ACADEMY OF ECONOMIC STUDIES
DOCTORAL SCHOOL OF FINANCE AND BANKING

THE FRACTAL MARKET HYPOTHESIS AND ITS IMPLICATIONS FOR THE STABILITY OF FINANCIAL MARKETS

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CONTENTS

I. PROBLEM REVIEW

- The efficient markets hypothesis has been a cornerstone of mainstream financial economics for decades, but it provides no testable predictions about extreme events and even considers them highly improbable (or even non-existent). So, the efficient market hypothesis gives an incorrect view of agents behavior.
- On the other side, the fractal market hypothesis takes into account that on the market heterogeneous agents co-exist who react to the inflowing information with respect to their investment horizon. What is considered a negative information and thus a selling signal for an investor with short horizon might be a buying opportunity for an investor with long horizon, and vice versa.

II. OBJECTIVES

GENERAL OBJECTIVES:

- Study if the prediction of the fractal markets hypothesis about a dominance of specific investment horizons during turbulent times holds.
- Characterize the efficiency of stock market in various countries (compute efficiency index and entropy for stock indexes).
- Compute the contagion correlations between different stock market data, taking into account different periods (before and after the global financial crisis).

INTERMEDIARY OBJECTIVES:

- Compute the multifractal spectrum for analyzed series.
- Compute the Hurst exponent (moving window =250 days) and see if the Hurst coefficient varies with the moment q, and if that the series are characterized by a multifractal spectrum.
- Analyze the volatility throughout a comparison between GARCH and Markov Switching Multifractal model.

III. LITERATURE REVIEW

- Various methods have been used and developed to analyze the fractal behavior in financial data: studies from Hurst (1951) on rescaled range statistical analysis and the modified rescaled range analysis of Lo (1991), the detrended fluctuation analysis (Ausloos, 2000; Bartolozzi et al., 2007; Di Matteo, 2007), or its generalization, multifractal detrended fluctuation analysis (Kantelhardt et al., 2002). To measure the efficiency of stock market, a measure has been introduced by Pincus (1991), namely approximate entropy, measuring the irregularity of time series data. Louis Bachelier, who anticipated many of the mathematical discoveries made later by Norbert Wiener and A.A.Markov, proposed the concept of entropic analysis of equity prices in 1900 (Reddy and Sebastin 2006).
- Regarding wavelet power and its connection to fractal market hypothesis, in 'Fractal Markets Hypothesis and the Global Financial Crisis: Wavelet Power Evidence' (2013), Ladislav Kristoufek, study if the most turbulent times of the Global Financial Crisis can be characterized by the dominance of short investment horizons.
- Analyzing the volatility and identifying a multifractal behavior in this process has been done through Markov Switching Multifractal model, introduced by Fisher and Calvet (2004). The Markov Switching Multifractal improves on the Multifractal Model of Assets Return's combinatorial construction by randomizing arrival times and hence guaranteeing a strictly stationary process and providing a pure regime-switching formulation of multifractal measures, which were pioneered by Benoit Mandelbrot.

IV. METHODOLOGY. DATA AND RESULTS

The set of data contains information beginning with September 1997 until may 2014. Data was obtained from www.reuters.com

Table 1. List with analyzed stock indexes

Index	Name
ATHEX	Athens Stock Exchange
BET	Bucharest Stock Exchange index
BUX	Budapest Stock Exchange Index
CAC40	Paris Stock Exchange Index
EURO STOXX 50	Euro Area Stock Market
FTSE	London Stock Exchange Index
IBEX35	Spain Stock Exchange Index
PSI	Portugal Stock Exchange Index
PX	Prague Stock Exchange Index
RTS	Russian Stock Exchange Index
SAX	Bratislava Stock Exchange Index
SOFIX	Bulgaria Stock Exchange Index
UX	Ukrainian index
WIG20	Warsaw Stock Exchange Index

IV.1. MULTIFRACTAL ANALYSIS

1.a. Multifractal Detrended Fluctuation Analysis

- Given the initial series, calculate the cumulative series X(i) = $\sum_{k=1}^{i}[x_k - \langle x \rangle]$; <x> is the mean of the initial series. The time series are divided into Ns= N/s (independent segments of same length s). A polynomial trend is fitted to X (stock exchange index) within each segment and m defines the order of the polynomial.
- Average over all the segments is made in order to obtain the q-th order fluctuation function Fq(s). The scale behavior of the fluctuations functions is obtained by log-regressing $F_q(s)$ over the scale s for several values of q.
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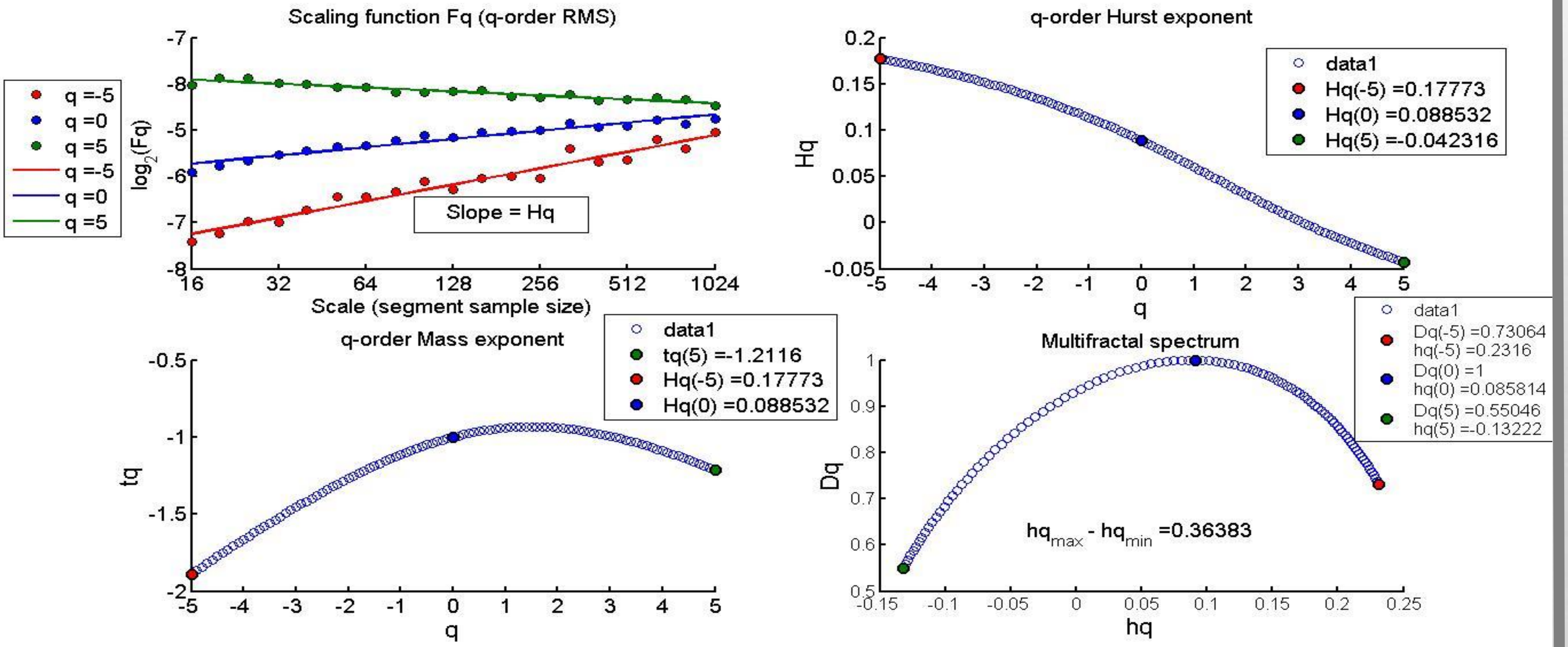
$$F_q(s)=\left\{\frac{1}{N_s}\sum_{v=1}^{N_s}[F^2(s,v)]^{q/2}\right\}^{1/q}$$

