Bucharest Academy of Economic Studies
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AN EMPIRICAL APPROACH TO REGIME SWITCHING VOLATILITY MODELS

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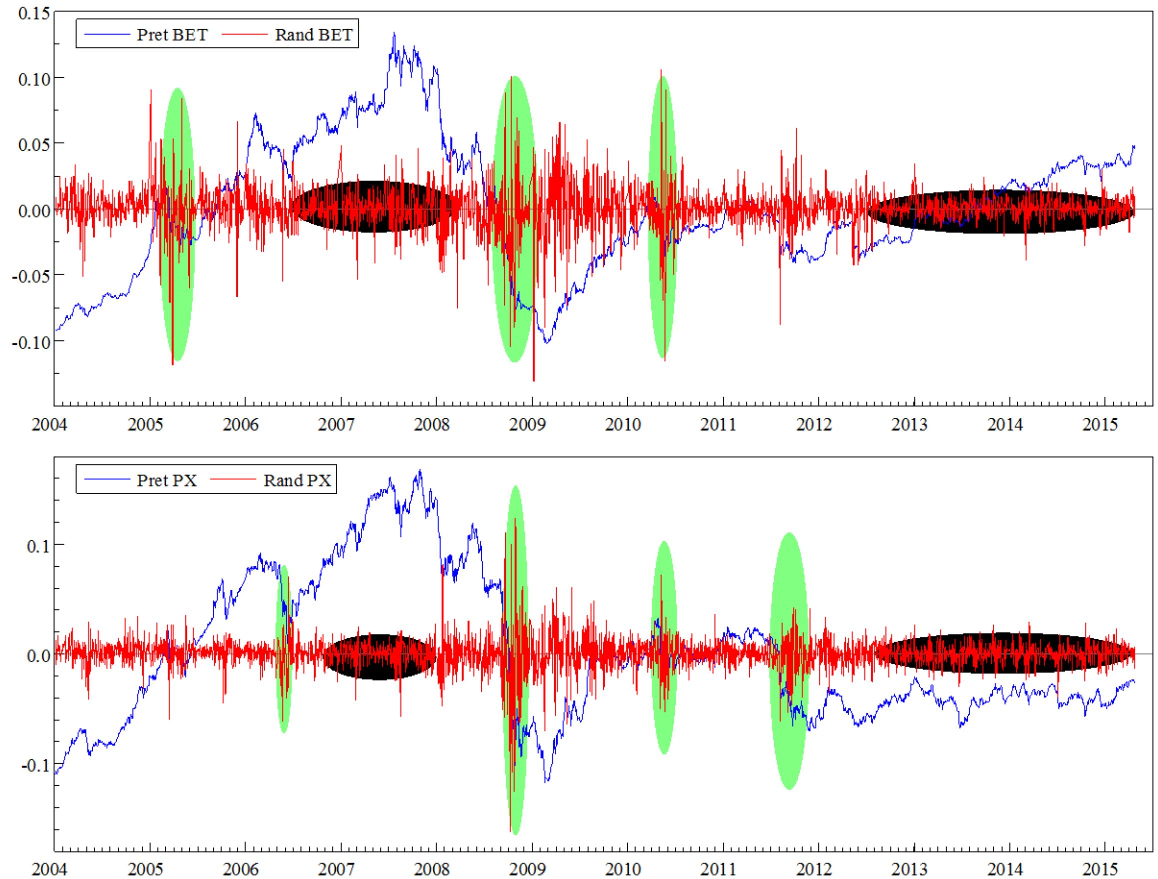
MOTIVATION
Why do we need to model volatility?

- ❑ Volatility clustering
- ❑ Leverage effects
- ❑ Structural breaks
- ❑ Volatility forecasting
  - ✓ Derivatives pricing
  - ✓ Optimal asset allocation
  - ✓ Dynamic hedging
  - ✓ Input for VaR models
  - ✓ Portfolio Risk-Management



MOTIVATION

Why using MS-GARCH models?

- Increased dependence between various Stock Exchanges across the world
- Financial time series exhibits volatility clustering and leverage effect (highlighted by the recent economic crisis)
- High persistence with GARCH models (structural breaks);
- Markov-switching ARCH [Hamilton and Susmel, 1994];
- Markov-switching GARCH [Gray, 1996; Klaassen, 2002; Marcucci, 2005];
- Very flexible models. Better volatility forecasts.

OUR CONTRIBUTION

Modelling volatility for CEE countries, **Romania** and **Czech Republic**

Modelling **structural breaks** through Markov Switching methodology

Ranking model performance using loss functions (**statistical** and **economical**) and three volatility proxies

In-sample and **out-of-sample** evaluation

Risk management perspective

Matlab advanced time-series analysis and **C++** mexing

OUTLINE

SINGLE REGIME GARCH MODEL
Conditional variance process

Conditional Mean Equation: $r_t = a + \eta_t h_t$

Conditional Variance Equation – three dimensions of GARCH (1,1):

GARCH (1,1) : $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2$

EGARCH(1,1): $\log(h_t^2) = \alpha_0 + \alpha_1 \left|\frac{\varepsilon_{t-1}}{h_{t-1}}\right| + \xi \left(\frac{\varepsilon_{t-1}}{h_{t-1}}\right) + \beta_1 \log(h_{t-1}^2)$

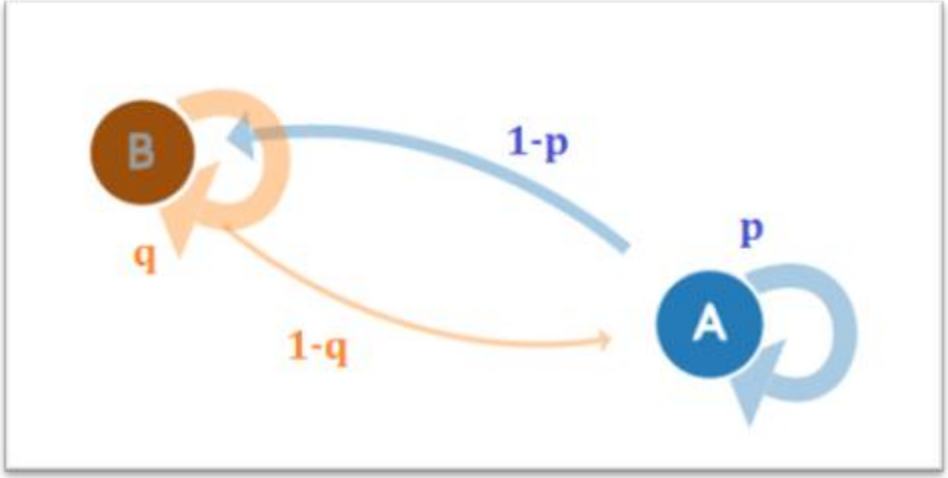
GJR-GARCH(1,1): $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 \left[1 - I_{\{\varepsilon_{t-1}>0\}}\right] + \xi \varepsilon_{t-1}^2 I_{\{\varepsilon_{t-1}>0\}} + \beta_1 h_{t-1}^2$

MS-GARCH (1, 1) MODEL
Conditional variance process

Ergodic Regular Markov Chain

$$P = \begin{pmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{pmatrix} = \begin{pmatrix} p & (1-q) \\ (1-p) & q \end{pmatrix} \xrightarrow{\text{“n” periods}} \begin{pmatrix} \pi_1 & \pi_2 \\ \pi_1 & \pi_2 \end{pmatrix}$$

Transition matrix functionality



Source: Own Computation

MS-GARCH (1, 1) MODEL
Conditional variance process

$$h_t^{2|(i)} = \alpha_0^{(i)} + \alpha_1^{(i)} \varepsilon_{t-1}^2 + \beta_1^{(i)} h_{t-1}^{2|(i)} \quad (1)$$

$$h_{t,t+1}^{2|(i)} = \Pr(s_t = 1|\xi_{t-1}) \left(\alpha_0^{(1)} + \alpha_1^{(1)} \varepsilon_t^2 + \beta_1^{(1)} h_t^{2|(1)} \right) + \Pr(s_t = 2|\xi_{t-1}) (\alpha_0^{(2)} + \alpha_1^{(2)} \varepsilon_t^2 + \beta_1^{(2)} h_t^{2|(2)}) \quad (2)$$

$$r_t|\xi_{t-1} \sim \begin{cases} f\left(\theta_t^{(1)}\right) \; with\ probability\ p_{1,t} \\ f\left(\theta_t^{(2)}\right) \; with\ probability\ (1 - p_{1,t}) \end{cases} \quad (3)$$

$$\theta_t^{(i)} = (\mu_t^{(i)}, h_t^{2|(i)}, v_t^{(i)}) \quad (4)$$

APPLICATION

Data set

- Daily observations between from **January 5, 2004** to **April 24, 2015** for : BET Index and PX Czech Index

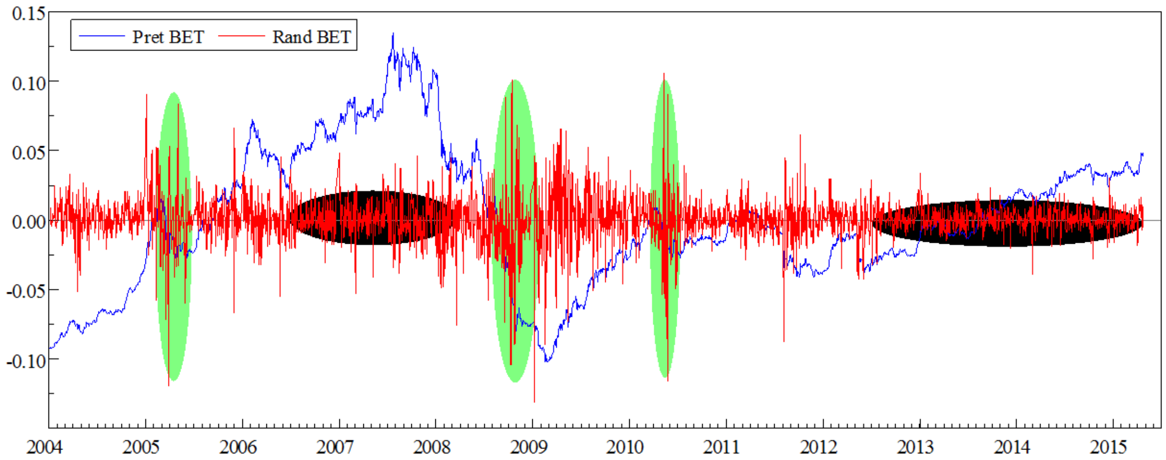
- **2826** observations for BET Index and **2844** observations for PX Index

