Multi-moment approximate option valuation models

- A general comparison -

MSc student: Olteanu Bogdan
Supervisor: PhD Professor Moisă Altăr

Bucharest University of Economic Studies

June 2014

Contents:

- Motivation
- Literature review
- Overview of the models
- Calibration method
- Empirical results
- Concluding remarks
- Selected references

Motivation

- A general overview of the option pricing models using statistical series expansions, specifically Gram-Charlier and Edgeworth expansions;
- Analysis of pricing and hedging performances, compared to those of the Black and Scholes (1973) and Heston (1993) models;
- Assess whether introducing additional stochastic factors to option valuation (e.g. volatility) or specifying a different form for the risk-neutral density of the underlying asset captures more information from the market;
- Prepare and improve the necessary background for the research of more general models, which consider, amongst others, elements of the microstructure of the markets they are applied on.

Literature review

- **Studies on option pricing models:** Harrison and Pliska (1981), Rubinstein (1985), Longstaff (1995), Bates (1996a), Bakshi, Cao, Chen (1997), Jurczenko, Maillet, Negrea (2002a, 2002b), Broadie and Detemple (2004), Levendorskiĭ (2012), Bauer (2012), Xiu (2013), Necula, Drimus, Farkas (2013).

Literature review

- **Studies on option pricing models:** Harrison and Pliska (1981), Rubinstein (1985), Longstaff (1995), Bates (1996a), Bakshi, Cao, Chen (1997), Jurczenko, Maillet, Negrea (2002a, 2002b), Broadie and Detemple (2004), Levendorskiĭ (2012), Bauer (2012), Xiu (2013), Necula, Drimus, Farkas (2013).
- **Numerical methods:** Carr, Madan (1999), Negrea (2001), Moodley (2005), Schmelzle (2005).

Literature review

- **Studies on option pricing models:** Harrison and Pliska (1981), Rubinstein (1985), Longstaff (1995), Bates (1996a), Bakshi, Cao, Chen (1997), Jurczenko, Maillet, Negrea (2002a, 2002b), Broadie and Detemple (2004), Levendorskiĭ (2012), Bauer (2012), Xiu (2013), Necula, Drimus, Farkas (2013).
- **Numerical methods:** Carr, Madan (1999), Negrea (2001), Moodley (2005), Schmelzle (2005).
- **Calibration methods:** Bakshi, Cao, Chen (1997), Jondeau, Rockinger (1999), Chiarella, Craddock, Hassan (2000), Mikhailov, Nogel (2003), Moodley (2005), Chen (2007), Wu, Xu & Li (2013).

Literature review

- **Studies on option pricing models:** Harrison and Pliska (1981), Rubinstein (1985), Longstaff (1995), Bates (1996a), Bakshi, Cao, Chen (1997), Jurczenko, Maillet, Negrea (2002a, 2002b), Broadie and Detemple (2004), Levendorskiĭ (2012), Bauer (2012), Xiu (2013), Necula, Drimus, Farkas (2013).
- **Numerical methods:** Carr, Madan (1999), Negrea (2001), Moodley (2005), Schmelzle (2005).
- **Calibration methods:** Bakshi, Cao, Chen (1997), Jondeau, Rockinger (1999), Chiarella, Craddock, Hassan (2000), Mikhailov, Nogel (2003), Moodley (2005), Chen (2007), Wu, Xu & Li (2013).
- **Monographies:** Shreve (2004), Negrea (2006), Chourdakis (2008), Necula (2009).

The Black and Scholes (1973) model - Benchmark (1)

By solving a PDE with boundary condition, Black and Scholes obtained the following form for a call option price:

$$C_t = S_t N(d_1) - Ke^{-r(T-t)} N(d_2),$$

where:

$$d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\left(\sigma\sqrt{T-t}\right)}$$

$$d_2 = d_1 - \left(\sigma\sqrt{T-t}\right)$$

Heston's SV (1993) model - Benchmark (2)

Heston solved a PDE using a method based on characteristic functions. He proposed a similar solution to the one of Black and Scholes:

$$C_t = S_t P_1 - K e^{-r(T-t)} P_2$$

Using the Inversion theorem, the two probabilities can be expressed as:

$$P_j = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \mathrm{Re}\left[\varphi_j(\phi) \frac{e^{-i\phi \ln(K)}}{i\phi} \right] d\phi,$$

where:

$$\varphi_j(\phi) = \exp\left[A_j(T-t,\phi) + B_j(T-t,\phi) V + i\phi \ln(S) \right].$$