- The Hurst exponent is the relationship between scale s and F(q);

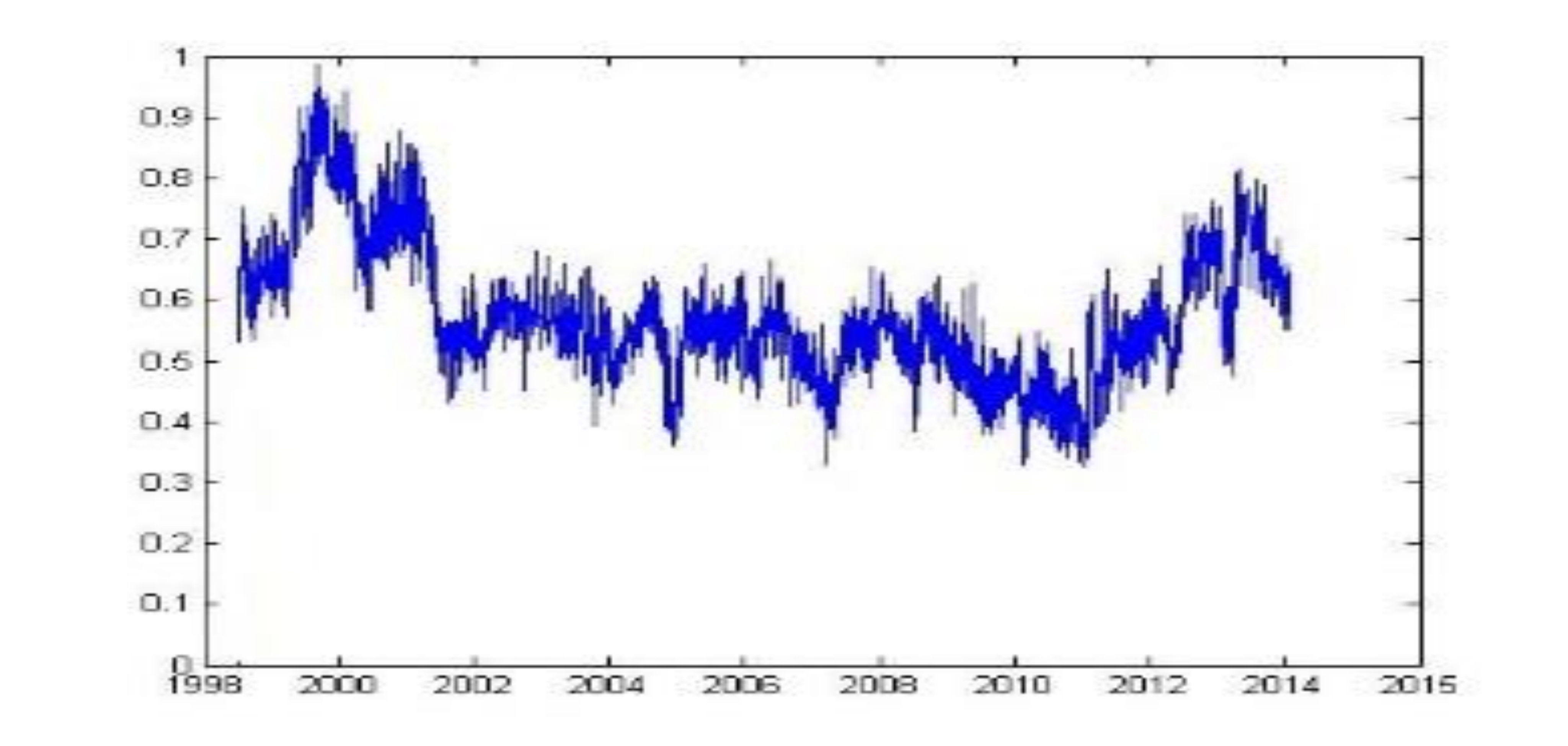
$$log_2(Fq(s)) = H(q) * log_2(s) + C$$

Figure 1. Multifractal Detrended Fluctuation Analysis for BET stock index



Source: Computation in Matlab

Figure 2. Hurst exponent for BET, computed using a moving window of 250 days



Source: Computation in Matlab

1.b. Source of Multifractality

The influence of correlation is quantified by:

H CORELATION (Q) = H(Q) – H SHUFLLED (Q)

RATIO = Δ H CORRELATION / Δ H SHUFFLED

It can be concluded the multifractality is mainly due to broad fat-tail distributions.