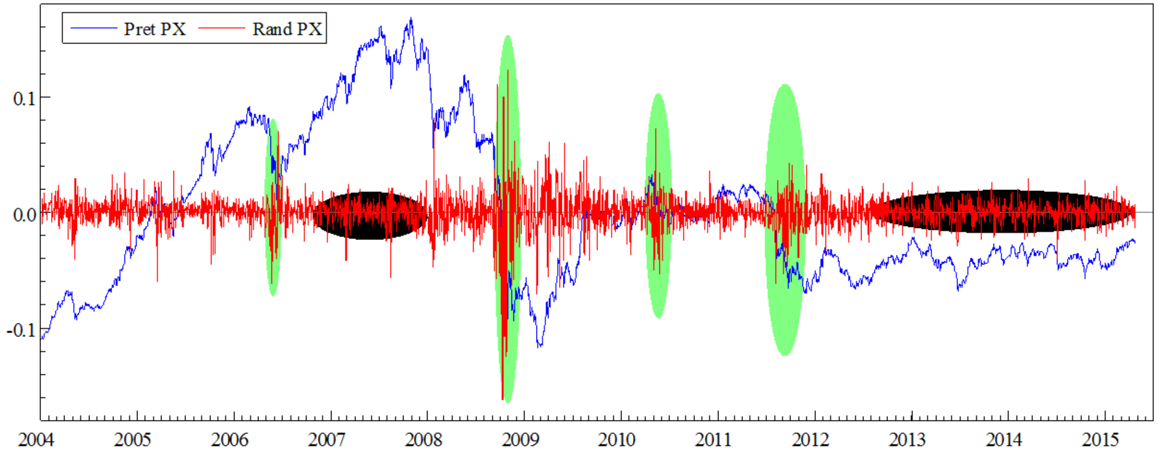
- **Out of sample** data include **122** daily observations for PX Index and **132** daily data points for BET Index. (5 minutes frequency intra-day prices)

Descriptive statistics	Min	Mean	Max	Std.dev	Skewness	Kurtosis	Jarque-Bera	ARCH(10)	Q(20)	Q*(20)
BET Index	-0.13117	0.000428	0.10565	0.016864	-0.555	11.2484	8153.4	54.575	66.9023	1451.08
							0.0000	0.0000	0.0000	0.0000
PX Index	-0.16185	0.000162	0.12364	0.014816	-0.560	17.947	26615	117.89	81.6504	3714.7
							0.0000	0.0000	0.0000	0.0000

Note: Descriptive statistcs are computed based on the entire sample.

APPLICATION
Data set





- Stationary time-series based on confirmatory ADF and KPSS tests
- Fat-tailed distribution is likely to suit the data better
- Presence of leverage effect and volatility clustering
- Excess of kurtosis for both time-series (PX and BET Index)

APPLICATION
Estimation

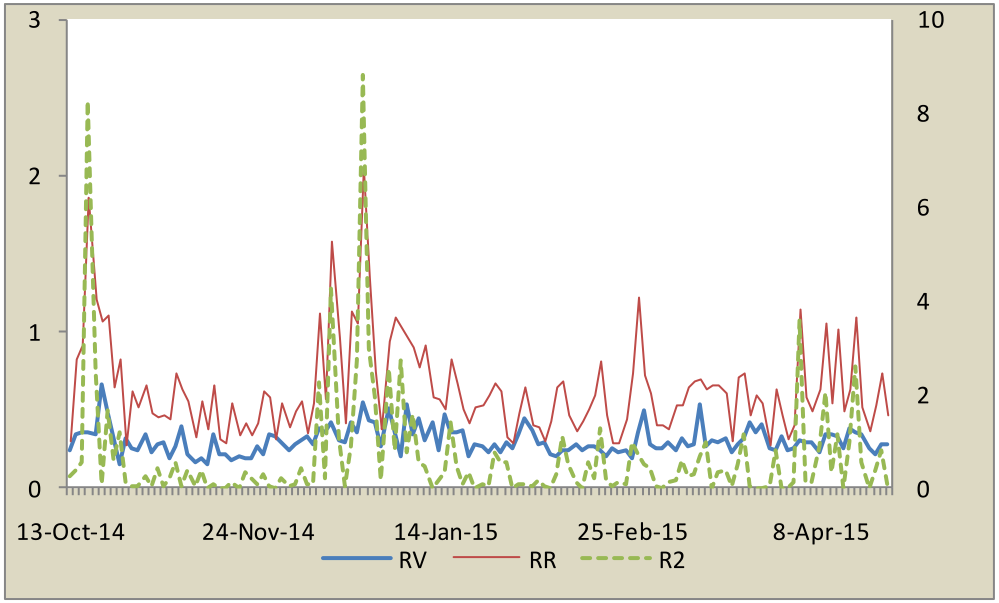
•In-sample and out-of-sample analysis on: **BET** and **PX**

•Estimated models in Matlab pursuing **Marcucci 's(2005)** methodology – Maximum Likelihood optimization through **BFGS algorithm** (*Broyden, Fetcher, Goldfarb and Shanno)*

•In-sample analysis and ranking of models using **statistical loss function** and **VaR** approach (economical loss functions) for out of sample evaluation

•Out of sample analysis based on a volatility proxies: **Realized Volatility**, **Realized Range** and **Squared Demeaned Returns**

APPLICATION
Volatility proxies



This thesis adopts three different measures of the actual volatility

1. **Realized Volatility**

$$RV_t = \sqrt{\sum_{i=1}^{k} r_{it}^2}$$

2. **Realized Range**

$$RR_t = \sqrt{\frac{1}{4\ln(2)} \ln(\frac{H_t}{L_t})^2}$$

3. **Squared Demeaned Returns**

$$R2_t = (r_t - \frac{1}{n}\sum_{i=1}^{n} r_i)^2$$

APPLICATION

Results – Single regime GARCH Estimates – BET Index

Parameters	GARCH-N	GARCH-t	GARCH-GED	EGARCH-N	EGARCH-t	EGARCH-GED	GJR-GARCH-N	GJR-GARCH-t	GJR-GARCH-GED
δ	0.0677	0.0763	0.0684	0.0594	0.0694	0.0613	0.0591	0.0692	0.0617
t-Stat	3.11	3.76	3.54	2.80	3.51	3.23	2.52	3.37	3.16
α(0)	0.0871	0.0792	0.0822	-0.2401	-0.1929	-0.2163	0.0934	0.0856	0.0886
t-Stat	11.01	6.00	6.16	-24.35	-10.39	-11.57	10.80	6.16	6.29
α(1)	0.2010	0.1776	0.1890	0.3622	0.2842	0.3177	0.2299	0.2157	0.2245
t-Stat	18.74	8.76	9.60	25.72	10.65	12.08	15.45	7.90	8.35
β (1)	0.7750	0.7933	0.7823	0.9575	0.9739	0.9673	0.7701	0.7855	0.7759
t-Stat	77.65	43.84	43.84	211.69	170.88	146.57	75.10	42.09	42.45
ξ				-0.0290	-0.0299	-0.0313	0.1774	0.1512	0.1620
t-Stat				-3.50	-2.03	-2.16	14.52	6.19	7.04
v		5.3820	1.2498		5.0726	1.2487		5.4033	1.2507
t-Stat		10.59	38.45		10.39	35.94		10.60	38.35
# of Parameters	4	5	5	5	6	6	5	6	6
LLF	**-4691.56**	**-4567.78**	**-4581.43**	**-4671.49**	**-4551.30**	**-4566.61**	**-4689.20**	**-4565.22**	**-4579.18**

Each of the considered models (GARCH, EGARCH, GJR-GARCH) was estimated using three different error distribution: Normal (N), a Student's t and a GED distribution. There are presented the MLE estimates togheter with their t-stat values which are compared against the following different critical values, dependending on the significance level. $\pm$ 2.57 (1%); $\pm$ 1.96 (5%); $\pm$ 1.645 (10%).

**Parameters are not statistically significant at a 5% level of confidence.*

- Significant in sample estimates
- Captured leverage effect through negative ξ and GJR ($\alpha_1 > \xi$)
- Conditional kurtosis displays presence of fat tails distributions

Conditional kurtosis	GARCH	EGARCH	GJR	Kurt (N)
Student	7.34	8.59	7.28	3.00
GED	5.16	5.19	5.14	3.00

APPLICATION

Results – MRS GARCH Estimates – BET Index

Parameters	MRS-N	MRS-t	MRS-t2	MRS-GED
δ (1)	0.0929	0.0877	0.0763	0.1146
t-Stat	2.12	1.06*	0.90*	1.27*
δ (2)	-0.0271	0.0659	0.0692	0.0464
t-Stat	-1.75*	1.39*	1.46*	0.95*
α (0,1)	0.2794	0.5714	0.5640	0.5528
t-Stat	2.65	1.89*	1.93*	2.14
α (0,2)	2.9850	0.0082	0.0101	0.0029
t-Stat	4.68	1.11*	1.06*	0.81*
α (1,1)	0.1518	0.4629	0.4742	0.4990
t-Stat	1.98	2.17	2.34	2.49
α (1,2)	0.3732	0.0825	0.0851	0.0687
t-Stat	3.93	2.78	2.74	2.52
β (1)	0.4700	0.3386	0.3149	0.3062
t-Stat	3.27	1.69*	1.63*	1.55*
β (2)	0.1954	0.9165	0.9140	0.9304
t-Stat	1.54*	37.28	33.51	43.42
p	0.9809	0.9803	0.9816	0.9698
t-Stat	83.76	47.91	52.15	36.46
q	0.9670	0.9911	0.9921	0.9852
t-Stat	61.55	122.49	131.69	97.32
v (1)		5.9165	7.6393	1.3422
t-Stat		4.43	1.36*	17.19
v(2)			5.5093	
t-Stat			3.75	
N. of Par.	10	12	11	11
Log(L)	-4601.49	-4540.72	-4540.31	-4553.11
pi1	63.3%	31.2%	30.1%	32.9%
pi2	36.7%	68.8%	69.9%	67.1%
Persistence 1	62.2%	80.1%	78.9%	80.5%
Persistence 2	56.9%	99.9%	99.9%	99.9%

