Multi-moment approximate option valuation models
- A general comparison -

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Contents:

- Motivation
- Literature review
- Overview of the models
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- Empirical results
- Concluding remarks
- Selected references
Motivation

- A general overview of the option pricing models using statistical series expansions, specifically Gram-Charlier and Edgeworth expansions;

- Analysis of pricing and hedging performances, compared to those of the Black and Scholes (1973) and Heston (1993) models;

- Assess whether introducing additional stochastic factors to option valuation (e.g. volatility) or specifying a different form for the risk-neutral density of the underlying asset captures more information from the market;

- Prepare and improve the necessary background for the research of more general models, which consider, amongst others, elements of the microstructure of the markets they are applied on.
Literature review

Literature review


- **Numerical methods:** Carr, Madan (1999), Negrea (2001), Moodley (2005), Schmelzle (2005).
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The Black and Scholes (1973) model - Benchmark (1)

By solving a PDE with boundary condition, Black and Scholes obtained the following form for a call option price:

\[
C_t = S_t N(d_1) - Ke^{-r(T-t)} N(d_2),
\]

where:

\[
d_1 = \frac{\ln \left( \frac{S_t}{K} \right) + \left( r + \frac{\sigma^2}{2} \right) (T-t)}{\sigma \sqrt{T-t}}\]

\[
d_2 = d_1 - \left( \sigma \sqrt{T-t} \right)
\]
Heston’s SV (1993) model - Benchmark (2)

Heston solved a PDE using a method based on characteristic functions. He proposed a similar solution to the one of Black and Scholes:

\[ C_t = S_t P_1 - K e^{-r(T-t)} P_2 \]

Using the Inversion theorem, the two probabilities can be expressed as:

\[ P_j = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \text{Re} \left[ \varphi_j(\phi) \frac{e^{-i\phi \ln(K)}}{i\phi} \right] d\phi, \]

where:

\[ \varphi_j(\phi) = \exp \left[ A_j(T-t,\phi) + B_j(T-t,\phi) V + i\phi \ln(S) \right]. \]
The Jarrow and Rudd (1982) model

Jarrow and Rudd derive an option pricing formula using a Gram - Charlier series expansion of the underlying security’s price distribution around a log-normal distribution:

\[ f(S_T) = l(S_T) - \frac{k_3}{3!} \frac{d^3[l(S_T)]}{dS_T^3} + \frac{k_4}{4!} \frac{d^4[l(S_T)]}{dS_T^4} + \varepsilon(S_T) \]

They obtain:

\[ C_{JR} = C_{BS} + [\gamma_1(f) - \gamma_1(l)] Q_3 + [\gamma_2(f) - \gamma_2(l)] Q_4, \]

where:

- \( C_{BS} \) = the Black-Scholes price of an European call
- \( \gamma_1(\cdot), \gamma_2(\cdot) \) = the R.Fisher parameters for skewness and kurtosis
- \( Q_3, Q_4 \) = the sensitivities of the price of the option to departures from log-normality
The Corrado and Su (1997) model

Corrado and Su use a Gram - Charlier Type A series expansion for the risk-neutral density of the underlying asset’s standardized return:

\[ f(z) = n(z) \left[ 1 + \frac{k_3(f)}{3!} H_3(z) + \frac{k_4(f)}{4!} H_4(z) \right] + \varepsilon(z) \]

The theoretical price of an European call option is thus:

\[ C_{CS} = C_{BS}^* + \gamma_1(f) Q_3' + \gamma_2(f) Q_4', \]

where:

\[ C_{BS}^* = C_{BS}(d^*) \]

\[ d^* = \left( \sigma \sqrt{T - t} \right)^{-1} \left[ \ln \left( \frac{S_t}{K} \right) + \left( r + \frac{\sigma^2}{2} \right) (T - t) - \ln(1 + w) \right] \]

\[ w = \frac{\gamma_1}{3!} \sigma^3 (T - t)^{\frac{3}{2}} + \frac{\gamma_2}{4!} \sigma^4 (T - t)^2 \]
The Rubinstein (1998) model

