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Application of Extreme Value Theory in Risk Management

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Bucharest, June 2014

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Objectives

- General objective:
  - Give a solution for inefficient allocation of capital by using the EVT as complementary risk measure of VaR, in the context of stock indices.

- Intermediary objectives:
  - Modeling indexes returns with ARMA – EGARCH approach to obtain standardized residuals;
  - Determining the value of potential losses from stock indexes positions using VaR and EVT;
  - Modeling VaR to incorporate EVT and use extreme VaR measures;
  - Backtesting the result and present the performance for every measures;
  - Determining how models should be used.

Literature review

- Hendricks (1996) analyses the performance of twelve different VaR models using historical data on exchange rate returns and finds that historical simulation performs better at 95% than at 99% or higher confidence levels and Exponentially Weighted Moving Average is more reliable with 0.94 decay factor for daily returns;
- McNeil (1997a, 1997b, 1998, 1999) studies the performance of the EVT in insurance and finance;
- Block Maxima is preferred in papers of Caserta and De Vries (2003), who differentiate their analysis for minima and maxima of AEX index;
- Younes Bensalah from Bank of Canada in 2000 presents the methods to estimate the threshold for a series of daily exchange rates of Canadian/U.S. dollars over a 5-year period (1995–2000);
- Embrechts et al. (1997) offer a very detailed analysis of Hill plots, Mean Excess Plots and QQ plots;
- Paul Embrechts, Sidney I. Resnick, and Gennady Samorodnitsky (1999) realized a study on California Earthquake (1971 – 1993) and they used EVT to determinate the risk;
- Gilli M. and Këllezi E. (2006) studied EVT for stock indices, using Dow Jones, FTSE 100, Nikkei 225, Swiss Market Index, S&P 500 and Hang Seng and conclude that EVT is a useful measure for VaR.

Methodology – Steps

- Process and analyze the data – assess stylized facts
- Data – autocorrelation and heteroscedasticity - produce i.i.d. series
- Compute VaR at 95% and 99% confidence levels:

  - point estimates for 1 and 10 days out of sample – Historical Simulation, EWMA
- Assess fat tails and pick threshold
- Estimate shape parameter; assess fit
- Apply Goodness of Fit Tests: KS, AD, Chi Squared
- Compute extreme VaR - point estimates 1 and 10 days out of sample
- Backtest - percentage of failures for EWMA measures

  - Unconditional Coverage, Independence Coverage and Conditional Coverage Tests

Methodology – The Models

Value-at-Risk: - (μ + σQ_α); μ sample mean, σ sample variance, Q_α α% quantile.

- is not a coherent measure of risk (Arztner 1997);

Conditional Value-at-Risk: CVaR(α) = E(R|R > VaR)

Models:

- **Historical Simulation:** pick the percentile from sorted historical data**;**
- **EWMA[1]: $\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1-\lambda) r_{t-1}^2$** account for past returns and past variance;
- **EVT – POT method[2]: excesses over a high threshold u (i.e. tails) ~ Generalized Pareto Distribution with shape parameter. $\xi > 0$ denotes fat tails. extreme VaR and ES**

$$VaR_p = u + \frac{\hat{\sigma}}{\hat{\xi}}\left[\left(\frac{n}{N_u}(1-p)\right)^{-\hat{\xi}} - 1\right] \qquad ES_p = \frac{VaR_p}{1-\xi} + \frac{\sigma - \xi u}{1-\xi}$$

where: λ decay factor (0.94 daily data), ξ shape parameter and σ scale parameter of GPD, n number of observations in the sample, N_u is the number of observations for the treshold u, p or α desired probability.

[1] Arztner (1997), **[2] Balkema-de Haan-Pickands**

Methodology – Generalized Pareto Distribution (I)

Peaks-Over-Treshold approach

- Consider a sample of observations $X_1, X_2, X_3, \ldots X_n$ with a distribution function $F(x)=P(X \leq x)$ and take a treshold value u, excesses over u are difened as: $y=X-u$

  Thus the distribution function of excesses is:

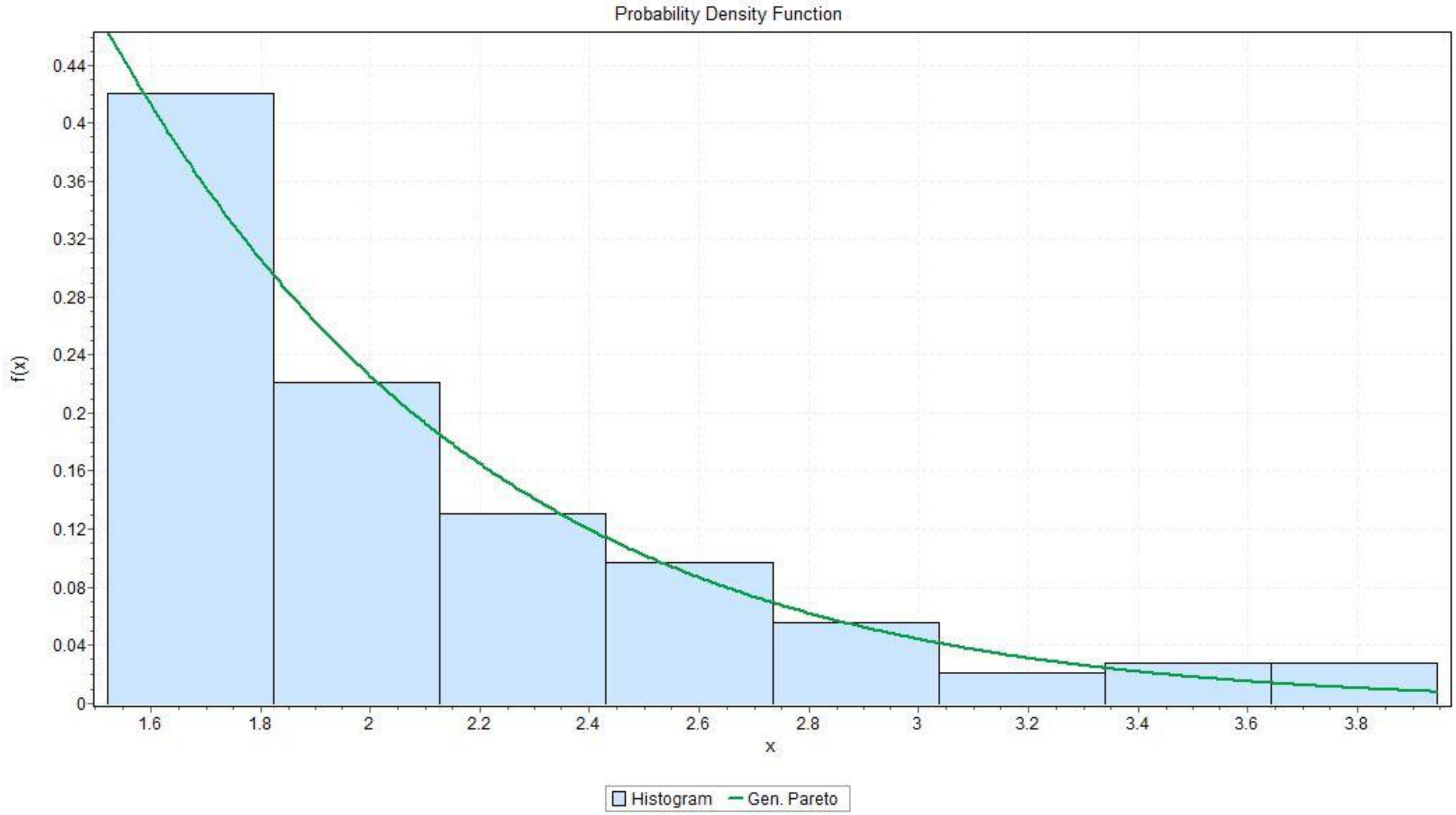
$$F_u(y) = P(X-u \mid X>u), \quad 0 \leq y \leq x_F - u$$

Balkema and de Haan (1974) and Pickands(1975) showed that for $u \to \infty$, the distribution function of the exceedances may be approximated by the **Generalized Pareto Distribution (GPD):**

$$G_{\xi,\sigma,\beta} = \begin{cases} 1-\left(1+\frac{\xi}{\sigma}y\right)^{-\frac{1}{\xi}}, \xi \neq 0 \\ 1-e^{-\frac{y}{\sigma}}, \xi = 0 \end{cases} \quad for\ y \in \begin{cases} [0, (x_F-u)], \xi \geq 0 \\ [0, -\frac{\sigma}{\xi}, \xi < 0 \end{cases}$$

where ξ is the tail index, β is location parameter and σ represents the scale parameter.

Methodology – Generalized Pareto Distribution (II)



Probability Distribution Function for upper BET – C tails (GPD)

Data and results – Input Data

- Input data

  - Four indexes from Central and Eastern Europe:
    - BET – C from Romania (BRD – 10%)
    - BUX from Hungary (OTP – 34.03%)
    - WIG from Poland (PKOBP – 10%)
    - PX from Czech Republic (KB, ERSTE -19%)