The Jarrow and Rudd (1982) model

Jarrow and Rudd derive an option pricing formula using a Gram - Charlier series expansion of the underlying security's price distribution around a log-normal distribution:

$$f(S_T) = l(S_T) - \frac{k_3}{3!}\frac{d^3[l(S_T)]}{dS_T^3} + \frac{k_4}{4!}\frac{d^4[l(S_T)]}{dS_T^4} + \varepsilon(S_T)$$

They obtain:

$$C_{JR} = C_{BS} + [\gamma_1(f) - \gamma_1(l)]\, Q_3 + [\gamma_2(f) - \gamma_2(l)]\, Q_4,$$

where:

C_{BS} = the Black-Scholes price of an European call

$\gamma_1(.), \gamma_2(.)$ = the R.Fisher parameters for skewness and kurtosis

Q_3, Q_4 = the sensitivities of the price of the option to departures from log-normality

The Corrado and Su (1997) model

Corrado and Su use a Gram - Charlier Type A series expansion for the risk-neutral density of the underlying asset's standardized return:

$$f(z) = n(z)\left[1 + \frac{k_3(f)}{3!}H_3(z) + \frac{k_4(f)}{4!}H_4(z)\right] + \varepsilon(z)$$

The theoretical price of an European call option is thus:

$$C_{CS} = C^*_{BS} + \gamma_1(f)Q'_3 + \gamma_2(f)Q'_4,$$

where:

$$C^*_{BS} = C_{BS}(d^*)$$

$$d^* = \left(\sigma\sqrt{T-t}\right)^{-1}\left[\ln\left(S_t/K\right) + \left(r + \sigma^2/2\right)(T-t) - \ln(1+w)\right]$$

$$w = \frac{\gamma_1}{3!}\sigma^3 (T-t)^{\frac{3}{2}} + \frac{\gamma_2}{4!}\sigma^4 (T-t)^2$$

The Rubinstein (1998) model

M. Rubinstein considers an Edgeworth series exp. as an approximation for the risk-neutral density of the underlying asset's standardized log-return:

$$f(z) = n(z)\left[1 + \frac{k_3(f)}{3!}H_3(z) + \frac{k_4(f)}{4!}H_4(z) + 10\frac{[k_3(f)]^2}{6!}H_6(z)\right] + \varepsilon$$

He then obtains the following pricing formula:

$$C_{Rub} = C^*_{BS} + \gamma_1(f)\, Q_3'' + \gamma_2(f)\, Q_4'' + [\gamma_1(f)]^2\, Q_5'',$$

where:

$$C^*_{BS} = C_{BS}(d^*)$$

$$d^* = \left(\sigma\sqrt{T-t}\right)^{-1}\left[\ln(S_t/K) + \left(r + \sigma^2/2\right)(T-t) - \ln(1+\phi)\right]$$

$$\phi = \frac{\gamma_1(f)}{3!}\sigma^3(T-t)^{\frac{3}{2}} + \frac{\gamma_2(f)}{4!}\sigma^4(T-t)^2 + 10\frac{[\gamma_1(f)]^2}{6!}\sigma^6(T-t)^3$$

Calibration method

For each of the models considered, we follow the approach of Whaley (1982) and estimate the vector of model parameters, Ω, by minimizing the sum of squared deviations between the observed market prices and theoretical option prices:

$$\min_{\Omega}\left(F\left(\Omega\right)\right)=\min_{\Omega}\left(\sum_{i=1}^{N}w_i\left[C_i^M-C_i^{\Omega}\right]^2\right),$$

$$w_i=\frac{1}{\left|Bid_i-Ask_i\right|}.$$

We use the optimisation algorithm proposed by Ingber (1993), called *Adaptive Simulated Annealing (ASA)*.

Data set used

- The option series used in this study contains European call and put options on the S&P500 Index, traded at the Chicago Board Options Exchange.
- Data were extracted from Reuters Datastream and contain daily closing prices for a sample period of three months, from March to May 2012.
- The risk-free interest rate was proxied by the three-months and six-months U.S. Treasury Bills rates.
- Due to reasons of brevity, we only focus on call options in this presentation.

Moneyness (S/K)	Days to maturity Short-term <60	Medium-term 60-120	Long-term >120	Total
Deep out of the money (M < 0.94)	223	206	232	661
Out of the money (0.94 <= M < 0.97)	119	165	116	400
At the money (0.97 <= M < 1.00)	246	116	194	556
At the money (1.00 <= M < 1.03)	233	198	192	623
In the money (1.03 <= M < 1.06)	138	107	197	442
Deep in the money (M >= 1.06)	256	368	284	908
Total	1,215	1,160	1,215	3,590

Table 1: Sample properties of S&P500 Index options. Data source: Reuters Datastream, own contribution.

