q)$$

- MES and $Beta$ return similar rankings, even on the relaxed hypothesis of the correlated system and institution innovations:

Table 5.2: Top 5 financial institutions identified by MES (siystemics) and Beta (systematics) (30.09.2014)

Ranks	Bank (MES)	MES(%)	Bank (Beta)	Beta
1	BCA dei Paschi	2,94%	BCA dei Paschi	1,72
2	BCA Popolare	2,78%	BCA Popolare	1,65
3	UBI BCA	2,38%	RBS	1,48
4	BCA Milano	1,99%	UBI BCA	1,42
5	RBS	1,93%	Mediobanca	1,37

Results (4)

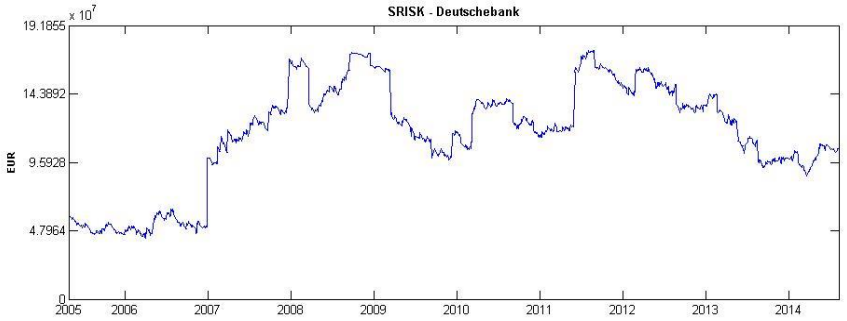
Table 5.3: Top 5 systemic financial institutions identified by SRISK (30.09.2014)

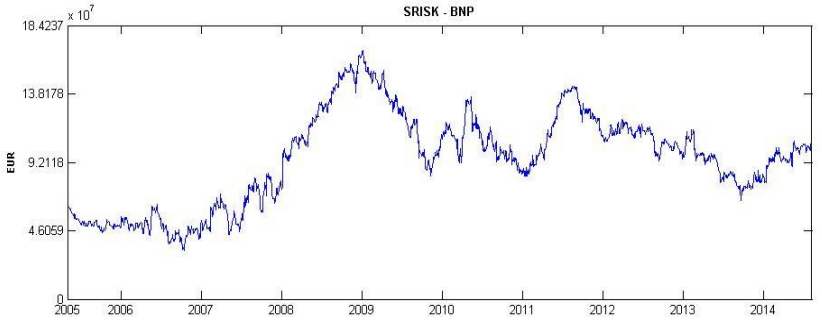
Ranks	Bank	SRISK (EUR)	SRISK (%)
1	Deutschebank	105.457.228	17,54%
2	BNP	101.177.567	16,82%
3	Credit Agricole	95.533.245	15,89%
4	Societe Generale	72.603.694	12,07%
5	ING	42.186.837	7,02%

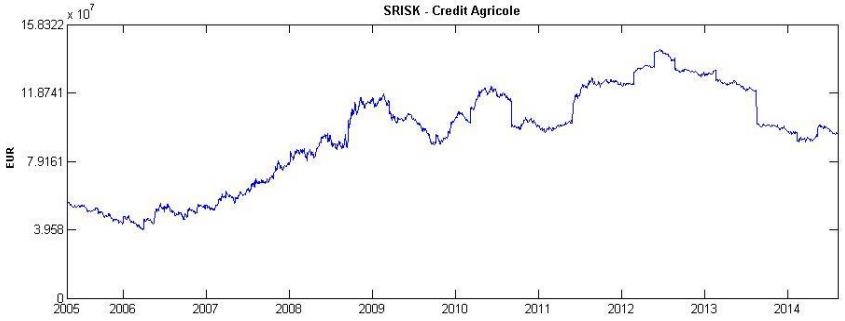
- $SRISK$ (%) can be interpreted as a contribution of the considered bank to systemic risk through its share of total decapitalisation of the market at day t, calculated as:

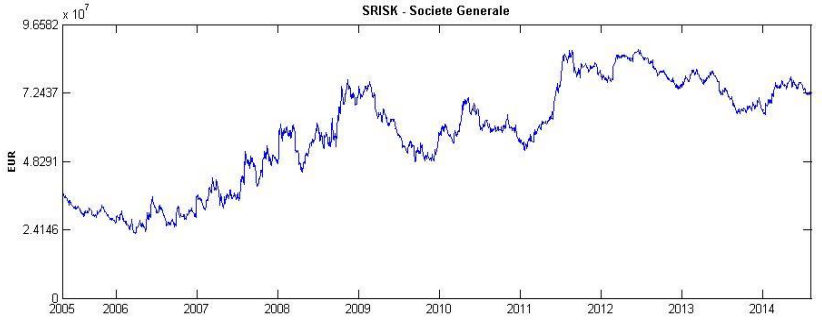
$$SRISK_t^i(\%) = \frac{SRISK_t^i}{\sum_j^N SRISK_t^j}$$

Results (5)


SRISK - Deutschebank
x 10^7
19.1855
14.3892
9.5928
4.7964
0
EUR
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014


SRISK - BNP
x 10^7
18.4237
13.8178
9.2118
4.6059
0
EUR
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014


SRISK - Credit Agricole
x 10^7
15.8322
11.8741
7.9161
3.958
0
EUR
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014


SRISK - Societe Generale
x 10^7
9.6582
7.2437
4.8291
2.4146
0
EUR
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014


SRISK - ING
x 10^7
10.9539
8.2155
5.477
2.7385
0
EUR
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014

Results (6)

- Table 5.4 shows the top 5 systemic financial institutions estimated both with DCC and $quantile\ regressions$ with respect to their contributions, calculated as an average on the analysed period:

Table 5.4: Top 5 systemic banks identified by ΔCoVaR DCC and ΔCoVaR Quant

Ranks	Bank	Average $\Delta CoVaR\ DCC$	Bank	Average $\Delta CoVaR\ Quant(\%)$
1	HSBC	3,50%	HSBC	3,66%
2	RBS	3,25%	UBI BCA	2,97%
3	Lloyds	3,07%	RBS	2,58%
4	Deutschebank	2,47%	Unicredit	2,45%
5	Unicredit	2,19%	Emilia Romagna	2,37%

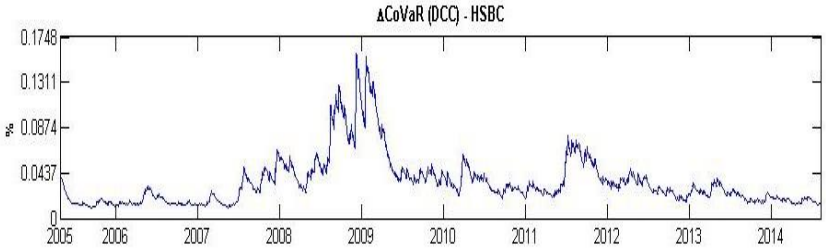
Results (7)

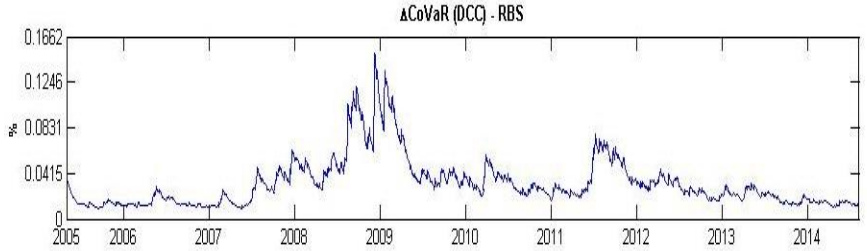
- For the sake of comparison, I will only focus on the $\Delta CoVaR\ DCC$ estimates, with respect to the common framework of dynamic conditional correlation. In Table 5.5 we can see the identified *SIFIs* rankings on the last day of the analysed period, 30.09.2014:

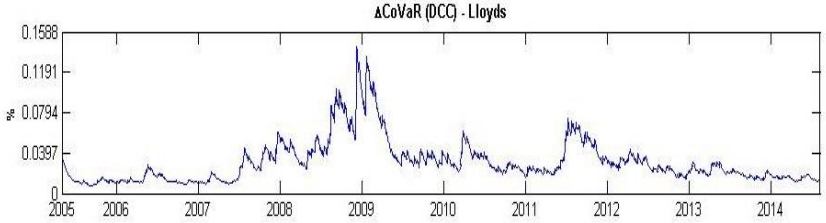
Table 5.5: Top 5 systemic financial institutions identified by $\Delta CoVaR\ DCC$ (30.09.2014)

Ranks	Bank	$\Delta CoVaR\ DCC(\%)$
1	HSBC	1,39%
2	RBS	1,26%
3	Lloyds	1,24%
4	Deutschebank	1,18%
5	Mediobanca	0,84%

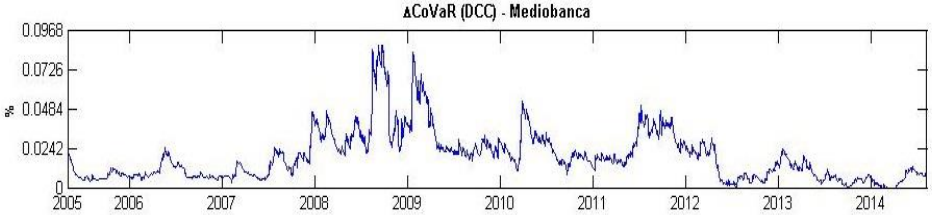
Results (8)