  - 2595 observations from the period January 2004 – April 2014

  - Source: Bloomberg

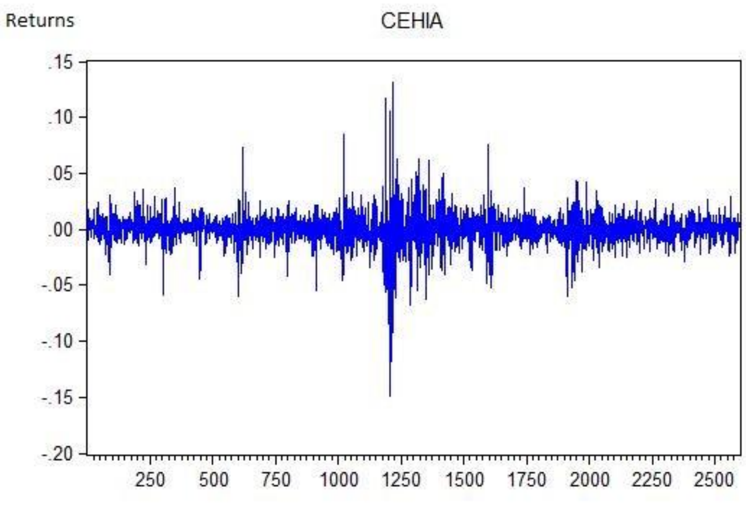
Data and results: Preliminaries

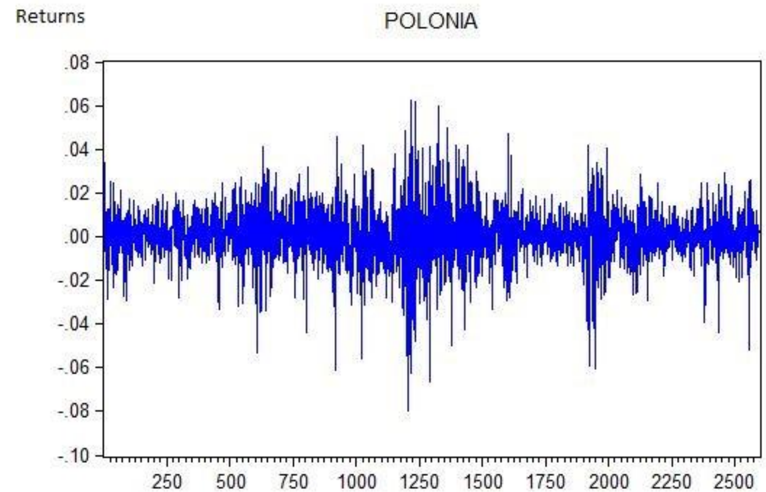
Data: Daily returns for every stock index: PX, WIG, BET – C and BUX, between January 2004 and April 2014.

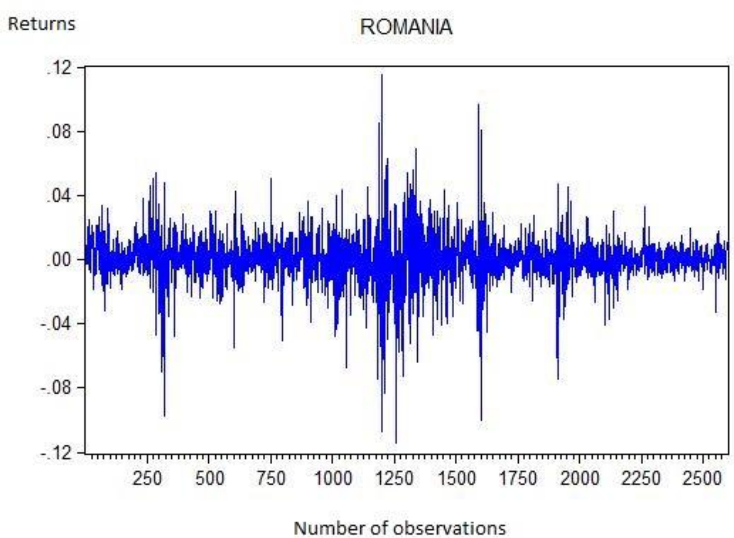
Main Statistics:

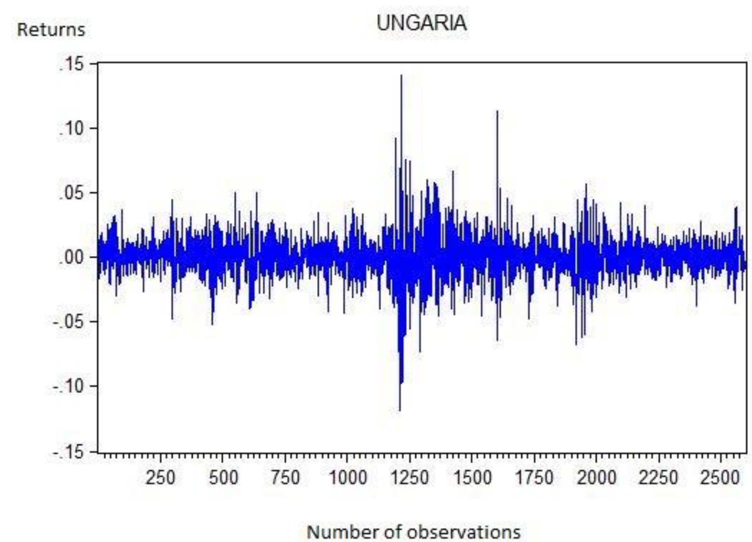
Indicators / Series	PX	WIG	BET - C	BUX
Number of observations	**2595**	**2595**	**2595**	**2595**
Mean	0.000279	0.000428	0.000443	0.000372
Median	0.000794	0.000811	0.000798	0.000431
Maximum	0.131609	0.062723	0.115058	0.140845
Minimum	-0.149435	-0.079546	-0.114129	-0.118817
Std. Dev.	0.015243	0.013023	0.015644	0.016693
Skewness	-0.191235	-0.378802	-0.547979	0.136745
Kurtosis	16.58597	6.362876	11.16924	9.572929
Jarque-Bera	19973.39	1284.838	7345.754	4679.454
Probability	0.000000	0.000000	0.000000	0.000000