M. Rubinstein considers an Edgeworth series exp. as an approximation for the risk-neutral density of the underlying asset’s standardized log-return:

\[ f(z) = n(z) \left[ 1 + \frac{k_3(f)}{3!} H_3(z) + \frac{k_4(f)}{4!} H_4(z) + 10 \frac{k_3(f)^2}{6!} H_6(z) \right] + \epsilon \]

He then obtains the following pricing formula:

\[ C_{Rub} = C_{BS}^* + \gamma_1(f) Q_3'' + \gamma_2(f) Q_4'' + [\gamma_1(f)]^2 Q_5'' \]

where:

\[ C_{BS}^* = C_{BS}(d^*) \]

\[ d^* = \left( \sigma \sqrt{T - t} \right)^{-1} \left[ \ln \left( \frac{S_t}{K} \right) + \left( r + \frac{\sigma^2}{2} \right) (T - t) - \ln(1 + \phi) \right] \]

\[ \phi = \frac{\gamma_1(f)}{3!} \sigma^3 (T - t)^{3/2} + \frac{\gamma_2(f)}{4!} \sigma^4 (T - t)^2 + 10 \frac{[\gamma_1(f)]^2}{6!} \sigma^6 (T - t)^3 \]
Calibration method

For each of the models considered, we follow the approach of Whaley (1982) and estimate the vector of model parameters, $\Omega$, by minimizing the sum of squared deviations between the observed market prices and theoretical option prices:

$$\min_{\Omega} (F(\Omega)) = \min_{\Omega} \left( \sum_{i=1}^{N} w_i \left[ C_i^M - C_i^\Omega \right]^2 \right),$$

$$w_i = \frac{1}{|Bid_i - Ask_i|}.$$

We use the optimisation algorithm proposed by Ingber (1993), called *Adaptive Simulated Annealing (ASA).*
Data set used

- The option series used in this study contains European call and put options on the S&P500 Index, traded at the Chicago Board Options Exchange.

- Data were extracted from Reuters Datastream and contain daily closing prices for a sample period of three months, from March to May 2012.

- The risk-free interest rate was proxied by the three-months and six-months U.S. Treasury Bills rates.

- Due to reasons of brevity, we only focus on call options in this presentation.
<table>
<thead>
<tr>
<th>Moneyness (S/K)</th>
<th>Days to maturity</th>
<th>Days to maturity</th>
<th>Days to maturity</th>
<th>Days to maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Short-term &lt;60</td>
<td>Medium-term 60-120</td>
<td>Long-term &gt;120</td>
<td>Total</td>
</tr>
<tr>
<td>Deep out of the money (M &lt; 0.94)</td>
<td>223</td>
<td>206</td>
<td>232</td>
<td>661</td>
</tr>
<tr>
<td>Out of the money (0.94 &lt;= M &lt; 0.97)</td>
<td>119</td>
<td>165</td>
<td>116</td>
<td>400</td>
</tr>
<tr>
<td>At the money (0.97 &lt;= M &lt; 1.00)</td>
<td>246</td>
<td>116</td>
<td>194</td>
<td>556</td>
</tr>
<tr>
<td>At the money (1.00 &lt;= M &lt; 1.03)</td>
<td>233</td>
<td>198</td>
<td>192</td>
<td>623</td>
</tr>
<tr>
<td>In the money (1.03 &lt;= M &lt; 1.06)</td>
<td>138</td>
<td>107</td>
<td>197</td>
<td>442</td>
</tr>
<tr>
<td>Deep in the money (M &gt;= 1.06)</td>
<td>256</td>
<td>368</td>
<td>284</td>
<td>908</td>
</tr>
<tr>
<td>Total</td>
<td>1,215</td>
<td>1,160</td>
<td>1,215</td>
<td>3,590</td>
</tr>
</tbody>
</table>

Table 1: Sample properties of S&P500 Index options. Data source: Reuters Datastream, own contribution.
Stylised facts

Figure 1: S&P500 Index daily return distribution for 2000-2012 period.

Figure 2: Q-Q plot for S&P500 Index daily returns, against a normal distribution.
Stylised facts (Cont’d)

Figure 3: S&P500 Index daily return evolution from 2000 to 2012.