Note: In-Sample estimates for MRS-GARCH models under various distributions
There are presented the MLE estimates togheter with their t-stat values which are compared against the following different critical values, dependending on the significance level. ± 2.57 (1%); ± 1.96 (5%); ± 1.645 (10%).
**Parameters are not statistically significant at a 5% level of confidence.*

Persistence	Normal	Student	Student	GED
MRS	0.60	0.94	0.94	0.94
GARCH	0.98	0.97	0.97	0.97

- Consistent MRS-GARCH – N mean equation (**HV** regime – **negative** estimate coefficient)
- Lower volatility **persistence** vs. GARCH models
- Unconditional probabilities
- Conditional kurtosis displays presence of fat tails distributions

Unconditional variance	MRS-N	MRS-t	MRS-t2	MRS-GED
State 1 (St=1)	0.74	2.88	2.67	2.84
State 2 (St=2)	6.92	8.03	11.33	3.47

$$Var(\varepsilon_t) = \frac{\alpha_0^{(i)}}{(1-\alpha_1^{(i)}-\beta_1^{(i)})}, where\ i = 1,2$$

APPLICATION

Results – In Sample Evaluation (Loss Functions)– BET Index

In-Sample	GARCH-N	GARCH-t	GARCH-GED	EGARCH-N	EGARCH-t	EGARCH-GED	GJR-N	GJR-t	GJR-GED	MRS-GARCH-N	MRS-GARCH-t2	MRS-GARCH-t	MRS-GARCH-GED
NUMPAR	4	5	5	5	6	6	5	6	6	10	12	11	11
PERS	0.98	0.97	0.97	0.96	0.97	0.97	0.97	0.97	0.97	0.62	1.00	1.00	1.00
AIC	3.487	3.396	3.406	3.473	3.385	3.396	3.486	3.395	3.405	3.425	3.381	3.380	3.390
Rank	13	7	9	11	3	6	12	5	8	10	2	1	4
BIC	3.496	3.407	3.417	3.484	3.398	3.409	3.497	3.408	3.418	3.447	3.407	3.405	3.414
Rank	12	3	8	11	1	6	13	5	9	10	4	2	7
LOGL	-4691.6	-4567.8	-4581.4	-4671.5	-4551.3	-4566.6	-4689.2	-4565.2	-4579.2	-4601.5	-4540.3	-4540.7	-4553.1
Rank	13	7	9	11	3	6	12	5	8	10	1	2	4
MSE1	1.569	1.535	1.538	1.491	1.496	1.480	1.560	1.530	1.532	1.497	1.831	1.780	1.830
Rank	10	7	8	2	3	1	9	5	6	4	13	11	12
MSE2	78.123	77.130	77.302	74.712	74.609	74.541	77.441	76.483	76.656	75.781	313.959	250.535	330.027
Rank	10	7	8	3	2	1	9	5	6	4	12	11	13
QLIKE	1.647	1.648	1.647	1.632	1.635	1.633	1.645	1.646	1.646	1.651	1.616	1.618	1.617
Rank	10	12	11	4	6	5	7	9	8	13	1	3	2
R2LOG	8.268	8.201	8.188	8.275	8.279	8.213	8.271	8.201	8.189	8.225	8.160	8.172	8.120
Rank	10	7	4	12	13	8	11	6	5	9	2	3	1
MAD2	3.274	3.214	3.222	3.177	3.183	3.156	3.272	3.222	3.227	3.222	3.544	3.489	3.539
Rank	10	4	5	2	3	1	9	7	8	6	13	11	12
MAD1	0.892	0.882	0.882	0.887	0.889	0.881	0.892	0.882	0.882	0.892	0.890	0.890	0.886
Rank	13	3	2	7	8	1	12	5	4	11	10	9	6
HMSE	7.049	7.351	7.388	6.360	6.805	6.859	7.239	7.724	7.708	8.176	6.525	6.474	6.530
Rank	7	9	10	1	5	6	8	12	11	13	3	2	4

$$MSE_1 = n^{-1}\sum_{t=1}^{n}(\sigma_{t+1} - h_{t+1|t})^2 \quad (1)$$

$$MSE_2 = n^{-1}\sum_{t=1}^{n}({\sigma_{t+1}}^2 - {h_{t+1|t}}^2)^2 \quad (2)$$

$$QLIKE = n^{-1}\sum_{t=1}^{n}\left(\log\left({h_{t+1|t}}^2\right) + {\sigma_{t+1}}^2{h_{t+1|t}}^{-2}\right) \quad (3)$$

$$R2LOG = n^{-1}\sum_{t=1}^{n}\left(\log\left(\mathrm{h}_{t+1|t}^2\sigma_{t+1}^2\right)\right)^2 \quad (4)$$

$$MAD_1 = n^{-1}\sum_{t=1}^{n}|\sigma_{t+1} - h_{t+1|t}| \quad (5)$$

$$MAD_2 = n^{-1}\sum_{t=1}^{n}|\sigma_{t+1}^2 - h_{t+1|t}^2| \quad (5)$$

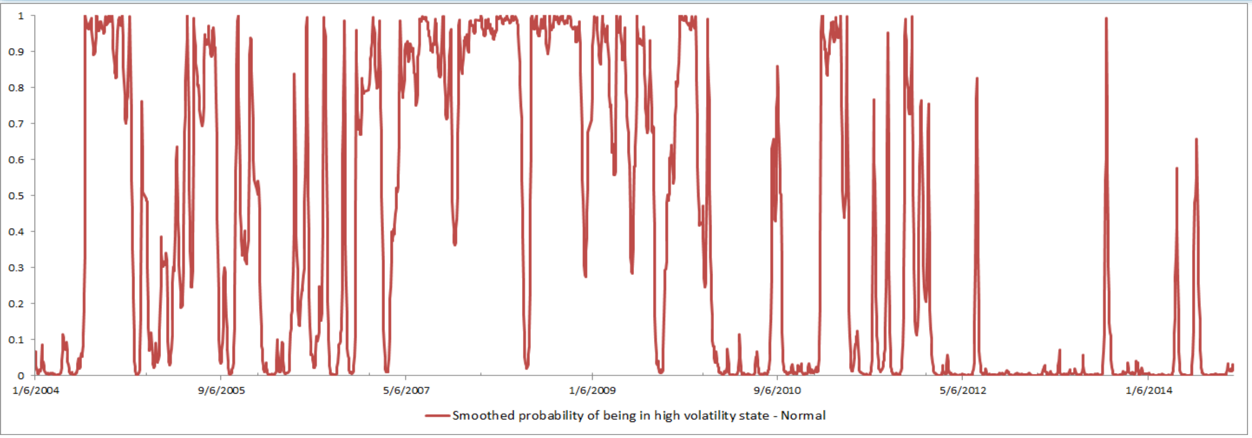
$$HMSE = T^{-1}\sum_{t=1}^{T}(\sigma_{t+1}^2 h_{t+1|t}^2 - 1)^2 \quad (7)$$

- EGARCH models best fits the in sample volatility process.

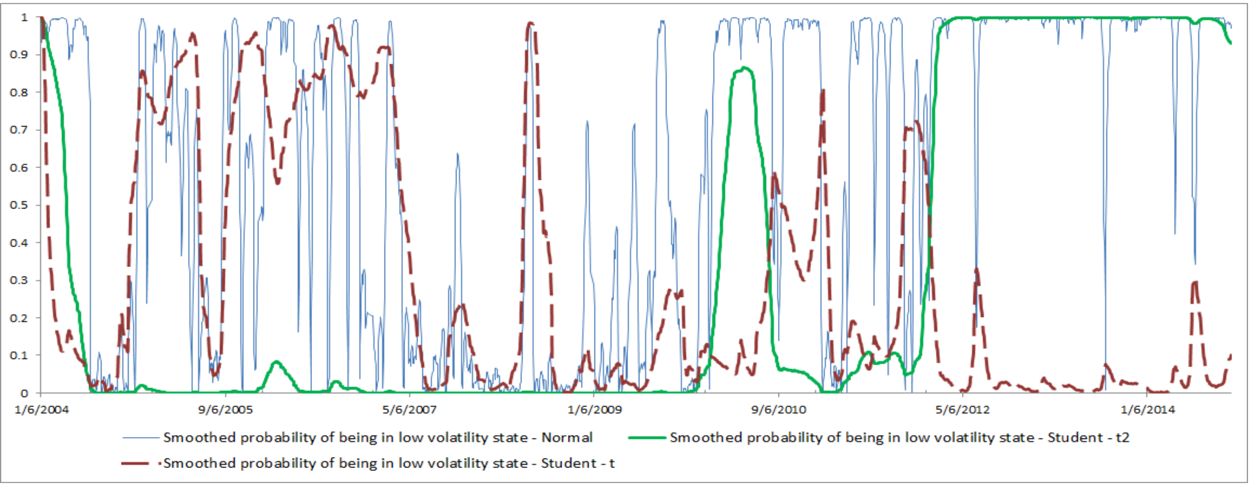
- MRS-GARCH models display lower losses under QLIKE and R2LOG *LF*

APPLICATION

Results – In Sample Evaluation (cont'd) – BET Index



- Use the entire information set (smoothing)
- Smoothed probabilities plots exhibits consistency with past economic events
- 2007-2009 period is described by HV regime



- Regime persistence relation with the leptokurtosis by a *t*-distribution
- Without this, the persistence of the, for example, low-volatility regime would have been lower, since then a large sudden change in the return rate would have been considered earlier as a shift to the high-volatility regime.