Data and results: Indices Returns









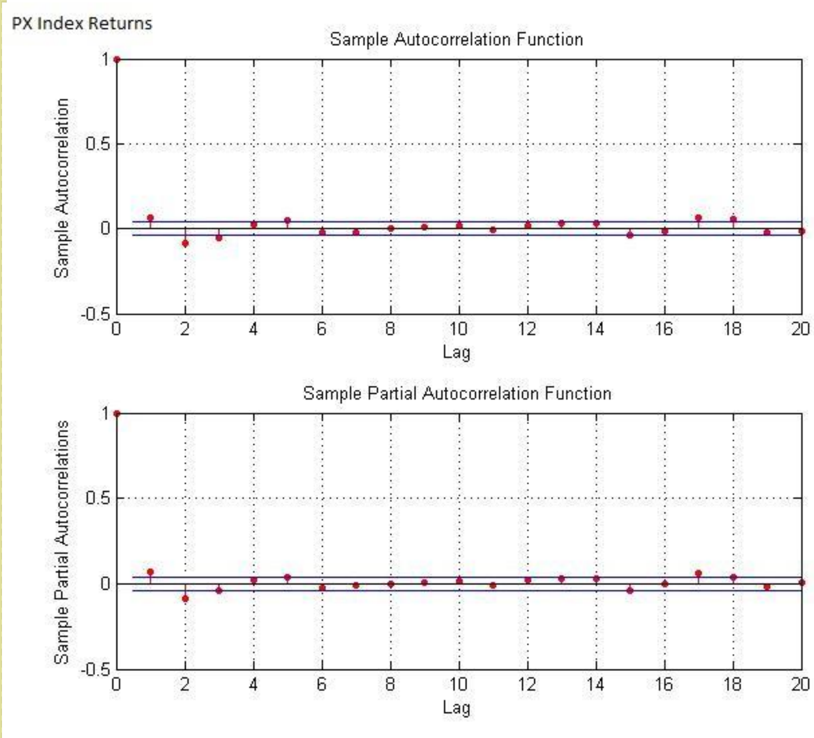
Data and results: Characteristics

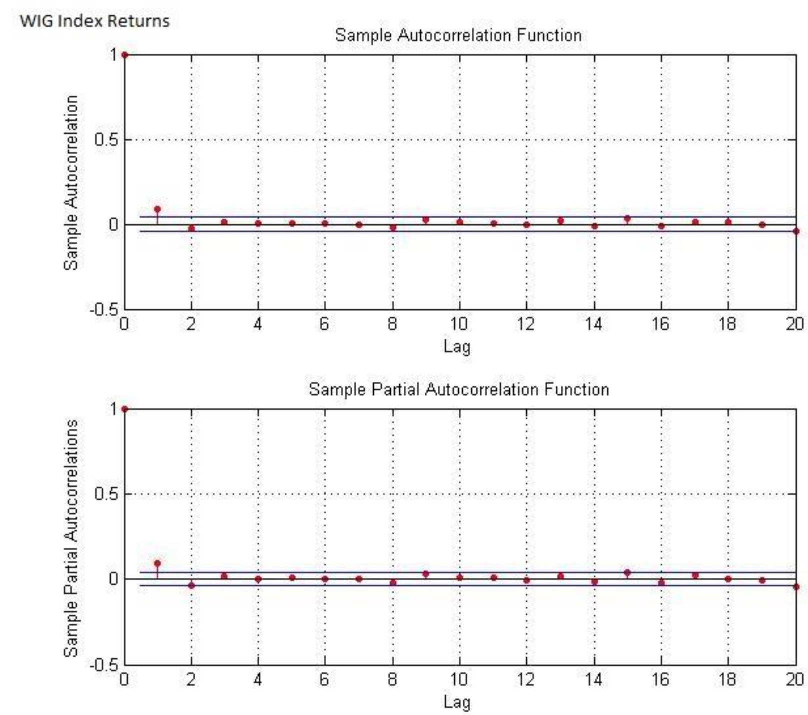
- Stationary – ADF and KPSS;
- Skewed – Skewness indicator ($s \neq 0$);
- Leptokurtic – Kurtosis indicator ($k > 3$);
- Autocorrelation – ACF, PACF;
- Heteroscedasticity – returns evolution;
- Volatility clustering – returns evolution.