Figure 4: Implied volatilities for S&P500 Index call options on the 22nd of March.
## Calibration results

<table>
<thead>
<tr>
<th>Parameters</th>
<th>All Options</th>
<th>Short-Term Options</th>
<th>At-the-money Options</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Heston</td>
<td>BS</td>
<td>CS</td>
</tr>
<tr>
<td>Kappa</td>
<td>2.700</td>
<td>2.517</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.077)</td>
<td>(1.089)</td>
<td></td>
</tr>
<tr>
<td>Theta</td>
<td>0.035</td>
<td>0.037</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.010)</td>
<td></td>
</tr>
<tr>
<td>SigmaV</td>
<td>0.375</td>
<td>0.376</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.138)</td>
<td>(0.129)</td>
<td></td>
</tr>
<tr>
<td>Rho</td>
<td>-0.822</td>
<td>-0.817</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.061)</td>
<td>(0.055)</td>
<td></td>
</tr>
<tr>
<td>Vu</td>
<td>0.017</td>
<td>0.017</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.006)</td>
<td></td>
</tr>
<tr>
<td>Sigma</td>
<td>0.133</td>
<td>0.153</td>
<td>0.143</td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.021)</td>
<td>(0.018)</td>
</tr>
<tr>
<td>Skewness</td>
<td>-1.173</td>
<td>-0.880</td>
<td>-1.116</td>
</tr>
<tr>
<td></td>
<td>(0.132)</td>
<td>(0.119)</td>
<td>(0.120)</td>
</tr>
<tr>
<td></td>
<td>(0.373)</td>
<td>(0.247)</td>
<td>(0.719)</td>
</tr>
<tr>
<td>Implied Volatility</td>
<td>0.131</td>
<td>0.144</td>
<td>0.147</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td>(0.031)</td>
<td>(0.033)</td>
</tr>
</tbody>
</table>

Table 2: Each day in the sample, the parameters of a given model are estimated by minimizing the sum of squared pricing errors between the market price and the model price for each option. The daily average is reported first, followed by the standard deviation, in brackets. Data source: own contribution.
Figure 5: Implied volatility from the Black and Scholes, Corrado and Su, Jarrow and Rudd and Rubinstein models. Data source: own contribution.
Figure 6: Implied skewness from the Corrado and Su, Jarrow and Rudd and Rubinstein models. Data source: own contribution.
Figure 7: Implied kurtosis from the Corrado and Su, Jarrow and Rudd and Rubinstein models. Data source: own contribution.
<table>
<thead>
<tr>
<th>Moneyness (S/K)</th>
<th>Short-term &lt;60</th>
<th>Medium-term 60-120</th>
<th>Long-term &gt;120</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deep out of the money (M &lt; 0.94)</td>
<td>11.96</td>
<td>11.77</td>
<td>11.04</td>
<td>11.41</td>
</tr>
<tr>
<td>Out of the money (0.94 &lt;= M &lt; 0.97)</td>
<td>12.18</td>
<td>12.11</td>
<td>12.33</td>
<td>12.19</td>
</tr>
<tr>
<td>At the money (0.97 &lt;= M &lt; 1.00)</td>
<td>12.93</td>
<td>14.00</td>
<td>13.38</td>
<td>13.29</td>
</tr>
<tr>
<td>At the money (1.00 &lt;= M &lt; 1.03)</td>
<td>14.28</td>
<td>15.06</td>
<td>14.29</td>
<td>14.46</td>
</tr>
<tr>
<td>In the money (1.03 &lt;= M &lt; 1.06)</td>
<td>15.11</td>
<td>16.30</td>
<td>15.09</td>
<td>15.48</td>
</tr>
<tr>
<td>Deep in the money (M &gt;= 1.06)</td>
<td>19.51</td>
<td>18.22</td>
<td>16.64</td>
<td>17.69</td>
</tr>
<tr>
<td>Total</td>
<td>14.00</td>
<td>15.07</td>
<td>13.94</td>
<td>14.38</td>
</tr>
</tbody>
</table>

Table 3: The implied volatility is obtained by inverting the Black and Scholes formula for each option in the sample.