Figure 3. Main root causes of multifractality

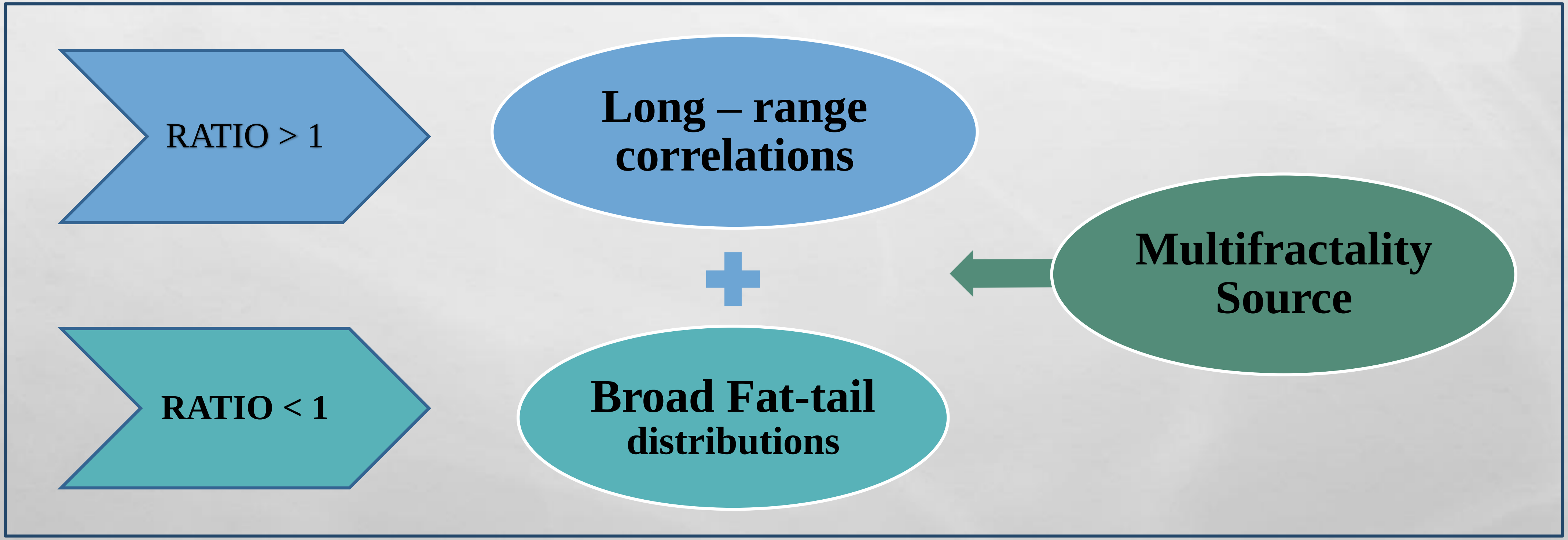


Table 2. Testing the multifractality source

Index	Hurst - DFA	Δ H original	Δα original series	ΔH shuffled series	Δα shuffled series	ΔH surrogate series	Δα surrogate series	ΔH corr	Ratio
ATHEX	0.58234	0.16254	0.27589	0.079878	0.1702	0.0886	0.17057	0.082662	1.034853
BET	0.59667	0.13541	0.36383	0.07017	0.18561	0.090155	0.19909	0.06524	0.929742
BUX	0.53593	0.18327	0.34302	0.120822	0.23491	0.11791	0.227	0.062448	0.51686
CAC40	0.51111	0.16818	0.32083	0.092482	0.18366	0.087167	0.17333	0.075698	0.818516
EUROSTOXX	0.53234	0.16381	0.30313	0.0840838	0.16325	0.093178	0.1832	0.079726	0.948176
FTSE	0.48666	0.17091	0.33006	0.096055	0.19708	0.090928	0.18103	0.074855	0.779293
IBEX35	0.50604	0.17338	0.32194	0.099649	0.20072	0.101213	0.20083	0.073731	0.739907
PSI	0.58103	0.25851	0.47633	0.113282	0.22791	0.126403	0.26147	0.145228	1.282004
PX	0.55807	0.35845	0.70762	0.119519	0.24737	0.133349	0.26982	0.238931	1.999105
RTS	0.61176	0.21291	0.37865	0.114517	0.21416	0.122965	0.23856	0.098393	0.8592
SAX	0.58448	0.22869	0.40497	0.161486	0.33139	0.15646	0.29818	0.067204	0.41616
SOFIX	0.5424	0.24393	0.49345	0.121923	0.26124	0.123257	0.25894	0.122007	1.000689
UX	0.63614	0.377	0.58118	0.268065	0.45615	0.27613	0.47298	0.108935	0.406375
WIG20	0.51647	0.17205	0.30568	0.0918826	0.17227	0.093894	0.18136	0.080167	0.872498