Index	PX Index		WIG Index		BET - C Index		BUX Index		Test critical values		
Test	Statistic	Prob.	Statistic	Prob.	Statistic	Prob.	Statistic	Prob.			
Augmented Dickey-Fuller test statistic (ADF)	t - Statistic		t - Statistic		t - Statistic		t - Statistic		1%	5%	10%
	-38.09171	0.0001	-46.60061	0.0001	-45.08475	0.0001	-37.72888	0.0001	-3.433	-2.862	-2.567
Kwiatkowski-Phillips-Schimidt-Shin test statistic	LM - Stat		LM - Stat		LM - Stat		LM - Stat		1%	5%	10%
	0.331892		0.187146		0.423888		0.298085		0.739	0.463	0.347

Data and results: ACF and PACF for returns (I)

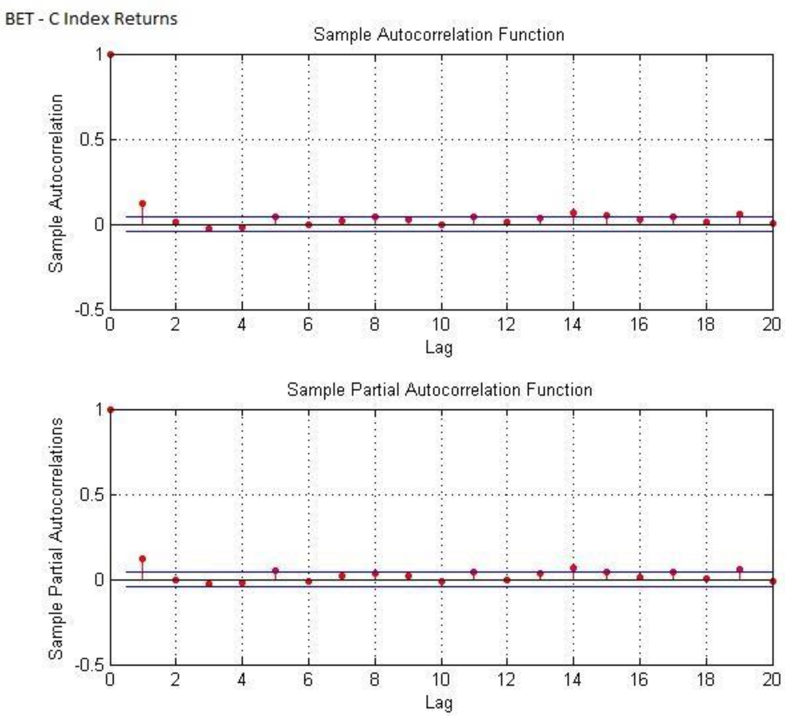
Left: PX – ARMA (2,2)
Right: WIG – ARMA (1,1)

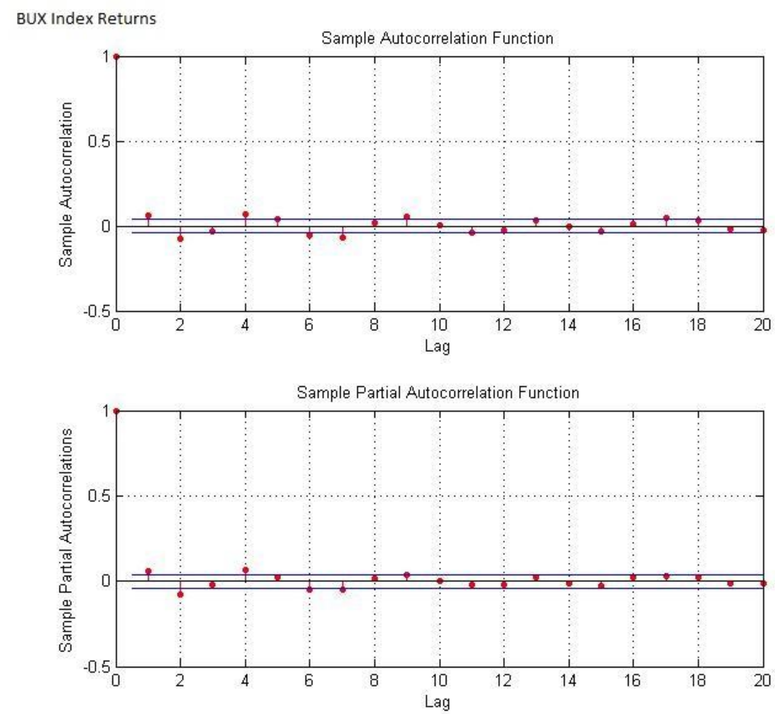




Data and results: ACF and PACF for returns (II)

Left: BET - C – ARMA (1,1)
Right: BUX – ARMA (2,2)