APPLICATION

Results – Out-of-sample (Loss functions) – BET Index

1 step ahead volatility - proxy: Realized Volatility

Model	MSE1	Rank	MSE2	Rank	QLIKE	Rank	R2LOG	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.5476	3	0.9137	3	-0.2832	3	9.9152	3	0.6975	3	0.803	3	0.898	2	0.67	3.4348**
GARCH-t	0.5362	2	0.862	1	-0.286	2	9.8745	2	0.6936	2	0.79	2	0.8997	3	0.67	3.4583**
GARCH-GED	0.5313	1	0.864	2	-0.3034	1	9.7777	1	0.6876	1	0.783	1	0.8965	1	0.67	3.6341**
EGARCH-N	0.709	12	1.4454	12	-0.0977	10	11.1416	12	0.7919	12	1.0012	12	0.9181	8	0.68	3.8553**
EGARCH-t	0.6632	8	1.2378	9	-0.1281	7	10.911	6	0.7716	8	0.9473	8	0.9164	7	0.65	3.0567**
EGARCH-GED	0.6692	10	1.2867	11	-0.1344	5	10.8823	5	0.7715	7	0.9536	10	0.9153	5	0.68	3.8553**
GJR-N	0.6724	11	1.2472	10	-0.0921	11	11.1095	10	0.7833	11	0.9605	11	0.9218	12	0.72	4.7619**
GJR-t	0.6499	6	1.1583	4	-0.113	9	10.9604	9	0.7718	9	0.9337	7	0.9203	10	0.7	4.2060**
GJR-GED	0.6489	5	1.1696	5	-0.1195	8	10.9213	8	0.7696	6	0.932	6	0.9196	9	0.7	4.3860**
MRS-GARCH-N	0.8499	13	2.3095	13	0.0323	13	12.0388	13	0.8587	13	1.1664	13	0.9299	13	0.64	2.7488**
MRS-GARCH-t2	0.6513	7	1.2011	6	-0.1284	6	10.9148	7	0.7671	5	0.9319	5	0.9161	6	0.6	1.8420*
MRS-GARCH-t	0.6479	4	1.2168	8	-0.141	4	10.8334	4	0.7625	4	0.9272	4	0.9151	4	0.61	2.1303*
MRS-GARCH-GED	0.664	9	1.2028	7	-0.0913	12	11.1287	11	0.7808	10	0.9498	9	0.9212	11	0.62	2.4568**

5 steps ahead volatility - proxy: Realized Volatility

Model	MSE1	Rank	MSE2	Rank	QLIKE	Rank	R2LOG	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	3.2525	6	27.6945	6	1.5029	6	12.7398	6	1.7361	6	4.6425	6	0.9226	6	0.7	5.0335**
GARCH-t	3.1006	5	25.2268	4	1.4707	5	12.5037	5	1.6981	5	4.4597	5	0.9204	5	0.7	4.9766**
GARCH-GED	3.0975	4	25.4812	5	1.4642	4	12.4652	4	1.6937	4	4.4535	4	0.9195	4	0.71	5.1895**
EGARCH-N	2.0903	2	14.1397	3	1.115	2	10.2576	2	1.3618	2	3.1806	2	0.8813	2	0.7	4.9266**
EGARCH-t	2.1228	3	14.1329	2	1.1409	3	10.3759	3	1.3809	3	3.2292	3	0.8856	3	0.7	4.7674**
EGARCH-GED	2.0519	1	13.5862	1	1.1058	1	10.1794	1	1.3517	1	3.1347	1	0.8805	1	0.7	4.9266**
GJR-N	3.908	12	36.9244	12	1.6635	12	13.9807	12	1.9129	12	5.4327	12	0.9352	12	0.71	5.1895**
GJR-t	3.6998	9	33.3111	7	1.6217	9	13.6723	9	1.8625	9	5.1837	9	0.9324	10	0.69	4.6080**
GJR-GED	3.7154	10	33.8289	8	1.6219	10	13.678	10	1.8642	10	5.2009	10	0.9323	9	0.7	4.9766**
MRS-GARCH-N	5.1159	13	69.4254	13	1.8391	13	15.3081	13	2.1523	13	6.8424	13	0.9452	13	0.65	4.0775**
MRS-GARCH-t2	3.6712	8	34.8337	9	1.5811	8	13.2993	8	1.8347	8	5.1441	8	0.9276	8	0.64	3.2420**
MRS-GARCH-t	3.6275	7	35.2469	11	1.5587	7	13.1589	7	1.8143	7	5.0849	7	0.9256	7	0.61	2.7964**
MRS-GARCH-GED	3.8213	11	35.127	10	1.65	11	13.7534	11	1.8956	11	5.3373	11	0.9343	11	0.61	2.7126**

- SR (Success Ratio) and DA (Directional Accuracy) tests for directional accuracy of volatility forecasts against volatility proxies
- $SR = m^{-1}\sum_{j=1}^{m} I_{\{\bar{\sigma}_{t+j}\bar{h}_{t+j|t+j-1}>0\}}$
- High SR (more than 55 % and an average of 70%)
- Highly significant DA test for all horizons (1, 5, 10, 22 steps)

APPLICATION

Results – Out-of-sample (Loss functions) – BET Index

10 steps ahead volatility - proxy: Realized Volatility																
Model	MSE1	Rank	MSE2	Rank	QLIKE	Rank	R2LOG	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	7.8736	6	138.179	6	2.3636	8	17.5675	8	2.7268	7	10.7597	6	0.938	8	0.58	3.3470**
GARCH-t	7.3179	4	121.127	4	2.306	4	17.1061	4	2.6299	4	10.1014	4	0.9344	5	0.6	3.5878**
GARCH-GED	7.3695	5	123.468	5	2.3085	5	17.1327	5	2.6361	5	10.1592	5	0.9344	6	0.59	3.4676**
EGARCH-N	2.5929	1	23.7594	1	1.4442	1	11.3635	1	1.4991	1	4.1467	1	0.8315	1	0.58	3.0690**
EGARCH-t	2.7718	3	26.1013	3	1.5019	3	11.6527	3	1.5609	3	4.3998	3	0.8428	3	0.58	2.9454**
EGARCH-GED	2.6038	2	23.8354	2	1.4496	2	11.3706	2	1.5046	2	4.1669	2	0.8329	2	0.58	3.0690**
GJR-N	9.2344	12	179.137	12	2.5001	12	18.8466	12	2.958	12	12.3387	12	0.9458	12	0.55	2.5719**
GJR-t	8.5696	9	156.551	7	2.4387	9	18.3488	10	2.8491	9	11.5574	9	0.9423	9	0.57	2.8215**
GJR-GED	8.6536	10	160.044	9	2.4449	10	18.4047	11	2.8611	10	11.6538	10	0.9426	10	0.57	2.8215**
MRS-GARCH-N	12.0717	13	318.145	13	2.6952	13	20.571	13	3.346	13	15.5702	13	0.9549	13	0.51	2.0972*
MRS-GARCH-t2	8.2195	8	158.657	8	2.3605	7	17.5161	7	2.7537	8	11.1656	8	0.9362	7	0.59	3.1921**
MRS-GARCH-t	8.0828	7	160.204	10	2.3296	6	17.3421	6	2.7122	6	10.9797	7	0.9338	4	0.56	2.6970**
MRS-GARCH-GED	8.7933	11	165.132	11	2.4573	11	18.2714	9	2.887	11	11.8536	11	0.9434	11	0.52	1.5517

22 steps ahead volatility - proxy: Realized Volatility																
Model	MSE1	Rank	MSE2	Rank	QLIKE	Rank	R2LOG	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	23.885	8	1021.41	8	3.4261	9	29.9102	8	4.8121	9	30.5871	8	0.9582	9	0.41	-0.078
GARCH-t	21.3149	5	833.431	4	3.3286	6	28.9676	6	4.5432	6	27.6308	5	0.9539	6	0.42	-0.1724
GARCH-GED	21.7271	7	864.644	5	3.3439	7	29.1112	7	4.5859	7	28.1056	7	0.9546	7	0.42	0.0323
EGARCH-N	2.7143	1	29.5899	1	1.6816	1	15.9611	1	1.492	1	4.6108	1	0.7261	1	0.4	-0.7903
EGARCH-t	3.0245	3	34.7806	3	1.7538	3	16.5149	3	1.5856	3	5.0489	3	0.7459	3	0.36	-1.7408
EGARCH-GED	2.7687	2	30.4459	2	1.6927	2	16.0837	2	1.5065	2	4.6812	2	0.7283	2	0.39	-1.095
GJR-N	26.8959	12	1255.95	12	3.5292	12	31.0273	12	5.1081	12	33.9837	12	0.962	12	0.39	-0.8203
GJR-t	24.1346	10	1033.43	9	3.4352	10	30.1052	9	4.8357	10	30.8296	10	0.9582	10	0.4	-0.5939
GJR-GED	24.5842	11	1070.33	11	3.4505	11	30.2508	11	4.88	11	31.3445	11	0.9589	11	0.39	-0.7064
MRS-GARCH-N	33.5698	13	1941.02	13	3.71	13	33.0061	13	5.6834	13	41.4157	13	0.9679	13	0.34	-1.1535
MRS-GARCH-t2	21.6459	6	939.168	7	3.2895	5	28.809	5	4.502	5	27.9129	6	0.9499	5	0.39	-1.095
MRS-GARCH-t	21.0553	4	936.386	6	3.2397	4	28.6135	4	4.3938	4	27.0877	4	0.9456	4	0.35	-1.9779
MRS-GARCH-GED	24.1235	9	1044.67	10	3.4185	8	30.2466	10	4.8036	8	30.656	9	0.956	8	0.31	-3.2938

- Single state models outperforms MRS-GARCH models under all periods and all statistical loss functions

APPLICATION

Results – Out-of-sample (DM test for EPA - Equal predictive ability) – BET Index

DM: Benchmark - **GARCH GED** - 1 step ahead volatility - proxy: Realized Volatility