Stylised facts

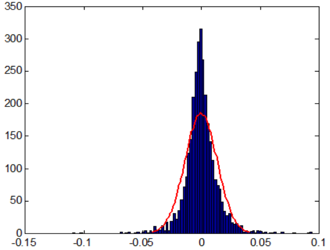


Figure 1: S&P500 Index daily return distribution for 2000-2012 period.

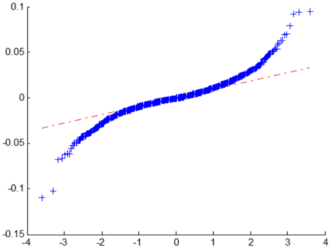


Figure 2: Q-Q plot for S&P500 Index daily returns, against a normal distribution.

Stylised facts (Cont'd)

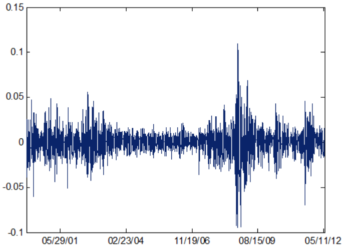


Figure 3: S&P500 Index daily return evolution from 2000 to 2012.

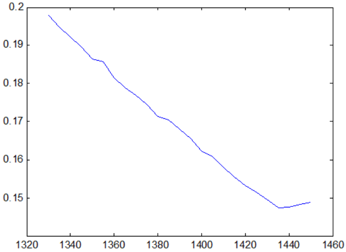


Figure 4: Implied volatilities for S&P500 Index call options on the 22nd of March.

Calibration results

	All Options					Short-Term Options					At-the-money Options				
Parameters	**Heston**	**BS**	**CS**	**JR**	**Rub**	**Heston**	**BS**	**CS**	**JR**	**Rub**	**Heston**	**BS**	**CS**	**JR**	**Rub**
Kappa	2.700					2.517					2.692				
	(1.077)					(1.089)					(1.164)				
Theta	0.035					0.037					0.039				
	(0.011)					(0.010)					(0.008)				
SigmaV	0.375					0.376					0.415				
	(0.138)					(0.129)					(0.111)				
Rho	-0.822					-0.817					-0.812				
	(0.061)					(0.055)					(0.047)				
V0	0.017					0.017					0.019				
	(0.006)					(0.006)					(0.005)				
Sigma		0.133	0.153	0.143	0.156		0.128	0.150	0.140	0.153		0.136	0.158	0.147	0.161
		(0.019)	(0.021)	(0.018)	(0.022)		(0.023)	(0.021)	(0.018)	(0.022)		(0.019)	(0.019)	(0.017)	(0.021)
Skewness			-1.173	-0.880	-1.116			-1.159	-0.858	-1.106			-1.200	-0.895	-1.140
			(0.132)	(0.119)	(0.120)			(0.132)	(0.119)	(0.120)			(0.110)	(0.099)	(0.104)
Kurtosis			4.225	6.792	5.824			4.255	6.794	5.804			4.294	6.755	5.955
			(0.373)	(0.247)	(0.719)			(0.373)	(0.247)	(0.719)			(0.385)	(0.244)	(0.677)
Implied Volatility	0.131	0.144	0.147	0.146	0.147	0.139	0.142	0.150	0.148	0.147	0.133	0.148	0.150	0.144	0.150
	(0.027)	(0.031)	(0.033)	(0.031)	(0.033)	(0.024)	(0.033)	(0.040)	(0.025)	(0.027)	(0.026)	(0.032)	(0.037)	(0.036)	(0.039)

Table 2: Each day in the sample, the parameters of a given model are estimated by minimizing the sum of squared pricing errors between the market price and the model price for each option. The daily average is reported first, followed by ithe standard deviation, in brackets. Data source: own contribution.

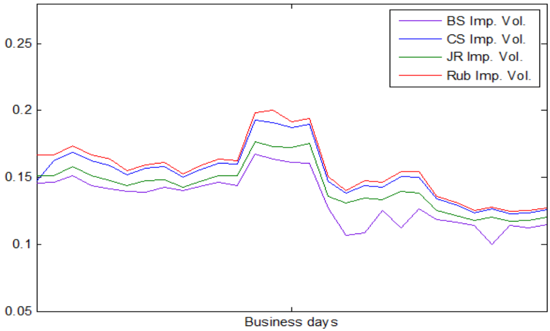


Figure 5: Implied volatility from the Black and Scholes, Corrado and Su, Jarrow and Rudd and Rubinstein models. Data source: own contribution.