Source: Computation in Matlab

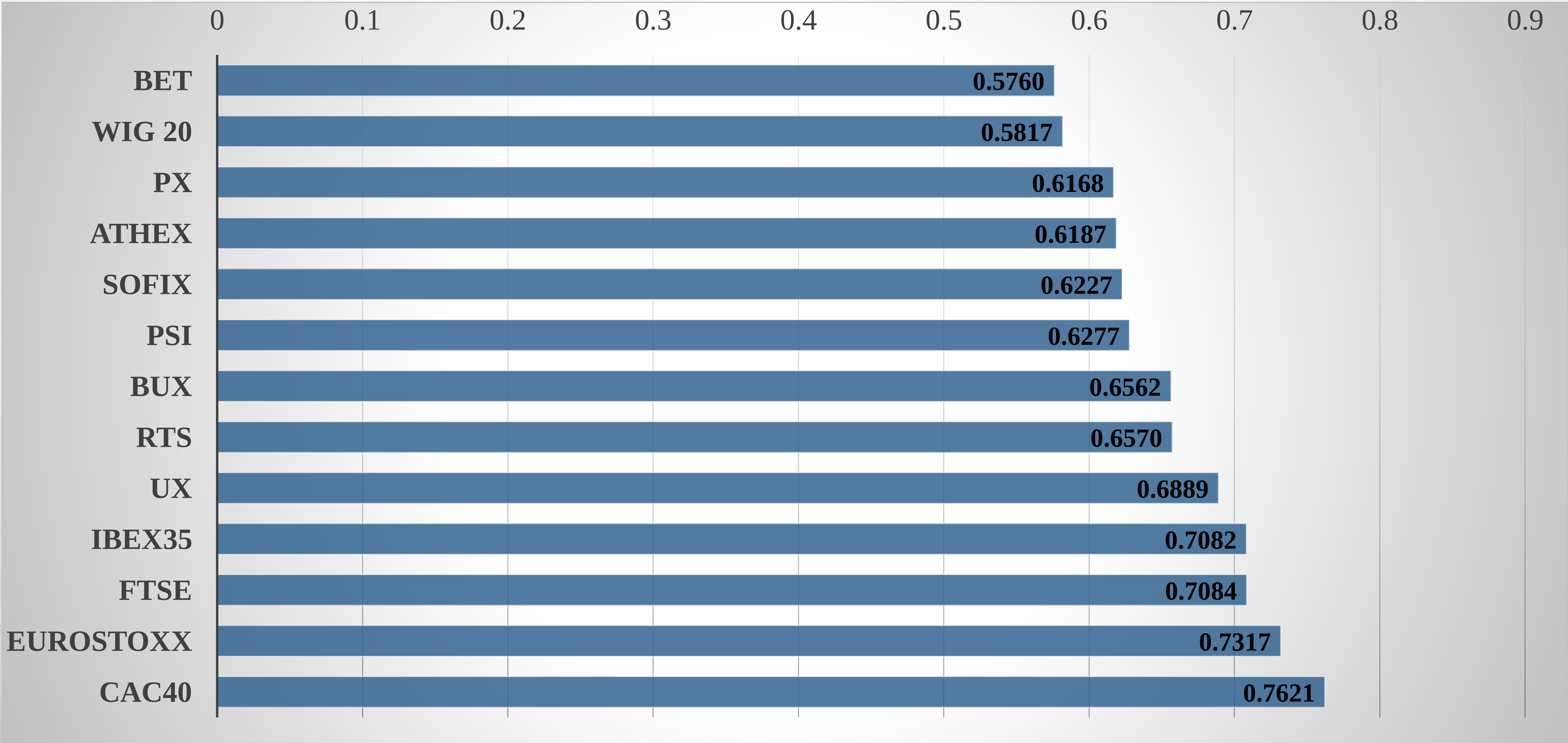
1.c. Efficiency Index

$$Efficiency\ Index = \sqrt{\sum_{i=1}^{k}\left(\frac{Mi - Mi*}{Ri}\right)^2}$$

$$\sum_{i=1}^{k}\left(\frac{Mi-Mi*}{Ri}\right)^2 = [(H\text{-}0.5)^2 + (D - 1.5\,)^2 + (D_b - 1.5\,)^2 + (D_g - 1.5\,)^2 + (D_{hw} - 1.5\,)^2 + ((\rho\,(1)/2)^2]\ ½$$

- D, D_b, D_g, D_{hw} are the estimated fractal dimensions based on the Hurst exponent method, Box Counting, Genton method and Hall–Wood method, respectively, and $\rho(1)$ is the first order autocorrelation. H is the estimated Hurst exponent