ΔCoVaR (DCC) - HSBC
0.1748
0.1311
0.0874
0.0437
0
%
2005 2006 2007 2008 2009 2010 2011 2012 2013 2014


ΔCoVaR (DCC) - RBS
0.1662
0.1246
0.0831
0.0415
0
%
2005 2006 2007 2008 2009 2010 2011 2012 2013 2014

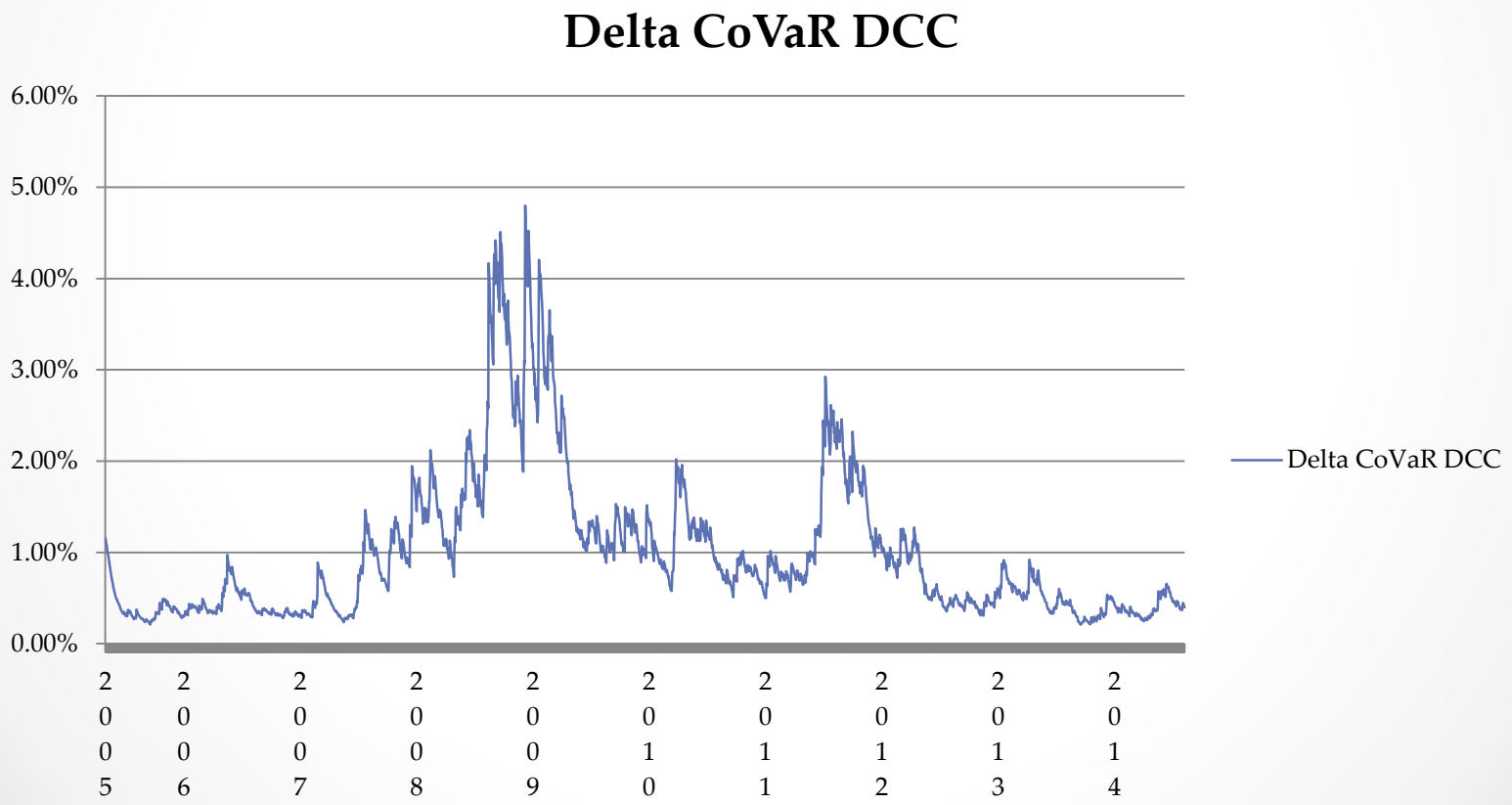

ΔCoVaR (DCC) - Lloyds
0.1588
0.1191
0.0794
0.0397
0
%
2005 2006 2007 2008 2009 2010 2011 2012 2013 2014


ΔCoVaR (DCC) - Deutschebank
0.1526
0.1145
0.0763
0.0382
0
%
2005 2006 2007 2008 2009 2010 2011 2012 2013 2014


ΔCoVaR (DCC) - Mediobanca
0.0968
0.0726
0.0484
0.0242
0
%
2005 2006 2007 2008 2009 2010 2011 2012 2013 2014

Results (9)

- **Evolution of *I* ΔCoVaR DCC:**



Results (10)

- **Evolution of *I* SRISK:**

Concluding remarks (1)

- The rankings based on different systemic risk measures are not identical;
- The reason for the different rankings is basically the difference in the estimation methodology;
- The empirical application returned a single financial institution identified as *SIFI* by both $\Delta CoVaR$ and $SRISK$ in the last day of the time horizon, namely *Deutschebank*. $SRISK$ ranks it the first with a contribution of 17,54% (approximately 105 mil. EUR), while $\Delta CoVaR$ ranks it the fourth with a contribution to the system's loss of 1,18%;
- The MES model is not a very reliable tool for systemic risk rankings because of the existance of strong correlation with the systematic risk ($Beta$), returning similar rankings;

Concluding remarks (2)

- The estimated systemic risk measures capture the co-movement of tail distributions and are increasing in crisis periods;
- However, these estimates are not sufficient for the regulators to take a decision on imposing capital requirements or liquidity buffers;
- It is necessary to model some other aspects, like interbank linkages which can transform into transmission channels for the shocks, for them to have a rich picture of financial fragility;
- Based on the constructed indicators and on the new progressively implementing regulations of Basel III, there is no strong evidence that indicates market pressure or the trigger of a new systemic crisis.

Thank You!