Data and results: Data for EVT

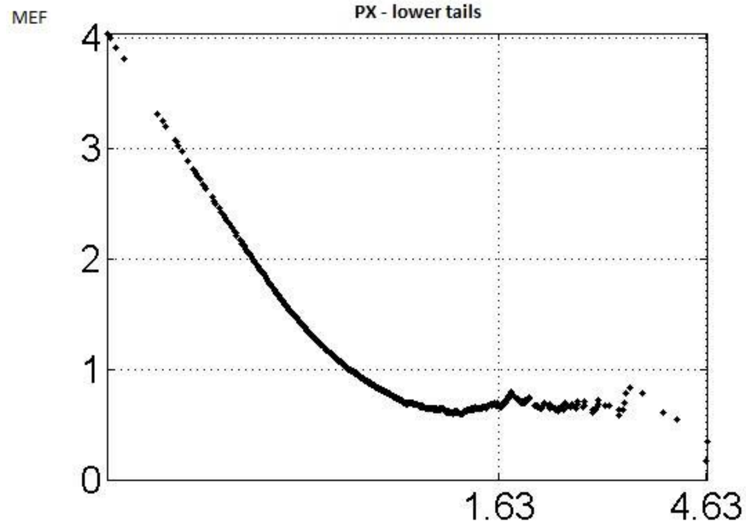
- Returns must be independent and identically distributed (i.i.d.) for EVT (homoskedasticity and no autocorrelation)

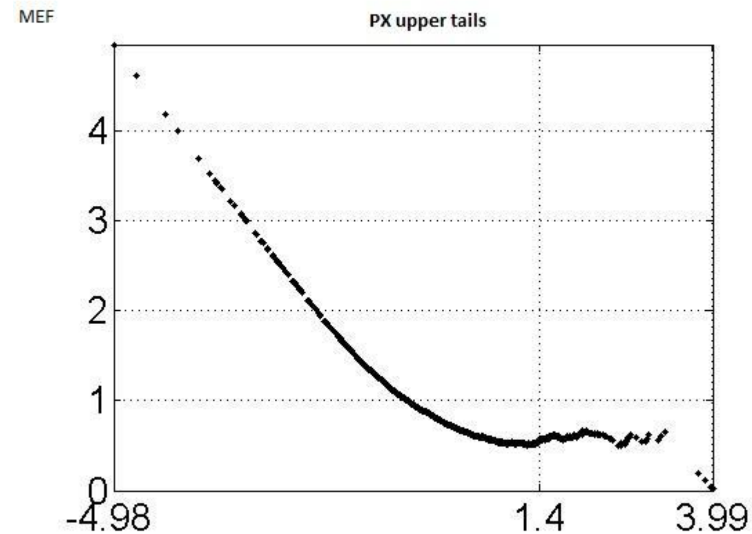
- Problem:
  - Autocorrelation – ARMA -> conditional mean: **ε(t)**
  - Heteroskedasticity – EGARCH -> conditional variance: **σ(t)**

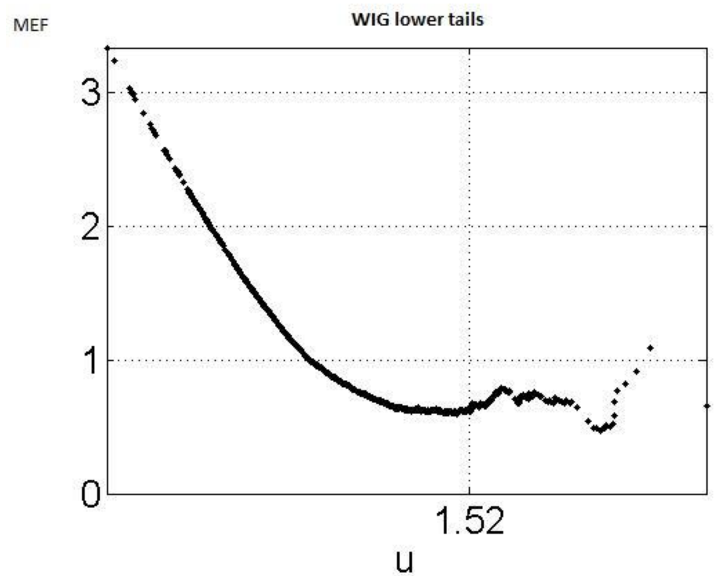
- Compute standardized residuals ~ i.i.d. returns: **r(t) = ε(t) / σ(t)**

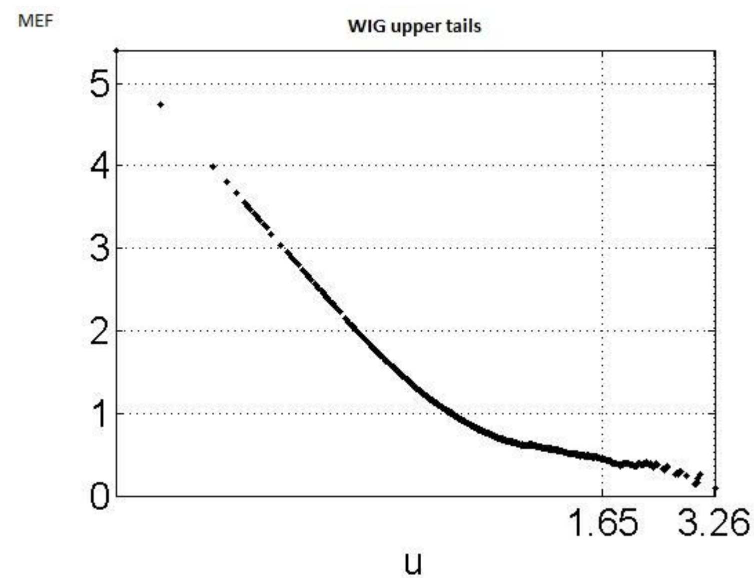
Data and results: Mean Excess Function / Plot (I)