Model	MSE1	MSE2	QLIKE	R2LOG	MAD2	MAD1	HMSE
GARCH-N	-5.33**	-2.96**	-8.88**	-9.25**	-5.68**	-8.28**	-3.70**
p-values	0.00	0.00	0.00	0.00	0.00	0.00	0.00
GARCH-t	-1.81	0.2	-5.05**	-4.16**	-2.13*	-3.72**	-4.25**
p-values	0.07	0.84	0.00	0.00	0.03	0.00	0.00
EGARCH-N	-3.69**	-2.76**	-4.35**	-4.34**	-3.78**	-4.23**	-3.84**
p-values	0.00	0.01	0.00	0.00	0.00	0.00	0.00
EGARCH-t	-3.51**	-2.68**	-4.07**	-3.98**	-3.60**	-3.96**	-3.88**
p-values	0.00	0.01	0.00	0.00	0.00	0.00	0.00
EGARCH-GED	-3.43**	-2.70**	-3.73**	-3.72**	-3.49**	-3.74**	-3.43**
p-values	0.00	0.01	0.00	0.00	0.00	0.00	0.00
GJR-N	-3.97**	-2.85**	-5.31**	-5.22**	-4.09**	-4.81**	-4.55**
p-values	0.00	0.00	0.00	0.00	0.00	0.00	0.00
GJR-t	-3.64**	-2.73**	-4.75**	-4.61**	-3.75**	-4.32**	-4.31**
p-values	0.00	0.01	0.00	0.00	0.00	0.00	0.00
GJR-GED	-3.48**	-2.64**	-4.54**	-4.40**	-3.58**	-4.12**	-4.16**
p-values	0.00	0.01	0.00	0.00	0.00	0.00	0.00
MRS-GARCH-N	-3.27**	-2.17*	-6.43**	-5.96**	-3.43**	-4.89**	-5.37**
p-values	0.00	0.03	0.00	0.00	0.00	0.00	0.00
MRS-GARCH-t2	-3.06**	-2.43*	-3.87**	-3.58**	-3.14**	-3.50**	-4.89**
p-values	0.00	0.02	0.00	0.00	0.00	0.00	0.00
MRS-GARCH-t	-2.77**	-2.29*	-3.54**	-3.25**	-2.85**	-3.19**	-4.37**
p-values	0.01	0.02	0.00	0.00	0.00	0.00	0.00
MRS-GARCH-GED	-5.03**	-3.68**	-5.44**	-5.20**	-5.20**	-5.47**	-4.93**
p-values	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Note: * and ** represent the DM statistics for which one can reject the null hypothesis of equal predictive accuracy at 5% and 1% respectively.
'+ and ++ represent the DM statistics for which one can reject the null at 5% and 1% respectively, but the sign of the statistics is positive, indicating that the benchmark implies a bigger loss.

- Diebold Mariano test for EPA

$d_t \equiv [g(e_{i,t}) - g(e_{j,t})]$.

- where g – loss function and $e_{i,t}$ - forecast error

$H_0: E(d_t) = 0\ \forall t$
$H_1: E(d_t) \neq 0\ \forall t$

DM-test statistic $DM = \frac{\bar{d}}{\sqrt{Var(\bar{d})}} \sim N(0,1)$

- Benchmark model (GARCH GED) significantly outperforms each one of the other models
- Negative DM statistics = benchmarks' loss is always lower than the competing model loss
- Modified DM lead to the same outcomes

APPLICATION

Results – Out-of-sample (Risk Management perspective) – BET Index

Risk management out-of-sample evaluation (VaR 95%)

Steps	1						5						10					
Model	TUFF	PF(%)	Rank	**LRPF**	**LRind**	**LRcc**	TUFF	PF(%)	Rank	**LRPF**	**LRind**	**LRcc**	TUFF	PF(%)	Rank	**LRPF**	**LRind**	**LRcc**
GARCH-N	3	4.545	3	0.059	1.33	1.389	0	4.688	3	0.027	22.927*	22.954*	36	1.626	5	3.952*	6.071*	10.024*
GARCH-t	3	3.03	2	1.247	0.252	1.5	38	3.906	2	0.347	25.544*	25.892*	37	0.813	1	6.891*	0.017	6.907*
GARCH-GED	3	4.545	3	0.059	1.33	1.389	0	4.688	3	0.027	22.927*	22.954*	36	1.626	4	3.952*	6.071*	10.024*
EGARCH-N	3	3.03	2	1.247	0.252	1.5	38	3.906	2	0.347	25.544*	25.892*	33	10.569	7	6.171*	41.261*	47.432*
EGARCH-t	3	3.03	2	1.247	0.252	1.5	38	3.906	2	0.347	25.544*	25.892*	33	8.13	6	2.151	29.331*	31.482*
EGARCH-GED	3	3.03	2	1.247	0.252	1.5	38	3.906	2	0.347	25.544*	25.892*	33	9.756	6	4.641*	37.414*	42.055*
GJR-N	3	3.03	2	1.247	0.252	1.5	38	3.906	2	0.347	25.544*	25.892*	36	1.626	4	3.952*	6.071*	10.024*
GJR-t	3	3.03	2	1.247	0.252	1.5	38	3.125	1	1.087	9.547*	10.634*	37	0.813	1	6.891*	0.017	6.907*
GJR-GED	3	3.03	2	1.247	0.252	1.5	38	3.906	2	0.347	25.544*	25.892*	36	1.626	3	3.952*	6.071*	10.024*
MRS-GARCH-N	3	3.03	2	1.247	0.252	1.5	38	3.906	2	0.347	25.544*	25.892*	37	0.813	1	6.891*	0.017	6.907*
MRS-GARCH-t2	3	2.273	1	2.572	0.141	2.712	38	3.906	2	0.347	25.544*	25.892*	37	0.813	1	6.891*	0.017	6.907*
MRS-GARCH-t	3	3.03	2	1.247	0.252	1.5	38	3.906	2	0.347	25.544*	25.892*	34	4.065	2	0.241	25.219*	25.460*
MRS-GARCH-GED	3	3.03	2	1.247	0.252	1.5	0	4.688	3	0.027	22.927*	22.954*	37	1.626	1	3.952*	0.067	4.019

Note: TUFF - time until first failure; PF - proportion of failures; LRPF - Likelihood ratio test for unconditional coverage (*H0 : PF = p*); LRind - Likelihood ratio test for independence; LRcc - Likelihood ratio test for conditional coverage; * - indicates significance at 5%

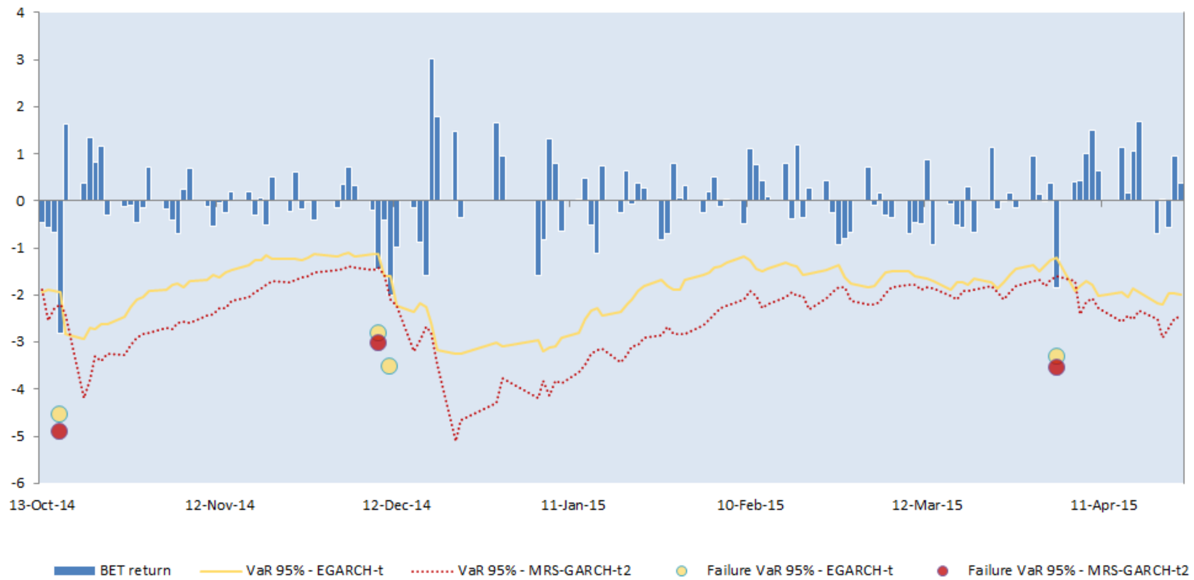
- $VaR_t^i = [n, \propto] = \mu_{t+n}^i + \phi(\alpha)\sqrt{h_{t+n}^i}$ - $\alpha = 1\%$ *and* 5%
- $LR_{UC} = LR_{PF} = -2log\frac{p^{n-1}(1-p)^{n_0}}{\hat{\pi}^{n_1}(1-\hat{\pi})^{n_o}} \sim X_{(1)}^n$

$H_0: p = \hat{\pi}$
$H_1: p \neq \hat{\pi}$

- The accuracy drops as the forecasting period increases
- On short term, the MRS-GARCH model outperform under VaR framework
- Results are consistent with Marcucci (2005)

APPLICATION

Results – Out-of-sample (Risk Management perspective) cont'd – BET Index



- 1 day ahead VaR limit under various GARCH specification (single and multiple regimes)
- MRS-GARCH models adapts faster to shocks in comparison with standard GARCH models
- EGARCH-GED 95% VaR displayed a double failure in a short span during December 2014 negative shocks (Ukraine conflict and tapering in USA)

APPLICATION

Results – Single regime GARCH Estimates – PX Index

Parameters	GARCH-N	GARCH-t	GARCH-GED	EGARCH-N	EGARCH-t	EGARCH-GED	GJR-GARCH-N	GJR-GARCH-t	GJR-GARCH-GED
δ	0.0698	0.0788	0.0808	0.0495	0.0662	0.0655	0.0462	0.0657	0.0649
t-Stat	3.48	4.10	4.24	2.41	3.48	3.43	2.22	3.39	3.34
α(0)	0.0743	0.0750	0.0756	-0.1870	-0.1808	-0.1846	0.0846	0.0818	0.0836
t-Stat	8.46	5.66	5.90	-13.70	-9.30	-9.70	8.56	5.89	6.12
α(1)	0.1493	0.1437	0.1466	0.2581	0.2470	0.2519	0.2063	0.1938	0.1994
t-Stat	12.04	7.76	8.23	13.83	9.33	9.72	11.75	7.72	8.26
β (1)	0.8072	0.8096	0.8067	0.9645	0.9688	0.9666	0.8008	0.8049	0.8014
t-Stat	56.15	40.13	40.07	220.50	145.92	153.18	50.32	38.18	37.53
ξ				-0.0702	-0.0590	-0.0641	0.0858	0.0882	0.0869
t-Stat				-8.24	-4.25	-5.08	6.00	4.22	4.31
v		7.0618	1.4447		7.2330	1.4703		7.3067	1.4672
t-Stat		8.19	31.05		7.87	30.69		7.73	30.14
# of Parameters	4	5	5	5	6	6	5	6	6
LLF	**-4322.91**	**-4261.42**	**-4276.53**	**-4299.01**	**-4243.63**	**-4258.89**	**-4307.17**	**-4252.87**	**-4266.54**

Each of the considered models (GARCH, EGARCH, GJR-GARCH) was estimated using three different error distribution: Normal (N), a Student's t and a GED distribution. There are presented the MLE estimates togheter with their t-stat values which are compared against the following different critical values, dependending on the significance level. ± 2.57 (1%); ± 1.96 (5%); ± 1.645 (10%).