Figure 4. Efficiency index ranking for the analyzed time series



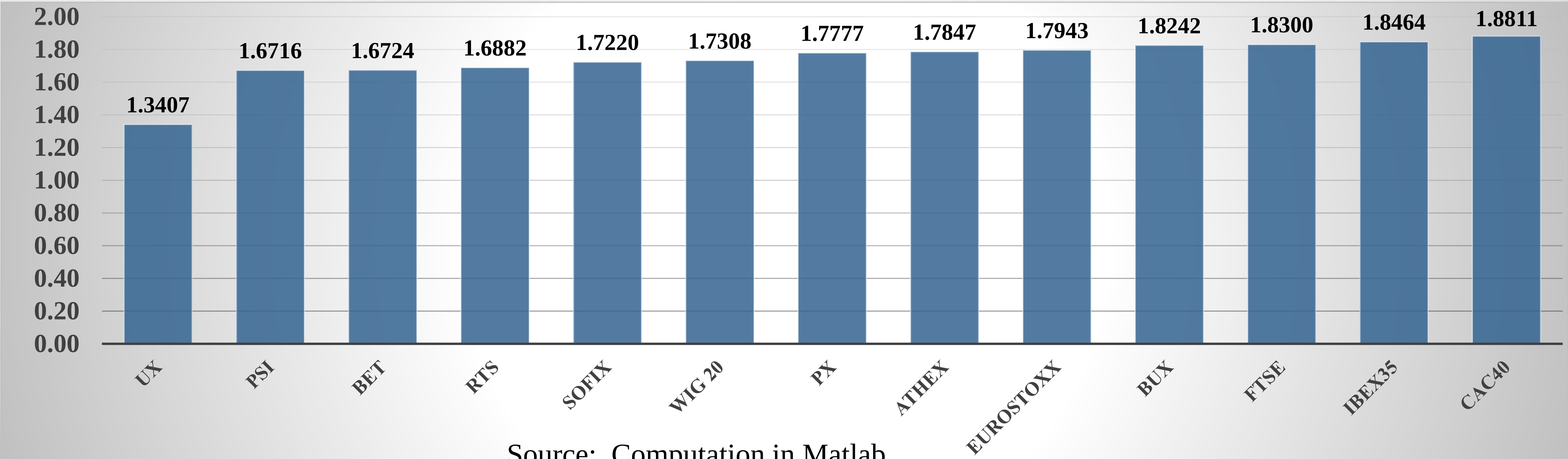
Source: Computation in Matlab

1.d. Approximate entropy

Randomness — • High Approximate Entropy value

Deterministic — • Low Approximate Entropy value

Figure 5. Randomness ranking from an entropy perspective for analyzed series



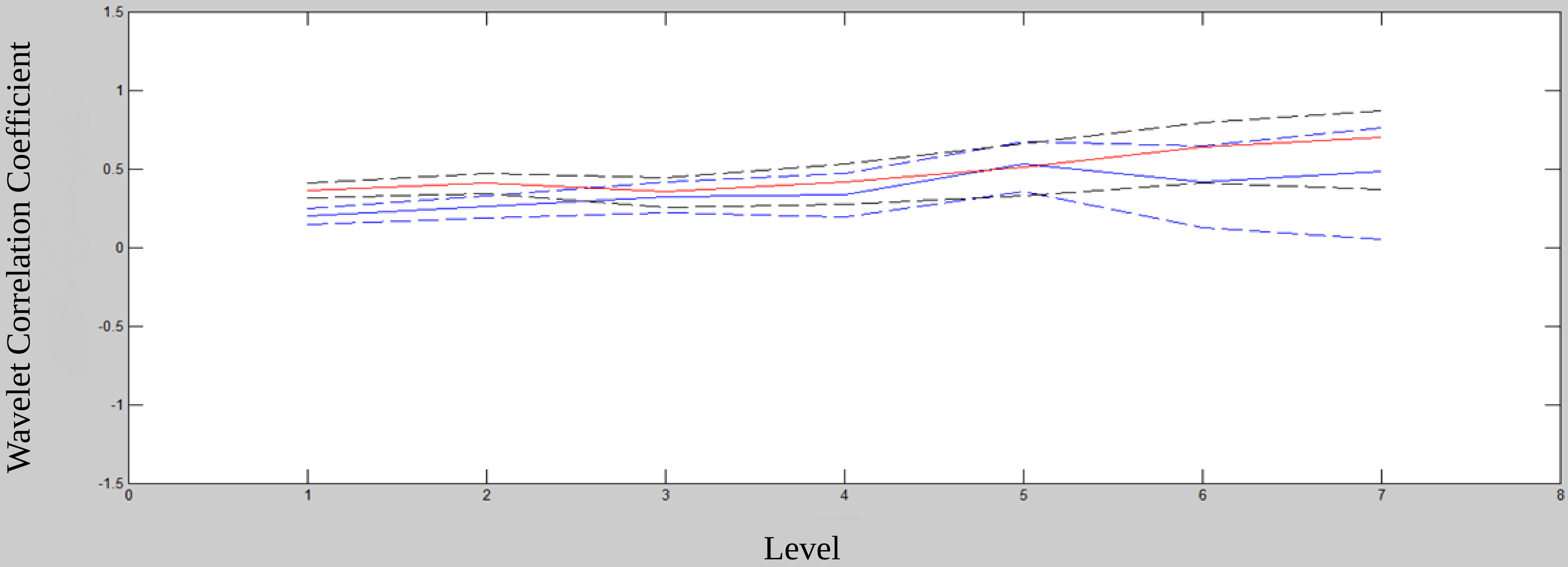
Source: Computation in Matlab

IV.2. Wavelet analysis

2.a. Analysis of contagion

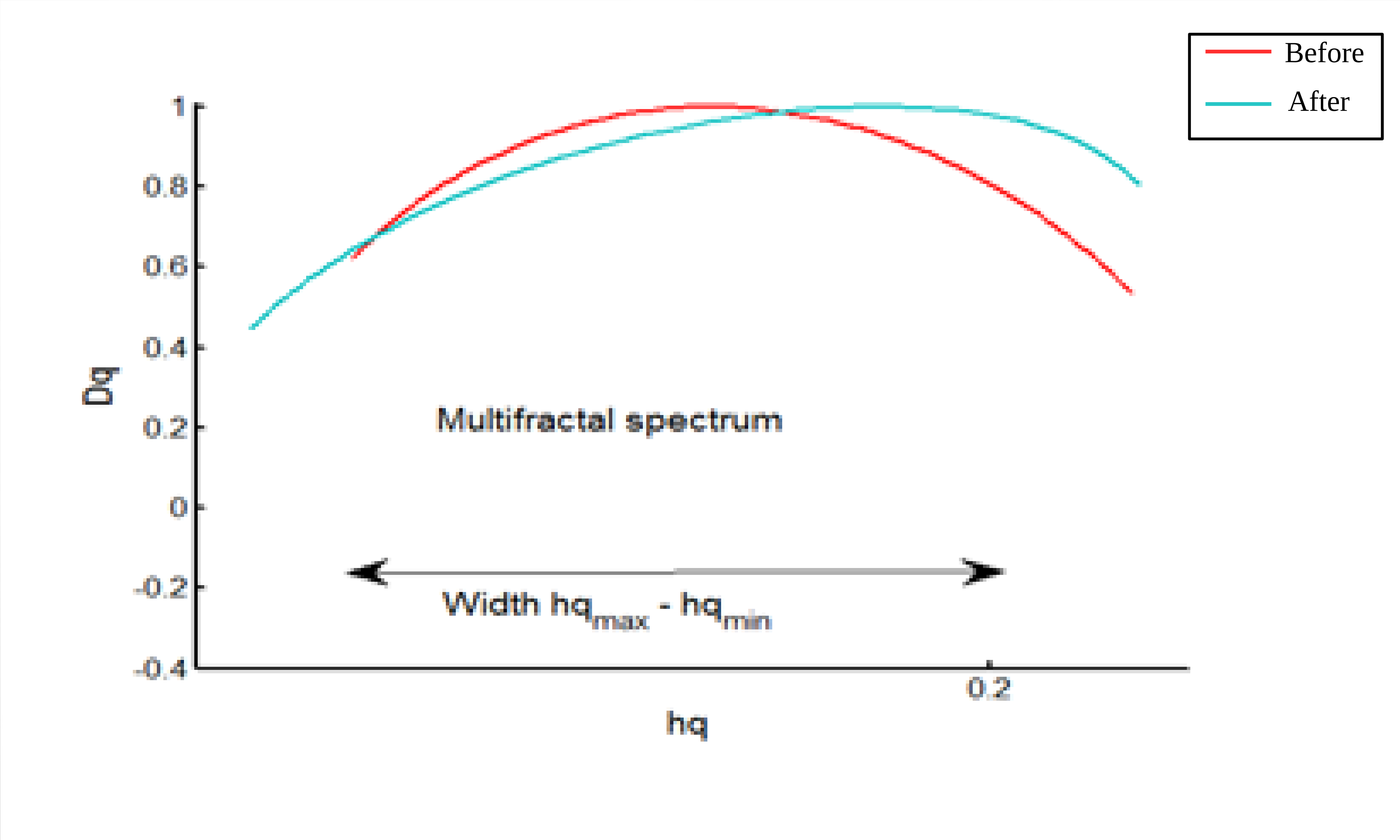
The analysis of contagion will be performed with the wavelet correlation difference before and after the bankruptcy of the Lehman Brothers in September 2008. The examined pairs consist of: BET, ATH, WIG20, BUX, PX. We estimate the wavelet correlation on two different time windows, the first window contains observations starting from **January 1st, 2003 and ending September 15, 2008**, the second window begins **September 16, 2008 and ends on 2nd of May, 2014**. Both windows have equal size of 1311 (daily data) and hence, the time frame makes them statistically comparable. Maximum overlap discrete wavelet transformation is used.

Figure 6. Time-frequency correlations* for BET-ATHEX pair



Source: Computation in Matlab

Figure 7. Multifractal spectrum before/after bankrupty of Lehman Brothers



Source: Computation in Matlab

2.b. Wavelet variance spectrum (1)

According to the Fractal Market Hypothesis, we observe increased variance at low scales (high frequencies) during the critical periods. Moreover, we observe a changing structure of variance across frequencies before the turbulences, due to the changing structure of investors' activity.

- Wavelet function: $\psi_{u,s}(t) = \frac{\Psi\left(\frac{t-u}{s}\right)}{\sqrt{s}}$

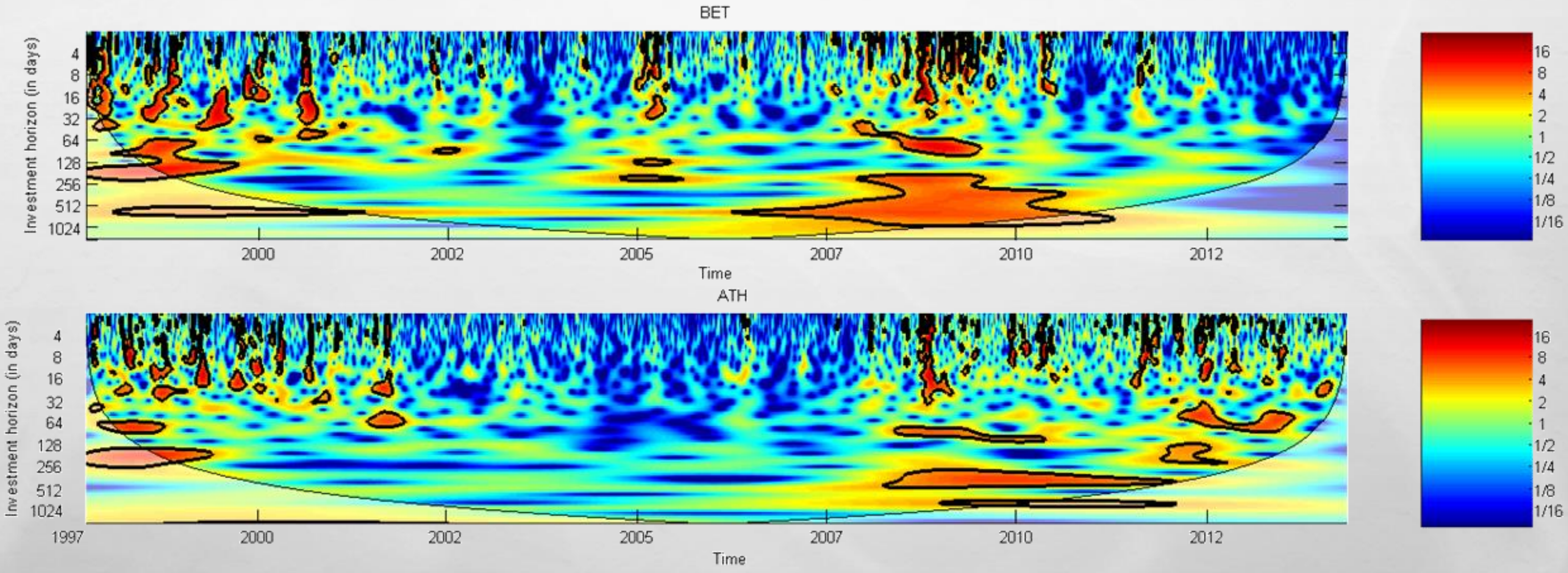
Continuous wavelet transform: $W_x(u,s) = \int_{-\infty}^{+\infty} \frac{r(t)\psi^*\left(\frac{t-u}{s}\right)dt}{\sqrt{s}}$; Where: ψ^* is the complex conjugate of ψ