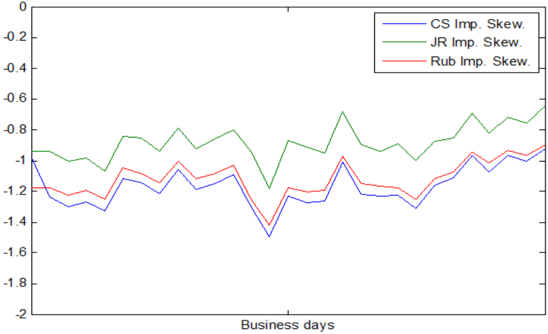


Figure 6: Implied skewness from the Corrado and Su, Jarrow and Rudd and Rubinstein models. Data source: own contribution.

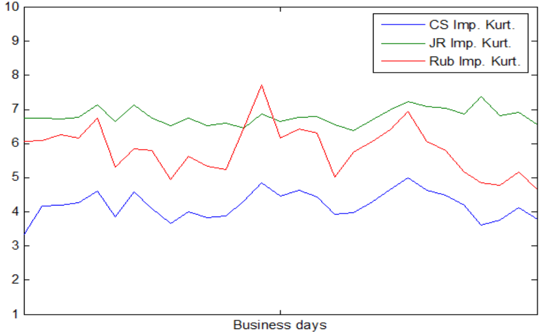


Figure 7: Implied kurtosis from the Corrado and Su, Jarrow and Rudd and Rubinstein models. Data source: own contribution.

Moneyness (S/K)	Days to maturity			
	Short-term <60	Medium-term 60-120	Long-term >120	Total
Deep out of the money (M < 0.94)	11.96	11.77	11.04	11.41
Out of the money (0.94 <= M < 0.97)	12.18	12.11	12.33	12.19
At the money (0.97 <= M < 1.00)	12.93	14.00	13.38	13.29
At the money (1.00 <= M < 1.03)	14.28	15.06	14.29	14.46
In the money (1.03 <= M < 1.06)	15.11	16.30	15.09	15.48
Deep in the money (M >= 1.06)	19.51	18.22	16.64	17.69
Total	14.00	15.07	13.94	14.38

Table 3: The implied volatility is obtained by inverting the Black and Scholes formula for each option in the sample.

The implied volatilites are then averaged across each moneyness - maturity category and days in the sample.

Data source: own contribution.

Out of sample performance

Moneyness (M=S/K)	Model	All-Option-Based Days to maturity			Maturity-Based Days to maturity			Moneyness-Based Days to maturity		
		<60	60-120	>120	<60	60-120	>120	<60	60-120	>120
Absolute pricing errors										
Deep out of the money (M < 0.97)	BS	2.576	3.604	3.813	2.688	3.886	3.709	2.659	3.552	3.726
	Heston	0.896	0.962	1.420	0.862	0.863	1.436	0.894	0.893	1.315
	CS	0.770	1.160	1.273	0.636	1.148	1.287	0.807	1.262	1.201
	JR	0.788	1.145	1.301	0.688	1.246	1.312	0.848	1.146	1.361
	Rub	0.766	1.090	1.261	0.640	1.130	1.261	0.759	1.029	1.143
Out of the money (0.94 <= M < 0.97)	BS	2.418	3.798	3.612	1.683	3.693	3.887	2.350	4.061	3.742
	Heston	1.391	2.025	2.881	1.471	2.218	2.573	1.513	1.983	2.923
	CS	1.578	2.394	2.204	1.557	2.351	2.313	1.504	2.342	2.010
	JR	1.586	2.407	2.310	1.419	2.451	2.310	1.522	2.426	2.414
	Rub	1.572	2.372	2.218	1.196	1.686	2.218	1.623	2.515	2.101
At the money (0.97 <= M < 1.03)	BS	2.470	4.114	4.250	2.911	4.234	4.267	2.490	3.757	4.673
	Heston	1.634	2.946	3.597	1.616	2.842	3.643	1.633	3.087	3.500
	CS	1.775	2.996	3.568	1.720	2.996	3.314	1.775	3.132	3.439
	JR	1.765	3.001	3.453	1.740	2.169	3.453	1.592	3.072	3.505
	Rub	1.887	3.015	3.364	1.575	2.137	3.364	1.751	3.265	3.091
In the money (1.03 <= M < 1.06)	BS	3.711	6.017	9.704	4.558	5.758	10.426	3.489	6.403	10.253
	Heston	1.512	2.469	3.626	1.608	2.609	3.299	1.605	2.542	3.450
	CS	1.742	2.548	3.704	1.641	2.559	3.919	1.713	2.331	3.615
	JR	1.462	2.394	3.309	1.655	1.889	2.375	1.398	2.215	3.421
	Rub	1.532	2.578	3.811	1.229	2.632	3.905	1.495	2.425	3.682
Deep in the money (M >= 1.06)	BS	0.968	4.579	7.574	1.443	4.579	7.160	1.023	4.434	8.183
	Heston	2.318	1.935	2.867	2.213	2.396	2.642	2.492	2.116	3.148
	CS	2.501	2.204	2.407	2.552	2.230	2.547	2.624	2.155	2.409
	JR	2.301	1.868	2.348	2.414	2.738	2.368	2.212	1.836	2.434
	Rub	2.625	2.424	2.713	2.626	2.457	2.845	2.391	2.299	2.507