**Parameters are not statistically significant at a 5% level of confidence.*

- Significant estimates
- Captured leverage effect through negative ξ and GJR $(\alpha_1 > \xi)$
- Conditional kurtosis displays presence of fat tails distributions

Conditional kurtosis	GARCH	EGARCH	GJR	Kurt (N)
Student	4.96	4.86	4.81	3.00
GED	4.43	4.24	4.26	3.00

APPLICATION

Results – MRS GARCH Estimates – PX Index

Parameters	MRS-N	MRS-t	MRS-t2	MRS-GED
δ (1)	-0.2950	0.0253	0.2017	0.1472
t-Stat	-5.64	0.86*	3.32	2.74
δ (2)	0.1164	0.1987	0.0244	-0.1854
t-Stat	2.62	3.21	0.86*	-3.89
α (0,1)	0.3653	0.0444	0.1407	0.1391
t-Stat	2.30	1.72*	1.46*	1.94*
α (0,2)	0.1415	0.1282	0.0437	0.1550
t-Stat	2.80	1.52*	1.77*	1.38*
α (1,1)	0.0727	0.1251	0.1615	0.0331
t-Stat	1.24*	2.99	1.55*	0.46*
α (1,2)	0.0178	0.1520	0.1233	0.1032
t-Stat	0.41*	1.68*	3.12	1.72*
β (1)	0.9226	0.8582	0.7147	0.7333
t-Stat	11.47	21.70	5.20	7.23
β (2)	0.7329	0.7163	0.8569	0.8897
t-Stat	11.12	5.38	22.15	10.65
p	0.8756	0.9997	0.9998	0.9611
t-Stat	14.85	607.14	1422.86	32.95
q	0.9568	0.9998	0.9997	0.9349
t-Stat	45.01	1427.19	609.81	20.82
v (1)		6.7889	4.9304	1.5821
t-Stat		3.94	2.73	11.39
v(2)			8.0353	
t-Stat			2.72	
N. of Par.	10	12	11	11
Log(L)	-4270.93	-4241.62	-4243.65	-4257.85
pi1	74.2%	54.4%	46.2%	37.4%
pi2	25.8%	45.6%	53.8%	62.6%
Persistence 1	99.5%	98.3%	87.6%	76.6%
Persistence 2	75.1%	86.8%	98.0%	99.3%

Note: In-Sample estimates for MRS-GARCH models under various distributions
There are presented the MLE estimates togheter with their t-stat values which are compared against the following different critical values, dependending on the significance level. ± 2.57 (1%); ± 1.96 (5%); ± 1.645 (10%).
**Parameters are not statistically significant at a 5% level of confidence.*

Persistence	Normal	Student	Student	GED
MRS - GARCH	0.93	0.93	0.93	0.91
GARCH	0.96	0.95	0.95	0.95

Unconditional variance	MRS-N	MRS-t	MRS-t2	MRS-GED
State 1 (St=1)	76.70	2.64	1.14	0.60
State 2 (St=2)	0.57	0.97	2.22	21.80

APPLICATION

Results – In Sample Evaluation (Loss Functions)– PX Index

In-Sample	GARCH-N	GARCH-t	GARCH-GED	EGARCH-N	EGARCH-t	EGARCH-GED	GJR-N	GJR-t	GJR-GED	MRS-GARCH-N	MRS-GARCH-t2	MRS-GARCH-t	MRS-GARCH-GED
NUMPAR	4	5	5	5	6	6	5	6	6	10	12	11	11
PERS	0.96	0.95	0.95	0.96	0.97	0.97	0.95	0.95	0.95	1.00	0.98	0.98	0.99
AIC	3.180	3.136	3.147	3.164	3.124	3.135	3.170	3.130	3.140	3.147	3.127	3.127	3.138
Rank	13	6	10	11	1	5	12	4	8	9	2	3	7
BIC	3.189	3.147	3.158	3.174	3.137	3.148	3.180	3.143	3.153	3.168	3.153	3.151	3.162
Rank	13	3	8	11	1	4	12	2	7	10	6	5	9
LOGL	-4322.9	-4261.4	-4276.5	-4299.0	-4243.6	-4258.9	-4307.2	-4252.9	-4266.5	-4270.9	-4241.6	-4243.7	-4257.8
Rank	13	7	10	11	2	6	12	4	8	9	1	3	5
MSE1	1.059	1.050	1.050	1.009	1.011	1.008	1.030	1.026	1.024	1.038	1.060	1.066	1.042
Rank	11	9	10	2	3	1	6	5	4	7	12	13	8
MSE2	65.369	65.346	65.308	64.434	64.836	64.700	62.382	62.621	62.473	67.712	65.662	65.802	66.649
Rank	9	8	7	4	6	5	1	3	2	13	10	11	12
QLIKE	1.339	1.338	1.338	1.323	1.323	1.323	1.329	1.328	1.328	1.344	1.332	1.334	1.333
Rank	12	11	10	1	3	2	6	5	4	13	7	9	8
R2LOG	6.690	6.666	6.663	6.643	6.639	6.625	6.646	6.631	6.624	6.599	6.656	6.647	6.580
Rank	13	12	11	7	6	4	8	5	3	2	10	9	1
MAD2	2.299	2.284	2.284	2.231	2.233	2.227	2.267	2.259	2.256	2.262	2.315	2.325	2.275
Rank	11	9	10	2	3	1	7	5	4	6	12	13	8
MAD1	0.722	0.718	0.718	0.712	0.712	0.710	0.716	0.714	0.714	0.713	0.720	0.720	0.711
Rank	13	10	9	4	3	1	8	7	6	5	11	12	2
HMSE	3.801	3.853	3.857	3.635	3.677	3.693	3.607	3.650	3.661	4.057	3.892	4.016	3.933
Rank	7	8	9	2	5	6	1	3	4	13	10	12	11

$$MSE_1 = n^{-1}\sum_{t=1}^{n}(\sigma_{t+1} - h_{t+1|t})^2 \quad (1)$$

$$MSE_2 = n^{-1}\sum_{t=1}^{n}({\sigma_{t+1}}^2 - {h_{t+1|t}}^2)^2 \quad (2)$$

$$QLIKE = n^{-1}\sum_{t=1}^{n}\left(\log\left({h_{t+1|t}}^2\right) + {\sigma_{t+1}}^2 {h_{t+1|t}}^{-2}\right) \quad (3)$$

$$R2LOG = n^{-1}\sum_{t=1}^{n}\left(\log\left(\mathrm{h}_{t+1|t}^2 \sigma_{t+1}^2\right)\right)^2 \quad (4)$$

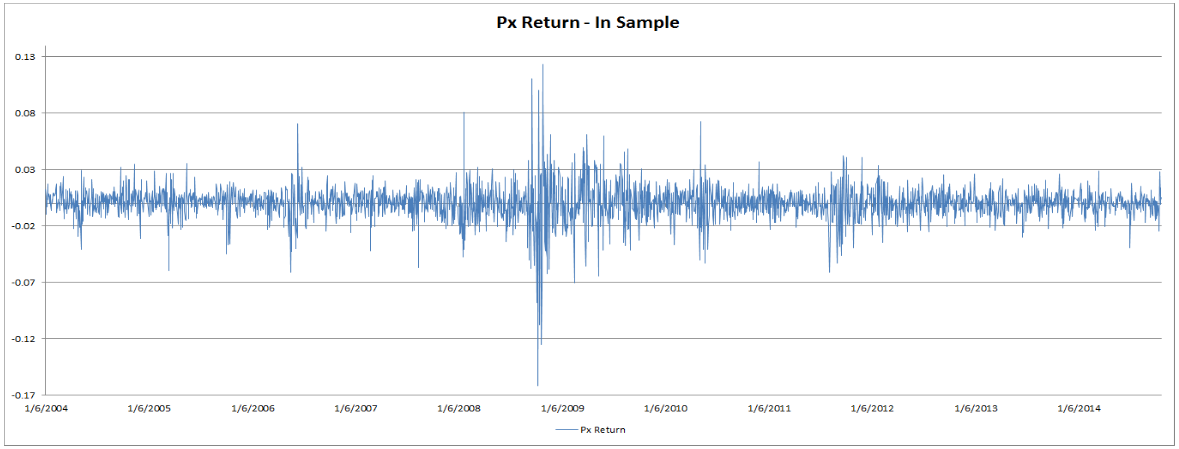
$$MAD_1 = n^{-1}\sum_{t=1}^{n}|\sigma_{t+1} - h_{t+1|t}| \quad (5)$$