$|W_x(u,s)|^2 =$ wavelet variance at scale s

$r(t)$ - return series

2.b. Wavelet variance spectrum (2)

Figure 8. Wavelet variance spectrum for BET and ATHEX



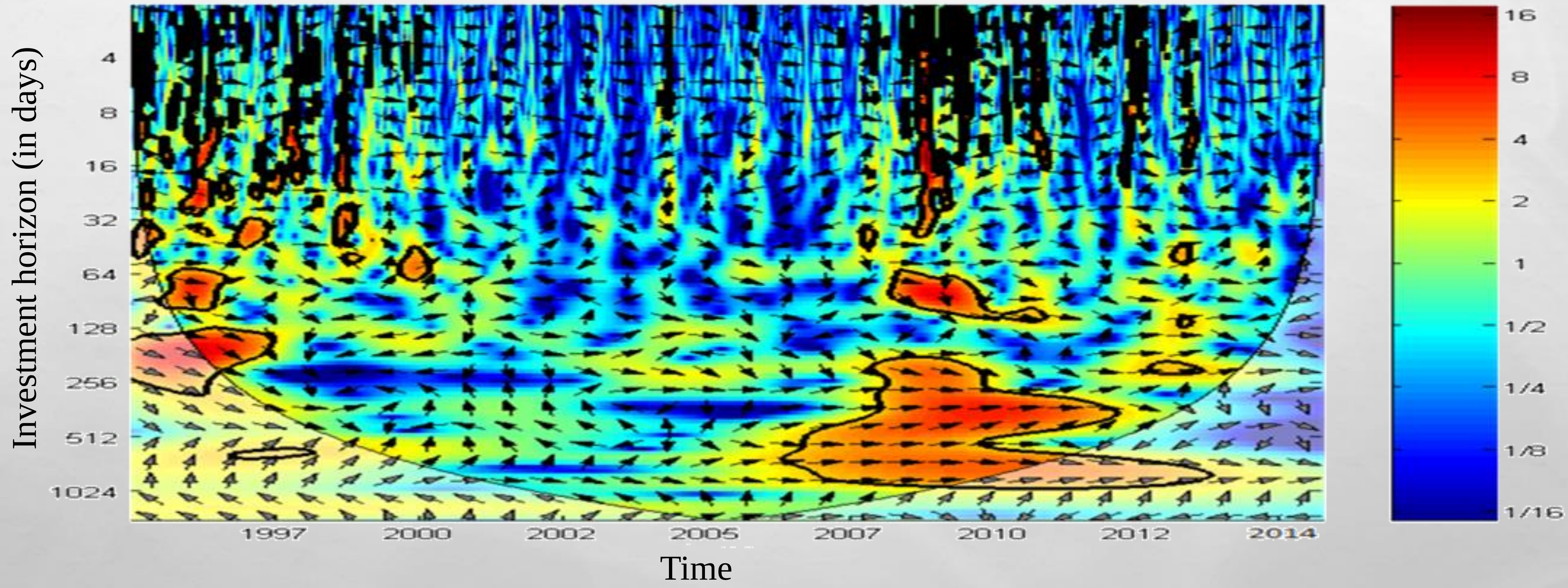
Source: Computation in Matlab

2.c. Cross wavelets transform

- finds regions in time frequency space where the two time series co-vary (but does not necessarily have high power)

$W_{x,y}(u,j) = W_x(u,s) * \overline{W_y(u,j)}$; $W_{x,y}(u,j)$ - cross wavelet transform