Table 4: Absolute pricing errors, computed as sample averages of the absolute difference between the market price and the model price for each call in a given moneyness-maturity category.

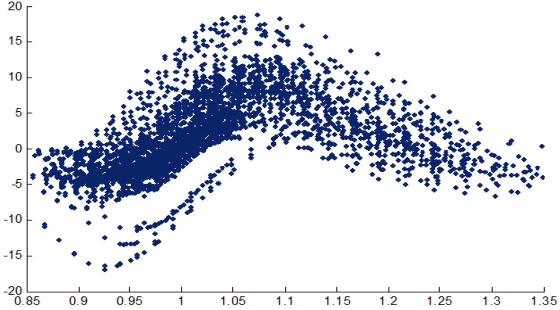
Data source: own contribuion.

Out of sample performance (Cont'd)

Moneyness (M=S/K)	Model	All-Option-Based Days to maturity <60	60-120	>120	Maturity-Based Days to maturity <60	60-120	>120	Moneyness-Based Days to maturity <60	60-120	>120
Percentage pricing errors										
Deep out of the money (M < 0.97)	BS	(0.877)	(0.815)	(0.758)	(0.085)	(0.875)	(0.763)	(0.912)	(0.894)	(0.815)
	Heston	0.295	0.012	(0.080)	0.265	0.061	(0.097)	0.305	0.013	(0.086)
	CS	0.179	0.107	(0.020)	0.166	(0.101)	(0.021)	0.179	0.098	(0.019)
	JR	0.222	(0.056)	0.014	0.236	(0.051)	0.013	0.208	(0.061)	0.014
	Rub	0.227	(0.050)	0.012	0.224	(0.046)	0.011	0.213	(0.047)	0.012
Out of the money (0.94 <= M < 0.97)	BS	(0.232)	(0.287)	(0.039)	(0.245)	(0.288)	(0.040)	(0.255)	(0.275)	(0.039)
	Heston	0.059	(0.060)	(0.031)	0.048	(0.032)	(0.033)	0.054	(0.055)	(0.030)
	CS	(0.114)	(0.158)	0.002	(0.121)	(0.159)	0.000	(0.119)	(0.156)	0.003
	JR	(0.112)	(0.155)	0.007	(0.120)	(0.156)	0.004	(0.113)	(0.163)	0.007
	Rub	(0.105)	(0.156)	0.005	(0.112)	(0.158)	(0.008)	(0.105)	(0.153)	0.004
At the money (0.97 <= M < 1.03)	BS	(0.005)	0.005	0.079	0.004	0.034	0.079	(0.005)	0.005	0.080
	Heston	0.011	(0.029)	0.000	0.003	0.040	0.007	0.011	(0.026)	4.242
	CS	(0.050)	0.004	0.032	0.005	0.036	0.003	(0.047)	0.003	0.034
	JR	0.051	0.040	0.027	0.058	0.004	0.027	0.049	0.038	0.026
	Rub	0.055	0.043	0.025	0.057	0.043	0.025	0.050	0.040	0.025
In the money (1.03 <= M < 1.06)	BS	0.051	0.008	0.098	0.056	0.076	0.098	0.048	0.007	0.101
	Heston	0.009	0.007	0.014	0.092	0.007	0.012	0.009	0.007	0.014
	CS	(0.020)	(0.004)	0.028	(0.010)	0.004	0.003	(0.019)	(0.004)	0.025
	JR	(0.015)	(0.003)	0.021	(0.001)	0.003	0.021	(0.014)	(0.003)	0.021
	Rub	(0.016)	0.007	0.028	0.001	0.007	0.028	(0.016)	0.007	0.028
Deep in the money (M >= 1.06)	BS	0.012	0.025	0.048	0.011	0.026	0.048	0.011	0.028	0.051
	Heston	(0.002)	0.003	0.001	0.010	0.002	0.003	(0.002)	0.002	0.001
	CS	(0.009)	0.007	0.000	(0.008)	0.006	0.000	(0.009)	0.006	1.867
	JR	(0.005)	(0.004)	(0.003)	(0.005)	(0.004)	(0.003)	(0.006)	(0.004)	(0.003)
	Rub	(0.009)	0.006	0.003	(0.006)	(0.005)	0.002	(0.009)	0.006	0.002