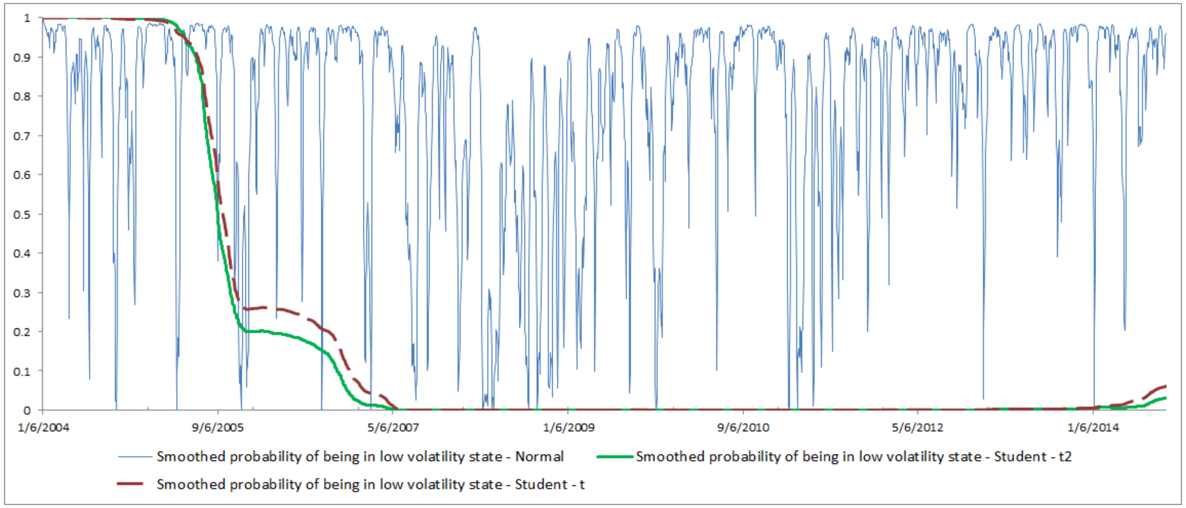
$$MAD_2 = n^{-1}\sum_{t=1}^{n}|\sigma_{t+1}^2 - h_{t+1|t}^2| \quad (5)$$

$$HMSE = T^{-1}\sum_{t=1}^{T}(\sigma_{t+1}^2 h_{t+1|t}^2 - 1)^2 \quad (7)$$

APPLICATION

Results – In Sample Evaluation (cont'd) – PX Index





APPLICATION

Results – Out-of-sample (Loss functions) – BET Index

Model	MSE1	Rank	MSE2	Rank	QLIKE	Rank	R2LOG	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.1374	3	0.3432	3	0.258	3	0.968	3	0.3411	3	0.5282	3	0.3525	3	0.56	1.1386
GARCH-t	0.1355	2	0.3361	2	0.2559	2	0.9601	2	0.3398	2	0.5247	2	0.3517	2	0.56	1.1386
GARCH-GED	0.1346	1	0.3338	1	0.2537	1	0.9537	1	0.3378	1	0.5215	1	0.3496	1	0.56	1.1386
EGARCH-N	0.1918	12	0.5561	12	0.3302	11	1.212	11	0.4011	11	0.6562	11	0.4013	11	0.57	1.2697
EGARCH-t	0.1883	10	0.5399	10	0.3267	10	1.1989	10	0.3983	10	0.6492	10	0.3995	10	0.58	1.4986
EGARCH-GED	0.1882	9	0.5414	11	0.3255	9	1.1958	9	0.3973	9	0.648	9	0.3982	9	0.57	1.3499
GJR-N	0.1722	7	0.4646	7	0.3145	8	1.1462	8	0.3889	8	0.6211	8	0.3974	8	0.57	1.3499
GJR-t	0.1693	6	0.4531	4	0.3107	7	1.133	7	0.3859	7	0.6146	6	0.395	7	0.57	1.3499
GJR-GED	0.1693	5	0.4542	5	0.3101	6	1.1316	6	0.3854	6	0.614	5	0.3944	6	0.57	1.3499
MRS-GARCH-N	0.1664	4	0.4606	6	0.3021	4	1.1039	4	0.3813	4	0.6088	4	0.39	5	0.61	1.8057*
MRS-GARCH-t2	0.1917	11	0.5222	9	0.3484	12	1.2564	12	0.4152	12	0.6711	12	0.4222	12	0.6	1.9545*
MRS-GARCH-t	0.2017	13	0.5579	13	0.3628	13	1.3052	13	0.4266	13	0.6946	13	0.4318	13	0.6	2.0341*
MRS-GARCH-GED	0.1724	8	0.4884	8	0.3046	5	1.1196	5	0.383	5	0.6167	7	0.3881	4	0.6	1.5018

5 steps ahead volatility - proxy: Realized Volatility																
Model	MSE1	Rank	MSE2	Rank	QLIKE	Rank	R2LOG	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.932	6	10.7254	6	1.9005	6	3.7076	6	0.8779	6	3.0216	6	0.3963	6	0.61	2.4996**
GARCH-t	0.9113	5	10.3739	5	1.8951	5	3.682	5	0.8689	5	2.9783	5	0.3931	5	0.63	2.8653**
GARCH-GED	0.9089	4	10.3496	4	1.8939	4	3.6776	4	0.8667	4	2.9705	4	0.392	4	0.62	2.6949**
EGARCH-N	0.5229	2	5.2505	2	1.7447	2	3.0774	2	0.603	2	1.9401	2	0.2674	2	0.66	3.5471**
EGARCH-t	0.5272	3	5.2748	3	1.7469	3	3.0899	3	0.6072	3	1.9536	3	0.2695	3	0.68	3.9278**
EGARCH-GED	0.5183	1	5.176	1	1.743	1	3.072	1	0.5998	1	1.9269	1	0.2659	1	0.68	3.9278**
GJR-N	1.0945	10	13.6687	9	1.9519	11	3.854	10	0.9743	11	3.4526	11	0.4372	11	0.68	3.9278**
GJR-t	1.0641	7	13.1231	7	1.9432	8	3.8227	8	0.9591	8	3.3827	8	0.4313	8	0.67	3.7374**
GJR-GED	1.0656	8	13.1729	8	1.9436	9	3.8206	7	0.96	9	3.3873	9	0.4316	9	0.68	3.9278**
MRS-GARCH-N	1.1115	11	14.2114	11	1.949	10	3.9023	11	0.9687	10	3.4428	10	0.4322	10	0.7	4.5006**
MRS-GARCH-t2	1.2031	12	15.2623	12	1.9787	12	4.0398	12	1.0159	12	3.6429	12	0.4506	12	0.66	3.3839**
MRS-GARCH-t	1.2697	13	16.4588	13	1.9962	13	4.1135	13	1.0464	13	3.7869	13	0.4616	13	0.65	3.2096**
MRS-GARCH-GED	1.0746	9	13.9573	10	1.928	7	3.8508	9	0.9344	7	3.3219	7	0.4135	7	0.66	3.5370**

APPLICATION

Results – Out-of-sample (Loss functions) – BET Index

10 steps ahead volatility - proxy: Realized Volatility																
Model	MSE1	Rank	MSE2	Rank	QLIKE	Rank	R2LOG	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	2.5294	6	54.2538	6	2.6285	6	8.1197	6	1.4369	6	6.8992	6	0.453	6	0.66	3.8580**
GARCH-t	2.4564	4	51.8646	4	2.619	4	8.0671	5	1.4145	4	6.7509	4	0.4475	5	0.68	4.1617**
GARCH-GED	2.4577	5	51.9634	5	2.619	5	8.065	4	1.4147	5	6.7543	5	0.4475	4	0.67	4.0095**
EGARCH-N	0.715	2	7.99	2	2.2853	2	6.3646	1	0.5369	2	2.0283	2	0.1661	2	0.66	3.7782**
EGARCH-t	0.7315	3	8.2342	3	2.2887	3	6.4004	3	0.5468	3	2.0732	3	0.1685	3	0.66	3.9275**
EGARCH-GED	0.7143	1	7.9495	1	2.285	1	6.3678	2	0.5357	1	2.0217	1	0.1654	1	0.66	3.9275**
GJR-N	2.8384	10	65.6246	10	2.6752	10	8.2494	10	1.5546	10	7.667	10	0.4865	10	0.64	3.4065**
GJR-t	2.7371	8	62.0935	7	2.6613	8	8.1906	8	1.5212	8	7.4467	8	0.4779	8	0.64	3.4065**
GJR-GED	2.7442	9	62.4301	8	2.6626	9	8.1874	7	1.5246	9	7.4695	9	0.479	9	0.64	3.4065**
MRS-GARCH-N	2.9755	11	71.6222	11	2.6917	11	8.2948	11	1.5953	12	7.9549	12	0.4969	12	0.61	3.2984**
MRS-GARCH-t2	3.075	12	72.3394	12	2.6949	12	8.5494	12	1.5951	11	7.9543	11	0.4906	11	0.66	3.9275**
MRS-GARCH-t	3.2552	13	78.8002	13	2.7164	13	8.6652	13	1.647	13	8.3104	13	0.5028	13	0.66	3.8580**
MRS-GARCH-GED	2.7281	7	64.7886	9	2.641	7	8.2071	9	1.4756	7	7.2574	7	0.4574	7	0.61	3.0572**

22 steps ahead volatility - proxy: Realized Volatility																
Model	MSE1	Rank	MSE2	Rank	QLIKE	Rank	R2LOG	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	8.7652	6	383.486	6	3.4791	7	20.2051	9	2.7063	7	18.8413	7	0.5595	7	0.35	-3.2509
GARCH-t	8.4299	4	359.352	4	3.4612	4	20.0731	4	2.6454	4	18.2257	4	0.5508	5	0.35	-3.2509
GARCH-GED	8.4701	5	362.422	5	3.4636	5	20.0846	5	2.6537	5	18.3094	5	0.5521	6	0.36	-3.1618
EGARCH-N	1.4467	1	14.3664	1	2.9222	1	14.7742	1	0.7891	2	2.9618	2	0.3277	2	0.36	-2.8553
EGARCH-t	1.4703	3	14.6684	3	2.9232	2	14.8446	3	0.7857	1	2.9485	1	0.3185	1	0.34	-3.3412
EGARCH-GED	1.4523	2	14.4303	2	2.9234	3	14.7885	2	0.7901	3	2.9642	3	0.3294	3	0.34	-3.3412
GJR-N	9.3114	10	428.566	10	3.5116	10	20.3257	10	2.8224	10	20.0252	10	0.5781	11	0.41	-1.0846
GJR-t	8.877	8	396.203	7	3.4884	8	20.1678	7	2.7426	8	19.2124	8	0.5667	8	0.41	-1.0846
GJR-GED	8.9166	9	399.371	8	3.491	9	20.1753	8	2.7518	9	19.3036	9	0.5683	9	0.41	-1.0846
MRS-GARCH-N	10.3217	12	509.88	12	3.5657	13	20.611	11	3.0122	13	21.9947	13	0.6059	13	0.39	0.0293
MRS-GARCH-t2	9.8878	11	466.844	11	3.5273	11	20.7301	12	2.8678	11	20.5755	11	0.5764	10	0.37	-2.766
MRS-GARCH-t	10.5655	13	520.305	13	3.5586	12	20.9868	13	2.9759	12	21.7302	12	0.5898	12	0.37	-2.766
MRS-GARCH-GED	8.7778	7	402.731	9	3.4668	6	20.1415	6	2.67	6	18.6935	6	0.5479	4	0.3	-2.3188