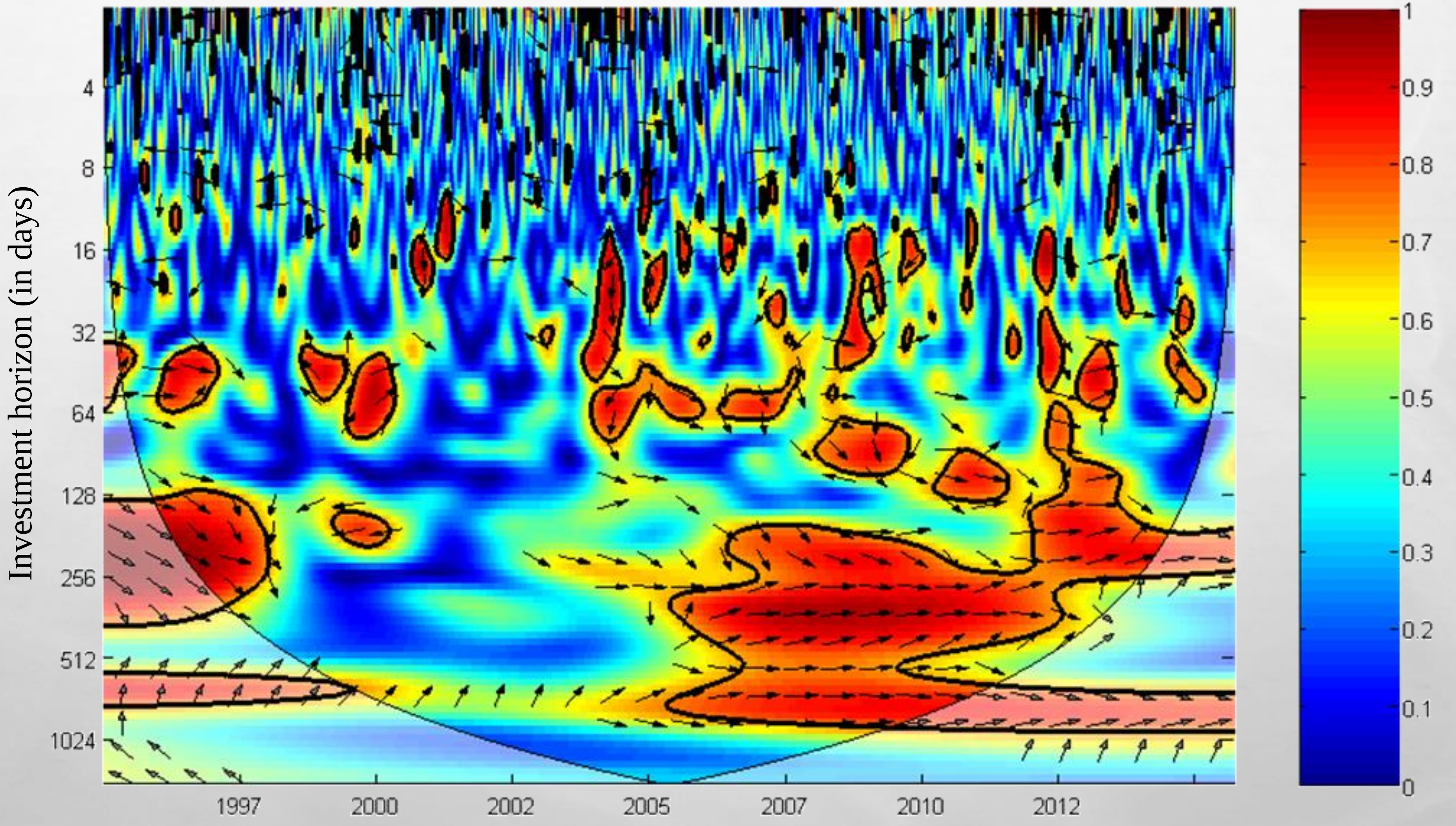
Figure 9. Cross Wavelet Transform for BET-ATHEX pair between 1997 – 2014



Source: Computation in Matlab

2.d. Wavelet coherence

Figure 10. Wavelet coherence for BET-ATHEX pair between 1997 – 2014 period



Source: Computation in Matlab

IV.3. MARKOV SWITCHING MULTIFRACTAL (MSM) VOLATILITY MODEL (1)

The Markov Switching Multifractal model presents the volatility as possessing a multifractal structure, which assumes that the volatility over a period will have a similar pattern as the volatility from another period of time.

The financial returns are modeled through the stochastic volatility:

$x_t = \sigma_t * \varepsilon_t$ (1)

Where:

σ_t – volatility modeled through a Markov switching process, having heterogeneous volatility components

ε_t – innovations from a standard normal distribution N (0,1)

IV.3. MARKOV SWITCHING MULTIFRACTAL (MSM) VOLATILITY MODEL (2)

$M_{k,t}$ drawn from a binomial distribution, with probability γ_k

$M_{k,t} = M_{k,t-1}$, with probability $1 - \gamma_k$

The stochastic volatility: $\sigma_t = \bar{\sigma} * (\prod_{k=1}^{k} M_{k,t})^{1/2}$

Transition probabilities : $\gamma_k = 1 - (1 - \gamma_1)^{b^{(k-1)}}$

- $w(r_t) = f\ (r_t / \sigma^2)$

Probabiliy density function of the volatility state: $\Pi_t = \frac{w(r_t) * (\Pi_{t-1} A)}{[w(r_t) * (\Pi_{t-1} A)] 1'}$

A= $(a_{i,j})$; $a_{i,j}$ is the probability that the volatility state in the t+1 moment will be m^j, taking into consideration that the present state is m^i.

The function to be maximed is the loglikelihood function (in order to obtain the parameters):

$$\ln L(r_1, r_2, \dots, r_t; \Phi) = \sum_{t=1}^{T} \ln[w(r_t) * (\Pi_{t-1} A)]$$

Table 3. Estimated parameters for the chosen models within the Maximum Likelihood method

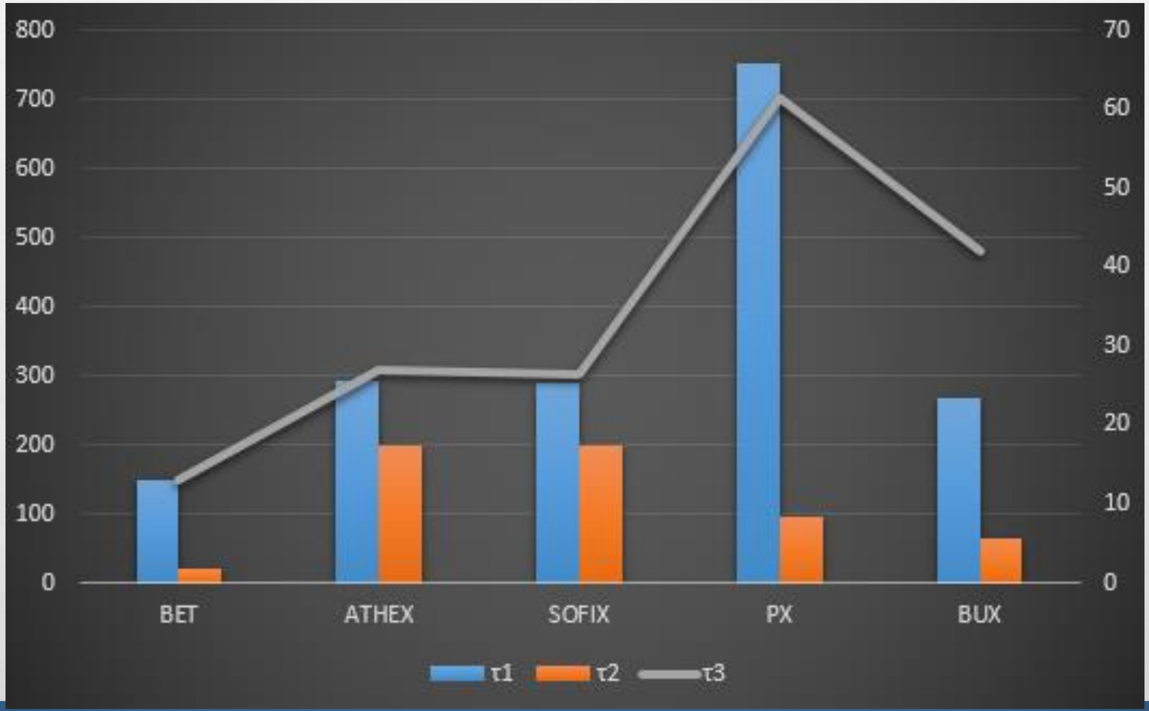
Parameter	BET	ATHEX	SOFIX	WIG20	PX	BUX
b	7.28	14.83	14.75	1.50	7.89	4.27
m_0	1.61	1.56	1.56	1.00	1.56	1.51
γ_k	0.30	0.07	0.072	0.99	0.07	0.07
$\bar{\sigma}$	0.31	0.44	0.44	0.23	0.33	0.39

Source: Computation in Matlab

Period or cycle (τ_k) indicator identifies the cycle of an index as the inverse of the frequency γ_k, with which the components $M_{k,t}$ are changing throughout the analyzed periods.