The implied volatilities are then averaged across each moneyness - maturity category and days in the sample.

Data source: own contribution.
Out of sample performance

<table>
<thead>
<tr>
<th>Moneyness (M=$/K)</th>
<th>All-Option-Based Days to maturity</th>
<th>Maturity-Based Days to maturity</th>
<th>Moneyness-Based Days to maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model</td>
<td>&lt;60</td>
<td>60-120</td>
</tr>
<tr>
<td></td>
<td>Absolute pricing errors</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deep out of the money (M &lt; 0.97)</td>
<td>BS</td>
<td>2.576</td>
<td>3.604</td>
</tr>
<tr>
<td></td>
<td>Heston</td>
<td>0.896</td>
<td>0.952</td>
</tr>
<tr>
<td></td>
<td>CS</td>
<td>0.770</td>
<td>1.160</td>
</tr>
<tr>
<td></td>
<td>JR</td>
<td>0.788</td>
<td>1.145</td>
</tr>
<tr>
<td></td>
<td>Rub</td>
<td>0.756</td>
<td>1.090</td>
</tr>
<tr>
<td>Out of the money (0.94 &lt;= M &lt; 0.97)</td>
<td>BS</td>
<td>2.418</td>
<td>3.798</td>
</tr>
<tr>
<td></td>
<td>Heston</td>
<td>1.391</td>
<td>2.025</td>
</tr>
<tr>
<td></td>
<td>CS</td>
<td>1.578</td>
<td>2.394</td>
</tr>
<tr>
<td></td>
<td>JR</td>
<td>1.586</td>
<td>2.407</td>
</tr>
<tr>
<td></td>
<td>Rub</td>
<td>1.572</td>
<td>2.372</td>
</tr>
<tr>
<td>At the money (0.97 &lt;= M &lt; 1.03)</td>
<td>BS</td>
<td>2.470</td>
<td>4.114</td>
</tr>
<tr>
<td></td>
<td>Heston</td>
<td>1.634</td>
<td>2.946</td>
</tr>
<tr>
<td></td>
<td>CS</td>
<td>1.775</td>
<td>2.996</td>
</tr>
<tr>
<td></td>
<td>JR</td>
<td>1.765</td>
<td>3.001</td>
</tr>
<tr>
<td>In the money (1.03 &lt;= M &lt; 1.06)</td>
<td>BS</td>
<td>3.711</td>
<td>6.017</td>
</tr>
<tr>
<td></td>
<td>Heston</td>
<td>1.512</td>
<td>2.469</td>
</tr>
<tr>
<td></td>
<td>CS</td>
<td>1.742</td>
<td>2.548</td>
</tr>
<tr>
<td></td>
<td>JR</td>
<td>1.462</td>
<td>2.394</td>
</tr>
<tr>
<td></td>
<td>Rub</td>
<td>1.532</td>
<td>2.578</td>
</tr>
<tr>
<td>Deep in the money (M &gt;= 1.06)</td>
<td>BS</td>
<td>0.968</td>
<td>4.579</td>
</tr>
<tr>
<td></td>
<td>Heston</td>
<td>2.318</td>
<td>1.935</td>
</tr>
<tr>
<td></td>
<td>CS</td>
<td>2.501</td>
<td>2.204</td>
</tr>
<tr>
<td></td>
<td>JR</td>
<td>2.301</td>
<td>1.888</td>
</tr>
<tr>
<td></td>
<td>Rub</td>
<td>2.623</td>
<td>2.424</td>
</tr>
</tbody>
</table>

Table 4: Absolute pricing errors, computed as sample averages of the absolute difference between the market price and the model price for each call in a given moneyness-maturity category.

Data source: own contribution.
### Table 5: Percentage pricing errors, computed as sample averages of the market price minus the model price, divided by the market price, for each call in a given moneyness-maturity category.