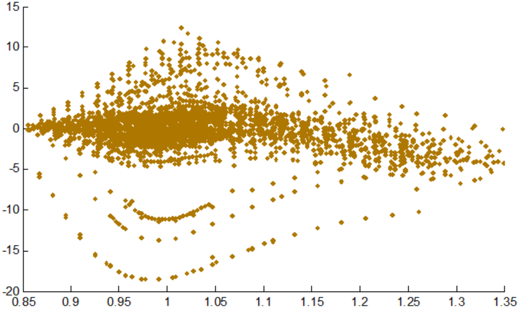
Table 5: Percentage pricing errors, computed as sample averages of the market price minus the model price, divided by the market price, for each call in a given moneyness-maturity category.

Data source: own contribuion.



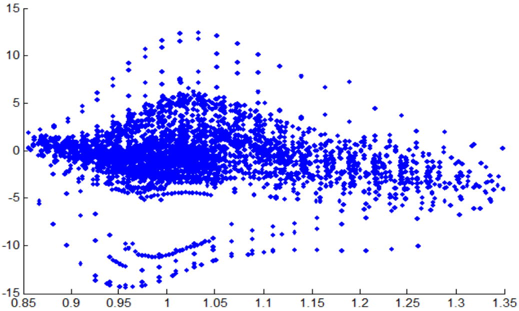
Figures 8: Dollar pricing errors for the **Black and Scholes model**. The pricing error is defined as the difference between the market price and the theoretical price. Moneyness is defined as the spot to strike ratio.

Data source: own contribution.



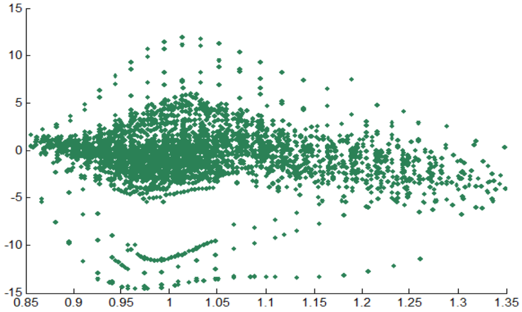
Figures 9: Dollar pricing errors for the **Heston model**. The pricing error is defined as the difference between the market price and the theoretical price. Moneyness is defined as the spot to strike ratio.

Data source: own contribution.



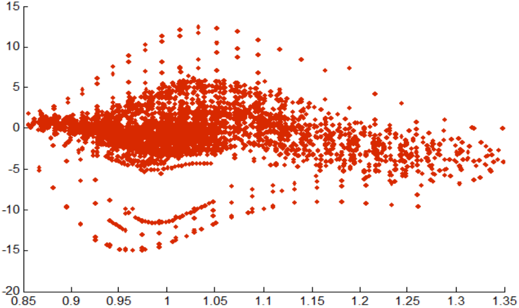
Figures 10: Dollar pricing errors for the **Corrado and Su model**. The pricing error is defined as the difference between the market price and the theoretical price. Moneyness is defined as the spot to strike ratio.

Data source: own contribution.



Figures 11: Dollar pricing errors for the **Jarrow and Rudd model**. The pricing error is defined as the difference between the market price and the theoretical price. Moneyness is defined as the spot to strike ratio.

Data source: own contribution.



Figures 12: Dollar pricing errors for the **Rubinstein model**. The pricing error is defined as the difference between the market price and the theoretical price. Moneyness is defined as the spot to strike ratio.

Data source: own contribution.