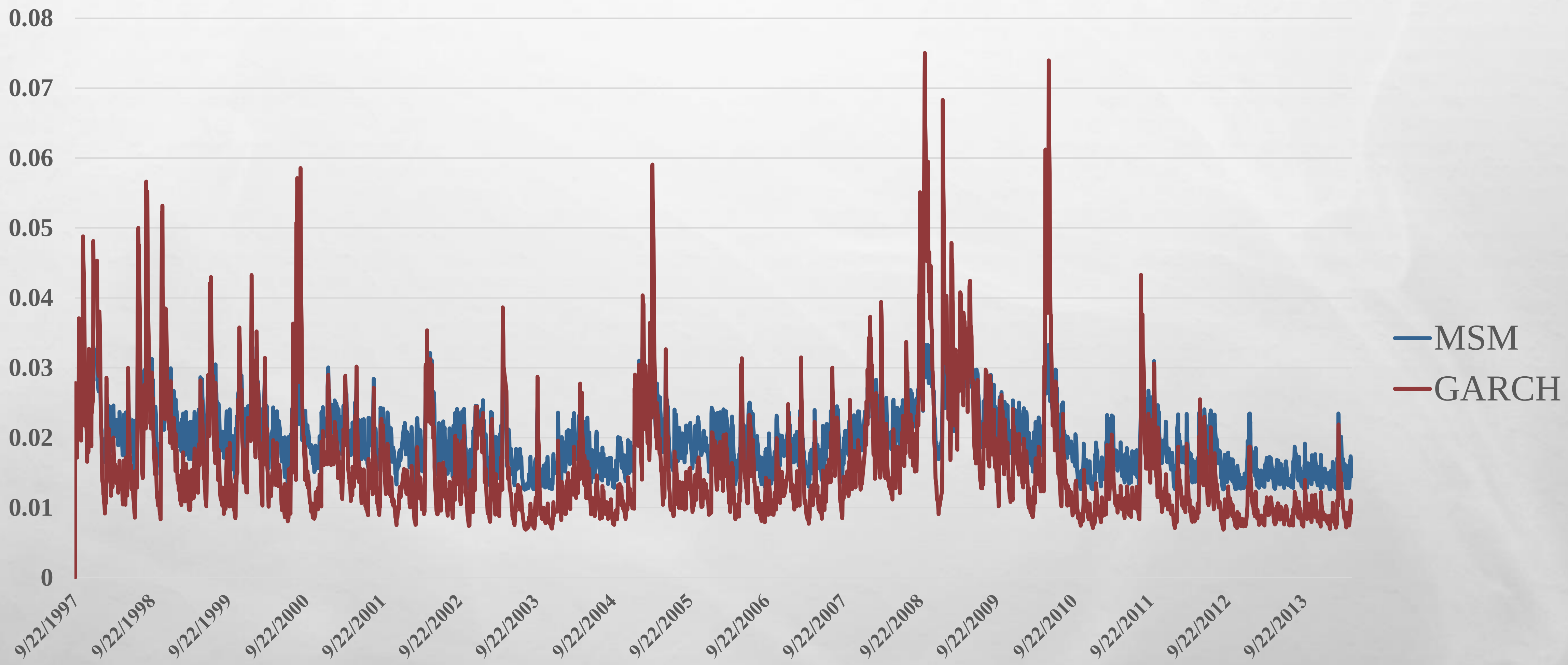
$$\tau_k = \frac{1}{\gamma^k} = = \frac{1}{1-(1-\gamma_1)^{b^{(k-1)}}}$$

Figure 11. Volatility cycles in days for MSM (3)



Source: Computation in Matlab

Figure 12. Conditional volatility for BET computed by MSM and GARCH (1,1) models



Source: Computation in Matlab

CONCLUSIONS (1)

- The paper has presented the analysis of several European stock returns within the Fractal Market hypothesis. The multifractal character of the stock return has been studied. The main findings of this first analysis are that the series are characterized by a multifractal spectrum. For the analyzed time series, the Hurst exponent is in the interval (0.5, 1), hence the time series have a long-range dependent structure. The source of multifractality has been analyzed by comparing the results from the initial time series with the shuffled time series.In the most cases, the source of multifractality are the broad fat-tail distributions of the returns.
- Based on the computation of Hurst exponent, fractal dimension and the first order autocorrelation coefficient, an efficiency index has been built. Taking into consideration the computed efficiency index, the most efficient stock indices are: BET, followed by WIG 20, PX, ATH, SOFIX, PSI. According entropy method, the least deterministic stock indexes are: CAC40, FTSE, EuroStoxx, PX, SOFIX.
- Within the wavelet discrete transform, a contagion analysis was made, were the contagion was composed by the changes in the correlation coefficients during critical events. The examined pairs consist of: BET, ATH, WIG20, BUX, PX. discrete wavelet analysis showed that the correlation between the most stock indexes increased after the bankruptcy of Lehman Brothers bank. This result reveals that the comovement between these two markets increased on short investment horizons.

CONCLUSIONS (2)

- ✓ The volatility of the analyzed stock returns has been studied throughout a comparison between GARCH and Markov Switching Multifractal model. This comparison allowed to observe that the conditional volatility modeled by GARCH has larger peaks than the one modeled by Markov Switching Multifractal model, especially in the periods with extreme events (the financial crisis, 2008 -2010). When the market is not affected by extreme events, the GARCH model underestimates the volatility (see the case for ATHEX and SOFIX between 2001-2007) .
- ✓ As the fractal market hypothesis predicts whether extreme events are related to short dominating investment horizon. During turbulent periods, the long-term investor panic and start to sell or stays out of the market, so he shortens its investment horizon. Due to the fact that the liquidity in decreasing in these situations, it can be concluded that the fractal market hypothesis is strongly related to the financial stability of the financial markets.

Future directions of the study:

As the univariate MSM model only characterize the volatility of a single stock return, it would be of great interest to compute a bivariate MSM model. As a future direction of study, a bivariate version Markov Switching Multifractal model will be performed in order to determine the probability of extreme co-movements, where a crisis will be defined as a joint peak in volatility components for all investment horizons. This bivariate model can help to determine the indicators that help understanding the nature of co-movements, cycles and correlations between the analyzed stock returns.

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THANK YOU !