APPLICATION

Results – Out-of-sample (DM test for EPA - Equal predictive ability) – BET Index

DM: Benchmark - **GARCH GED** - 1 step ahead volatility - proxy: Realized Volatility

Model	MSE1	MSE2	QLIKE	R2LOG	MAD2	MAD1	HMSE
GARCH-N	-5.53**	-4.60**	-6.82**	-6.54**	-5.81**	-6.34**	-7.29**
p-values	0.00	0.00	0.00	0.00	0.00	0.00	0.00
GARCH-t	-3.97**	-2.85**	-5.65**	-5.11**	-5.20**	-5.81**	-6.36**
p-values	0.00	0.00	0.00	0.00	0.00	0.00	0.00
EGARCH-N	-2.50*	-2.30*	-2.77**	-2.71**	-2.62**	-2.78**	-2.92**
p-values	0.01	0.02	0.01	0.01	0.01	0.01	0.00
EGARCH-t	-2.51*	-2.33*	-2.77**	-2.69**	-2.65**	-2.81**	-2.95**
p-values	0.01	0.02	0.01	0.01	0.01	0.00	0.00
EGARCH-GED	-2.43*	-2.26*	-2.66**	-2.60**	-2.54*	-2.68**	-2.79**
p-values	0.02	0.02	0.01	0.01	0.01	0.01	0.01
GJR-N	-2.41*	-2.15*	-2.85**	-2.72**	-2.63**	-2.89**	-3.15**
p-values	0.02	0.03	0.00	0.01	0.01	0.00	0.00
GJR-t	-2.37*	-2.11*	-2.80**	-2.67**	-2.61**	-2.86**	-3.12**
p-values	0.02	0.03	0.01	0.01	0.01	0.00	0.00
GJR-GED	-2.28*	-2.04*	-2.67**	-2.56*	-2.49*	-2.73**	-2.98**
p-values	0.02	0.04	0.01	0.01	0.01	0.01	0.00
MRS-GARCH-N	-1.72	-1.66	-1.95	-1.85	-1.91	-2.07*	-2.24*
p-values	0.08	0.10	0.05	0.06	0.06	0.04	0.03
MRS-GARCH-t2	-7.16**	-5.41**	-11.24**	-10.00**	-8.88**	-11.08**	-12.58**
p-values	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MRS-GARCH-t	-7.45**	-5.56**	-12.17**	-10.65**	-9.32**	-11.88**	-13.95**
p-values	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MRS-GARCH-GED	-1.78	-1.75	-2.00*	-1.88	-1.99*	-2.14*	-2.35*
p-values	0.07	0.08	0.05	0.06	0.05	0.03	0.02

Note: * and ** represent the DM statistics for which one can reject the null hypothesis of equal predictive accuracy at 5% and 1% respectively.
'+ and ++ represent the DM statistics for which one can reject the null at 5% and 1% respectively, but the sign of the statistics is positive, indicating that the benchmark implies a bigger loss.

- Diebold Mariano test for EPA

$d_t \equiv [g(e_{i,t}) - g(e_{j,t})]$.

- where g – loss function and $e_{i,t}$ - forecast error

$H_0: E(d_t) = 0\ \forall t$
$H_1: E(d_t) \neq 0\ \forall t$

DM-test statistic $DM = \frac{\bar{d}}{\sqrt{\widehat{Var}(\bar{d})}} \sim N(0,1)$

APPLICATION

Results – Out-of-sample (Risk Management perspective) – BET Index

Risk management out-of-sample evaluation (VaR 95%)

Steps	1						5						10					
Model	TUFF	PF(%)	Rank	**LRPF**	**LRind**	**LRcc**	TUFF	PF(%)	Rank	**LRPF**	**LRind**	**LRcc**	TUFF	PF(%)	Rank	**LRPF**	**LRind**	**LRcc**
GARCH-N	27	5.738	5	0.134	0.861	0.994	30	3.39	3	0.723	18.924*	19.647*	26	3.54	4	3.952*	6.071*	10.024*
GARCH-t	27	1.639	2	3.883*	0.067	3.95	30	2.542	2	1.816	12.621*	14.438*	26	2.655	3	6.891*	0.017	6.907*
GARCH-GED	27	5.738	5	0.134	0.861	0.994	30	3.39	3	0.723	18.924*	19.647*	25	4.425	5	3.952*	6.071*	10.024*
EGARCH-N	27	4.098	4	0.222	0.432	0.653	27	4.237	4	0.152	14.515*	14.667*	24	7.965	8	6.171*	41.261*	47.432*
EGARCH-t	27	1.639	2	3.883*	0.067	3.95	29	3.39	3	0.723	18.924*	19.647*	25	6.195	7	2.151	29.331*	31.482*
EGARCH-GED	27	4.098	4	0.222	0.432	0.653	27	4.237	4	0.152	14.515*	14.667*	24	7.965	7	4.641*	37.414*	42.055*
GJR-N	27	3.279	3	0.862	0.274	1.136	30	2.542	2	1.816	12.621*	14.438*	25	3.54	4	3.952*	6.071*	10.024*
GJR-t	27	1.639	2	3.883*	0.067	3.95	30	2.542	2	1.816	12.621*	14.438*	26	1.77	2	6.891*	0.017	6.907*
GJR-GED	27	4.098	4	0.222	0.432	0.653	30	2.542	2	1.816	12.621*	14.438*	25	3.54	4	3.952*	6.071*	10.024*
MRS-GARCH-N	27	3.279	3	0.862	0.274	1.136	30	2.542	2	1.816	12.621*	14.438*	25	2.655	4	6.891*	0.017	6.907*
MRS-GARCH-t2	122	0	1	12.516*	NaN	NaN	118	0	1	12.105*	NaN	NaN	27	0.885	1	6.891*	0.017	6.907*
MRS-GARCH-t	27	1.639	2	3.883*	0.067	3.95	30	2.542	2	1.816	12.621*	14.438*	26	1.77	2	0.241	25.219*	25.460*
MRS-GARCH-GED	27	4.098	4	0.222	0.432	0.653	30	2.542	3	1.816	12.621*	14.438*	25	5.31	6	3.952*	0.067	4.019

Note: TUFF - time until first failure; PF - proportion of failures; LRPF - Likelihood ratio test for unconditional coverage (*H0 : PF = p*); LRind - Likelihood ratio test for independence; LRcc - Likelihood ratio test for conditional coverage; * - indicates significance at 5%

- $VaR_t^i = [n, \propto] = \mu_{t+n}^i + \phi(\alpha)\sqrt{h_{t+n}^i}$ - $\alpha = 1\%$ *and* 5%

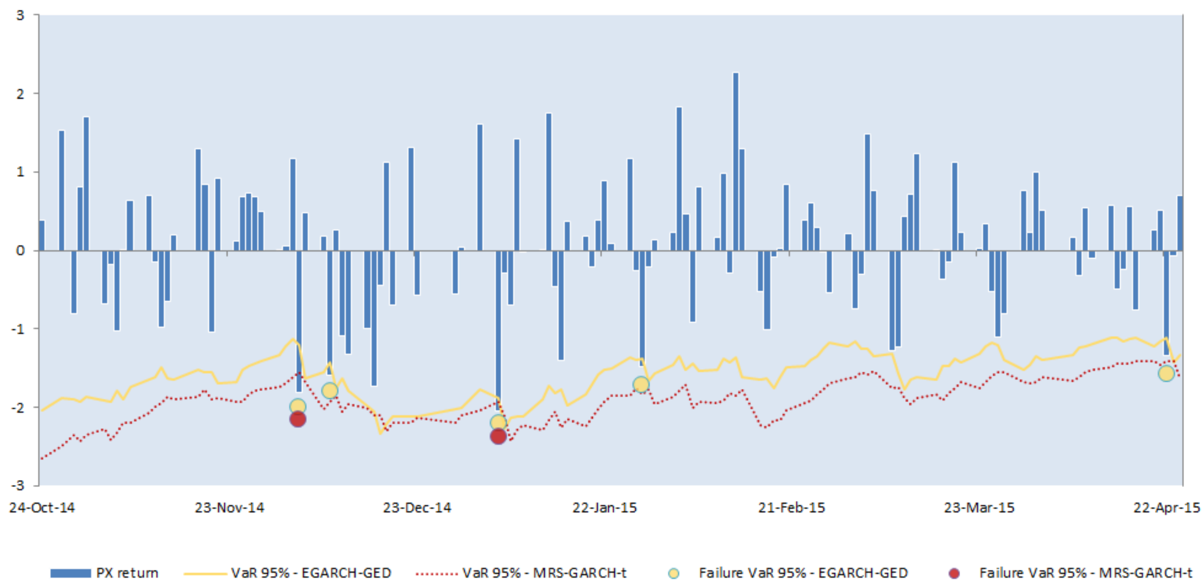
- $LR_{UC} = LR_{PF} = -2log\frac{p^{n-1}(1-p)^{n_0}}{\hat{\pi}^{n_1}(1-\hat{\pi})^{n_o}} \sim X_{(1)}^n$

$H_0 : p = \hat{\pi}$
$H_1 : p \neq \hat{\pi}$

- The accuracy drops as the forecasting period increases
- On short term, the MRS-GARCH model outperform under VaR framework
- Results are consistent with Marcucci (2005)

APPLICATION

Results – Out-of-sample (Risk Management perspective) cont'd – BET Index



- 1 day ahead VaR limit under various GARCH specification (single and multiple regimes)

CONCLUSIONS

- The in sample evaluation fail to provide a “star” volatility model for neither countries.

- However, the in sample performance of EGARCH class model should be noted, with sensible differences among innovation’s conditional distribution.

- Tesults also confirm Dacco and Satchell’s (1999) arguments regarding that more parsimonious models perform better in a loss function framework

- From OOS evaluation the outcomes show that MRS-GARCH models underperform in terms of volatility forecasting (not consistent with our theoretical expectations of enhanced perfomance)

- VaR results outlines the fact that MRS-GARCH models adapts faster to shocks in contrast with standard GARCH models.

- Using a *t*-distribution instead of a normal one for the error term helps make the regimes more stable

- Future research: understand "the economics of volatility" and further replace the unobservable volatility measure with a realized variable (RV or High Freq. Data)

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Thank you! ☺

“You cannot stop the waves, but you can learn to surf”
Jon Kabat-Zinn