<table>
<thead>
<tr>
<th>Moneyness (M=S/K)</th>
<th>All-Option-Based</th>
<th>Maturity-Based</th>
<th>Moneyness-Based</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model</td>
<td>Days to maturity</td>
<td>Model</td>
</tr>
<tr>
<td></td>
<td>&lt;60</td>
<td>60-120</td>
<td>&gt;120</td>
</tr>
<tr>
<td>Deep out of the money (M &lt; 0.97)</td>
<td>BS</td>
<td>(0.877)</td>
<td>(0.815)</td>
</tr>
<tr>
<td></td>
<td>Heston</td>
<td>0.295</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>CS</td>
<td>0.179</td>
<td>(0.047)</td>
</tr>
<tr>
<td></td>
<td>JR</td>
<td>0.222</td>
<td>(0.058)</td>
</tr>
<tr>
<td></td>
<td>Rub</td>
<td>0.227</td>
<td>(0.050)</td>
</tr>
<tr>
<td>Out of the money (0.94 &lt;= M &lt; 0.97)</td>
<td>BS</td>
<td>(0.232)</td>
<td>(0.287)</td>
</tr>
<tr>
<td></td>
<td>Heston</td>
<td>0.059</td>
<td>(0.060)</td>
</tr>
<tr>
<td></td>
<td>CS</td>
<td>(0.114)</td>
<td>(0.158)</td>
</tr>
<tr>
<td></td>
<td>JR</td>
<td>(0.112)</td>
<td>(0.155)</td>
</tr>
<tr>
<td></td>
<td>Rub</td>
<td>(0.105)</td>
<td>(0.158)</td>
</tr>
<tr>
<td>At the money (0.97 &lt;= M &lt; 1.03)</td>
<td>BS</td>
<td>(0.005)</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>Heston</td>
<td>0.011</td>
<td>(0.029)</td>
</tr>
<tr>
<td></td>
<td>CS</td>
<td>(0.050)</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>JR</td>
<td>0.051</td>
<td>0.040</td>
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<tr>
<td></td>
<td>Rub</td>
<td>0.055</td>
<td>0.043</td>
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<td>In the money (1.03 &lt;= M &lt; 1.06)</td>
<td>BS</td>
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<td>0.008</td>
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<tr>
<td></td>
<td>Heston</td>
<td>0.009</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>CS</td>
<td>(0.020)</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>JR</td>
<td>(0.015)</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>Rub</td>
<td>(0.016)</td>
<td>0.007</td>
</tr>
<tr>
<td>Deep in the money (M &gt;= 1.06)</td>
<td>BS</td>
<td>0.012</td>
<td>0.025</td>
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<td>Heston</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>CS</td>
<td>(0.009)</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>JR</td>
<td>(0.005)</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>Rub</td>
<td>(0.009)</td>
<td>0.006</td>
</tr>
</tbody>
</table>

Data source: own contribution.
Figures 8: Dollar pricing errors for the **Black and Scholes model**. The pricing error is defined as the difference between the market price and the theoretical price. Moneyness is defined as the spot to strike ratio.

Data source: own contribution.
Figures 9: Dollar pricing errors for the **Heston model**. The pricing error is defined as the difference between the market price and the theoretical price. Moneyness is defined as the spot to strike ratio.

Data source: own contribution.
Figures 10: Dollar pricing errors for the **Corrado and Su model**. The pricing error is defined as the difference between the market price and the theoretical price. Moneyness is defined as the spot to strike ratio.

Data source: own contribution.
Figures 11: Dollar pricing errors for the Jarrow and Rudd model. The pricing error is defined as the difference between the market price and the theoretical price. Moneyness is defined as the spot to strike ratio.

Data source: own contribution.
Figures 12: Dollar pricing errors for the **Rubinstein model**. The pricing error is defined as the difference between the market price and the theoretical price. Moneyness is defined as the spot to strike ratio.

Data source: own contribution.
Hedging performance

- All hedges of call options use only the underlying index as the hedging instrument;
- All parameters implied by all options of the previous day are used to establish the current's day hedges, which are liquidated the day after;
- For each option, the hedging error is defined as:

\[ H_{j,t+1} = (C_{j,t+1}^M - \Delta_t S_{t+1}) - (C_{j,t}^M - \Delta_t S_t) e^{r/252}. \]
### Table 6: All hedges of call options use only the underlying asset as the hedging instrument.