Hedging performance

- All hedges of call options use only the underlying index as the hedging instrument;
- All parameters implied by all options of the previous day are used to establish the current's day hedges, which are liquidated the day after;
- For each option, the hedging error is defined as:

$$H_{j,t+1} = \left(C^M_{j,t+1} - \Delta_t S_{t+1} \right) - \left(C^M_{j,t} - \Delta_t S_t \right) e^{r/252}.$$

Hedging performance (Cont'd)

Moneyness (M=S/K)	Model	1-Day Revision Days to maturity <60	60-120	>120
Absolute hedging errors				
Deep out of the money (M < 0.97)	BS	0.668	0.680	0.699
	Heston	0.809	0.675	1.188
	CS	0.532	0.792	1.067
	JR	0.431	0.426	0.512
	Rub	0.476	0.675	0.914
Out of the money (0.94 <= M < 0.97)	BS	0.655	1.363	0.436
	Heston	0.522	1.346	0.450
	CS	0.718	1.896	0.439
	JR	0.683	0.904	0.568
	Rub	0.600	1.346	0.365
At the money (0.97 <= M < 1.03)	BS	0.738	1.058	1.525
	Heston	1.168	0.982	1.877
	CS	0.975	1.517	1.674
	JR	0.838	0.971	1.417
	Rub	0.700	1.017	1.142
In the money (1.03 <= M < 1.06)	BS	0.998	1.769	2.330
	Heston	0.935	1.624	2.500
	CS	1.189	1.852	2.779
	JR	1.704	2.321	2.766
	Rub	1.003	1.624	2.231
Deep in the money (M >= 1.06)	BS	1.430	1.439	1.887
	Heston	1.492	1.470	1.343
	CS	1.431	1.359	1.618
	JR	1.438	1.382	1.544
	Rub	1.439	1.470	1.842

Table 6: All hedges of call options use only the underlying asset as the hedging instrument.

The average absolute hedging errors are reported for each moneyness-maturity class.

Data source: own contribution.

Concluding remarks

- Even though the Black and Scholes model offers good pricing performances for at-the-money options, both the statistical series expansions models and the stochastic volatility model outperform it for all other option categories;
- The Gram-Charlier and Edgeworth expansions around the standardized normal distribution perfom better than the Gram-Charlier expansion around the lognormal distribution;
- The Corrado and Su, the Jarrow and Rudd, the Rubinstein and the Heston model are able to correct the systematic biases of the Black and Scholes model;
- The Black and Scholes offers similar hedging performances for a 1-day revision horizon as the statistical series expansions pricing models, but the Rubinstein models perform the best overall.

Thank your for your attention!

Olteanu Bogdan

Selected references

- Bakshi, G., Cao, C. & Chen, Z. (1997), "Empirical performance of alternative option pricing models", Journal of Finance 52: 2003-2049.
- Bates, D. (1996a), "Testing Option Pricing Models", Handbook of Statistics 14: Statistical Methods in Finance, Maddala and Rao (eds), 567-611.
- Bates, D. (1996b), "Jumps and Stochastic Volatility: Exchange Rate Process Implicit in Deutsche Mark Options", Review of Financial Studies 9 (1), 69-107.
- Bates, D. (2000), "Post '87 Crash Fears in the S&P500 Futures Option Market", Journal of Econometrics 94, 181-238.
- Broadie, M. and J. Detemple, "Option Pricing: Valuation Models and Applications", Management Science 50 (9), 1145-1177.
- Brown, C. and Robinson, D. (2002), "Skewness and Kurtosis Implied by Option Prices: A Correction", Journal of Financial Research 25, 279-282.
- Carr, P., Madan, D. B. (1999), "Option evaluation the using fast fourier transform", Journal of Computational Finance 2(4): 61–73.

- Chiarella, C., Craddock, M., El-Hassan, N. (2000), “The calibration of stock option pricing models using inverse problem methodology”, Research Paper Series 39, Quantitative Finance Research Centre, University of Technology, Sydney.
- Corrado, C. and T. Su (1996a), "S&P500 Index Option Tests of Jarrow and Rudd's Approximate Option Valuation Formula", Journal of Future Markets 16 (6), 611-629.
- Corrado, C. and T. Su (1996b), "Skewness and Kurtosis in S&P500 Index Returns Implied by Option Prices", Journal of Financial Research 19 (2), 175-192.
- Corrado, C. and T. Su (1997a), "Implied Volatility Skews and Stock Index Skewness and Kurtosis Implied by S&P500 Index Option Prices", Journal of Derivatives 4, 8-19.
- Corrado, C. and T. Su (1997b), "Implied Volatility Skews and Stock Return Skewness and Kurtosis Implied by Stock Option Prices", European Journal of Finance 3, 73-85.
- Cox, J. C., Ingersoll, J. E. and Ross, S. A. (1985), “A theory of the term structure of interest rates”, Econometrica 53: 385–407.
- Cox, J. C. and Ross, S. A. (1976), “The Valuation of Options for Alternative Stochastic Processes”, Journal of Financial Economics 3: 145–166.