- Mean Excess Function (MEF): $$e_n(u) = \frac{\sum_{i=1}^{n}(X_i - u)}{\sum_{i=1}^{n} I_{\{X_i > u\}}}$$
- Mean Excess Plot: $\{(X_{k,n}, e_n(X_{k,n}))| \ k=1,2,\dots,n\}$

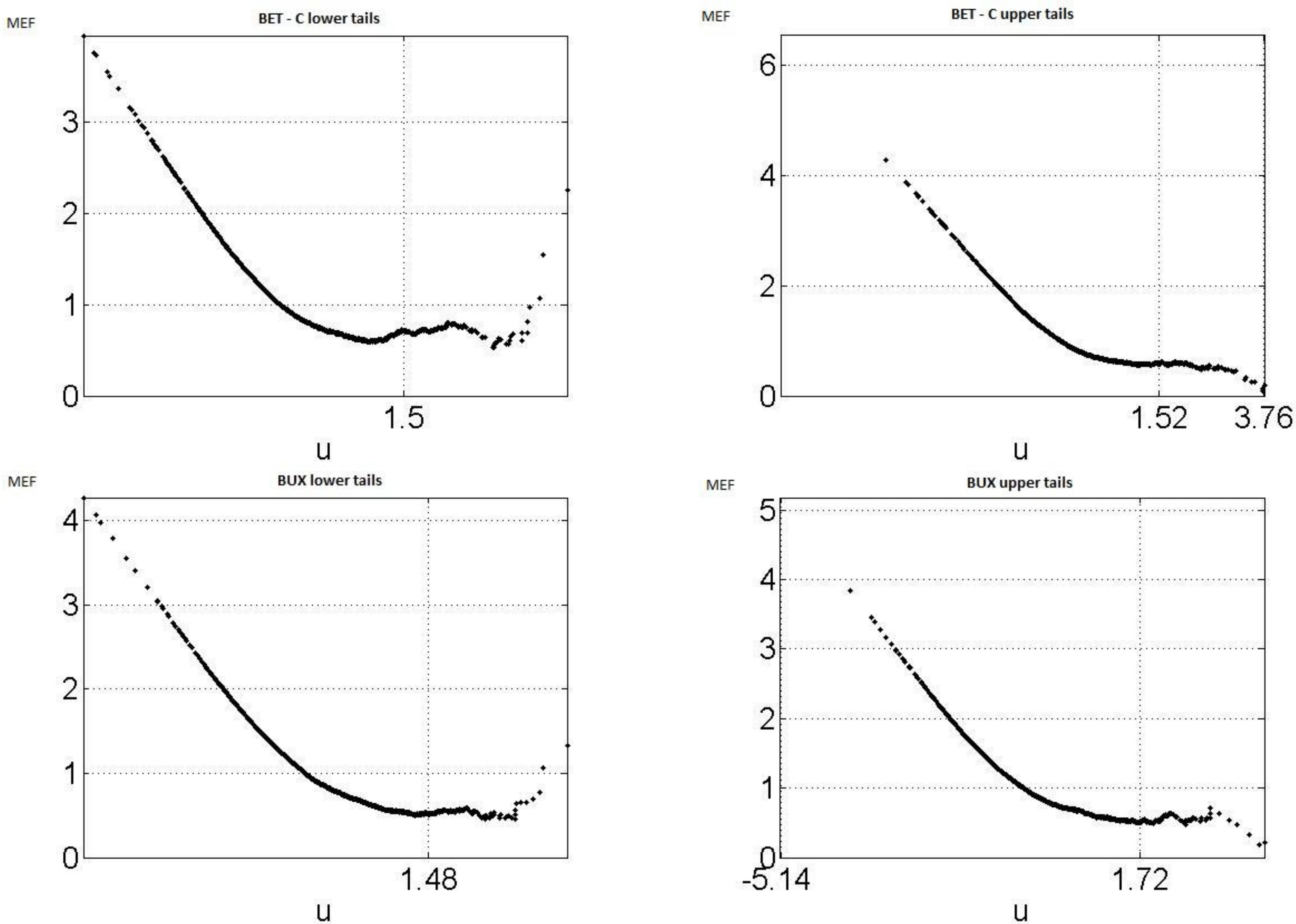








Data and results: Mean Excess Function / Plot (II)