The average absolute hedging errors are reported for each moneyness-maturity class.

Data source: own contribution.

<table>
<thead>
<tr>
<th>Moneyness (M=S/K)</th>
<th>1-Day Revision</th>
<th>Model</th>
<th>&lt;60</th>
<th>60-120</th>
<th>&gt;120</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Absolute hedging errors</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Deep out of the money (M &lt; 0.97)</strong></td>
<td></td>
<td>BS</td>
<td>0.668</td>
<td>0.680</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>Heston</td>
<td>0.809</td>
<td>0.675</td>
<td>1.188</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CS</td>
<td>0.532</td>
<td>0.792</td>
<td>1.067</td>
</tr>
<tr>
<td></td>
<td></td>
<td>JR</td>
<td>0.431</td>
<td>0.426</td>
<td>0.512</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rub</td>
<td>0.476</td>
<td>0.675</td>
<td>0.914</td>
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<tr>
<td><strong>Out of the money (0.94 &lt;= M &lt; 0.97)</strong></td>
<td></td>
<td>BS</td>
<td>0.855</td>
<td>1.363</td>
<td>0.436</td>
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<tr>
<td></td>
<td></td>
<td>Heston</td>
<td>0.522</td>
<td>1.346</td>
<td>0.450</td>
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<tr>
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<td></td>
<td>CS</td>
<td>0.718</td>
<td>1.896</td>
<td>0.439</td>
</tr>
<tr>
<td></td>
<td></td>
<td>JR</td>
<td>0.683</td>
<td>0.904</td>
<td>0.568</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rub</td>
<td>0.600</td>
<td>1.346</td>
<td>0.365</td>
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<tr>
<td><strong>At the money (0.97 &lt;= M &lt; 1.03)</strong></td>
<td></td>
<td>BS</td>
<td>0.738</td>
<td>1.058</td>
<td>1.525</td>
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<tr>
<td></td>
<td></td>
<td>Heston</td>
<td>1.168</td>
<td>0.982</td>
<td>1.877</td>
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<tr>
<td></td>
<td></td>
<td>CS</td>
<td>0.975</td>
<td>1.517</td>
<td>1.674</td>
</tr>
<tr>
<td></td>
<td></td>
<td>JR</td>
<td>0.838</td>
<td>0.971</td>
<td>1.417</td>
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<td></td>
<td>Rub</td>
<td>0.700</td>
<td>1.017</td>
<td>1.142</td>
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<tr>
<td><strong>In the money (1.03 &lt;= M &lt; 1.06)</strong></td>
<td></td>
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<td>0.998</td>
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<tr>
<td></td>
<td></td>
<td>Heston</td>
<td>0.935</td>
<td>1.624</td>
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<tr>
<td></td>
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<td>1.189</td>
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</tr>
<tr>
<td></td>
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<td>JR</td>
<td>1.704</td>
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<tr>
<td><strong>Deep in the money (M &gt;= 1.06)</strong></td>
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<tr>
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<td></td>
<td></td>
<td>Rub</td>
<td>1.439</td>
<td>1.470</td>
<td>1.842</td>
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</table>
Concluding remarks

- Even though the Black and Scholes model offers good pricing performances for at-the-money options, both the statistical series expansions models and the stochastic volatility model outperform it for all other option categories;

- The Gram-Charlier and Edgeworth expansions around the standardized normal distribution perform better than the Gram-Charlier expansion around the lognormal distribution;

- The Corrado and Su, the Jarrow and Rudd, the Rubinstein and the Heston model are able to correct the systematic biases of the Black and Scholes model;

- The Black and Scholes offers similar hedging performances for a 1-day revision horizon as the statistical series expansions pricing models, but the Rubinstein models perform the best overall.
Thank your for your attention!

Olteanu Bogdan
Selected references


Motivation

Literature review

Overview of the models

Calibration method

Empirical results

Concluding remarks


Schogl, E. (2013), "Option pricing where the underlying asset follow a Gram/Charlier density or arbitrary order", Journal of Economic Dynamics & Control 37, 61-632.