- Derman, E. (1999), "Regimes of volatility", RISK 12, 55-59.
- Dumas, B., Fleming, J., Whaley, R.E. (1998), “Implied volatility functions:empirical tests”, Journal of Finance 6: 2059-2107.
- Hald, A. (2000), "The Early History of the Cumulants and the Gram-Charlier Series", International Statistical Review 68 (2), 137-153.
- Harrison, J. and D. Kreps (1979), "Martingales and arbitrage in multiperiod security markets", Journal of Economic Theory 2, 381-408.
- Harrison, J. and S. Pliska (1981), "Martingales and Stochastic Integrals in the Theory of Continuous Trading", Stochastic Processes and their Applications 11, 215-260.
- Ingber, A. L. (1995), “Adaptive simulated annealing (asa): Lessons learned”, Control and Cybernetics.
- Jarrow, R. and A. Rudd (1982), "Approximate Option Valuation for Arbitrary Stochastic Processes", Journal of Financial Economics 10, 347-369.
- Jondeau, E. and M. Rockinger (2001), "Estimating Gram-Charlier Expansions with Positivity Constraints", Journal of Economic Dynamics and Control 25 (10), 1457-1483.

- Jurczenko, E., Maillet, B. and Negrea, B. (2002a), "Multi-moment Approximate Option Pricing Models: A General Comparison (Part 1)", Working Paper, CNRS - Université Paris 1 Panthéon-Sorbonne.
- Jurczenko, E., Maillet, B. and Negrea, B. (2002b), "Skewness and Kurtosis Implied by Option Prices: A Second Comment", Working Paper, CNRS - Université Paris 1 Panthéon-Sorbonne.
- Longstaff, F (1995), "Option Pricing and the Martingale Restriction", Review of Financial Studies 8 (4), 1091-1124.
- MacBeth, J. and L. Merville (1979), "An Empirical Examination of the Black-Scholes Call Option Pricing Model", Journal of Finance 34, 1173-1186.
- Merton, R. (1976), "Option Pricing when the Underlying Stock Returns are Discontinuous". Journal of Financial Economics 4, 125-144.
- Mikhailov, S., Nogel, U. (2003), "Heston's stochastic volatility model implementation, calibration and some extensions", Wilmott Magazine: 74-94.
- Necula, C. (2009), Evaluarea opțiunilor financiare. Volumul I: Modelul Black - Scholes - Merton, București: Editura ASE.
- Necula, C, G. Drimus and W. Farkas (2013), "A General Closed Form Option Pricing Formula", available at SSRN: http://dx.doi.org/10.2139/ssrn.2210359

- Negrea, B. (2006), Evaluarea activelor financiare: o introducere în teoria proceselor stocastice aplicată în finanțe, București: Editura Economică.
- Negrea, B. (2001), “Evaluation des options par la transformeé de Fourier”, Cahiers de la MSE.
- Rubinstein, M. (1985), "Nonparametric Tests of Alternative Option Pricing Models Using All Reported Trades and Quotes on the 30 Most Active CBOE Option Classes From August 23, 1976 Through August 31, 1978", Journal of Finance 40, 455-480.
- Rubinstein, M. (1998), "Edgeworth Binomial Trees", Journal of Derivatives 5 (3), 20-27.
- Schogl, E. (2013), "Option pricing where the underlying assest follow a Gram/Charlier density or arbitrary order", Journal of Economic Dynamics&Control 37, 61-632.
- Schoutens, W. , Simons, E. and Tistaert, J. (2003), “A Perfect Calibration! Now What?”, UCS Technical Report, Catholic University Leuven.
- Shu, J. & Zhang, J. E. (2004), “Pricing S&P 500 index options under stochastic volatility with the indirect inference method”, Journal of Derivatives Accounting 1(2): 1–16.
- Shreve, S. (2004), Stochastic Methods in Finance v2: Continuous Time Models, New York, NY: Springer-Verlag.