Data and results: Parameters estimation

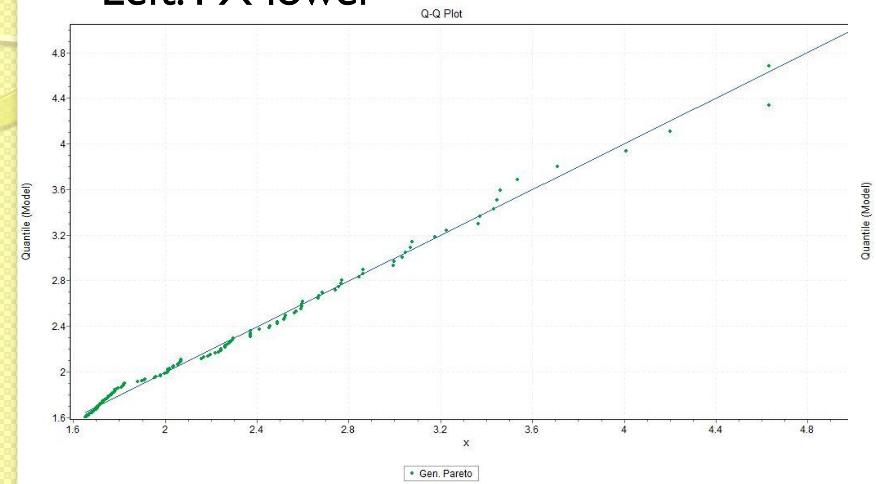
Index	Indicator	Threshold	k	xi	Sigma
PX	Lower	1.63%	130	0.009318	0.664258
	Upper	1.40%	151	0.031089	0.545768
WIG	Lower	1.52%	146	0.133127	0.53709
	Upper	1.65%	126	-0.21676	0.54091
BET - C	Lower	1.50%	138	0.008536	0.701483
	Upper	1.52%	145	-0.09265	0.635686
BUX	Lower	1.48%	152	0.044258	0.492824
	Upper	1.72%	120	0.032974	0.496052

Number of observations: 2592

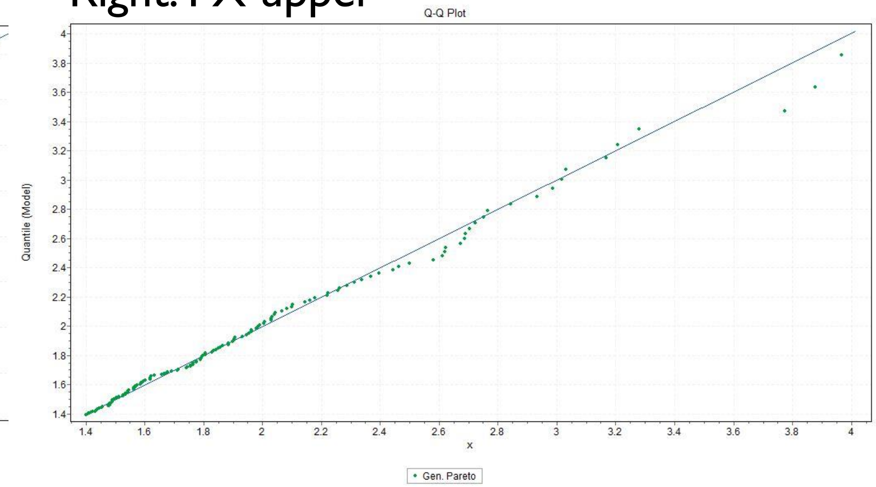
Index	Lower / Upper	Test	Statistic	Critical Value	p - value	Deg. of freedom
PX	Lower	Kolmogorov Smirnov	0.08372	0.14287	0.3047	-
		Anderson Darling	0.9937	3.9074	-	-
		Chi - Squared	5.5341	18.475	0.59508	7
PX	Upper	Kolmogorov Smirnov	0.04246	0.13257	0.9375	-
		Anderson Darling	0.33378	3.9074	-	-
		Chi - Squared	6.9811	18.475	0.43085	7
WIG	Lower	Kolmogorov Smirnov	0.04694	0.13482	0.88939	-
		Anderson Darling	0.47124	3.9074	-	-
		Chi - Squared	3.4614	18.475	0.8393	7
WIG	Upper	Kolmogorov Smirnov	0.04854	0.14512	0.91364	-
		Anderson Darling	0.41595	3.9074	-	-
		Chi - Squared	1.074	16.812	0.98265	6
BET - C	Lower	Kolmogorov Smirnov	0.04339	0.13867	0.94726	-
		Anderson Darling	0.26362	3.9074	-	-
		Chi - Squared	2.924	18.475	0.89194	7
BET - C	Upper	Kolmogorov Smirnov	0.03814	0.13528	0.97905	-
		Anderson Darling	0.27234	3.9074	-	-
		Chi - Squared	5.0726	18.475	0.6511	7
BUX	Lower	Kolmogorov Smirnov	0.036	0.13213	0.98529	-
		Anderson Darling	0.28026	3.9074	-	-
		Chi - Squared	2.0686	18.475	0.95595	7
BUX	Upper	Kolmogorov Smirnov	0.06016	0.14871	0.7547	-
		Anderson Darling	0.36699	3.9074	-	-
		Chi - Squared	3.5074	16.812	0.74299	6

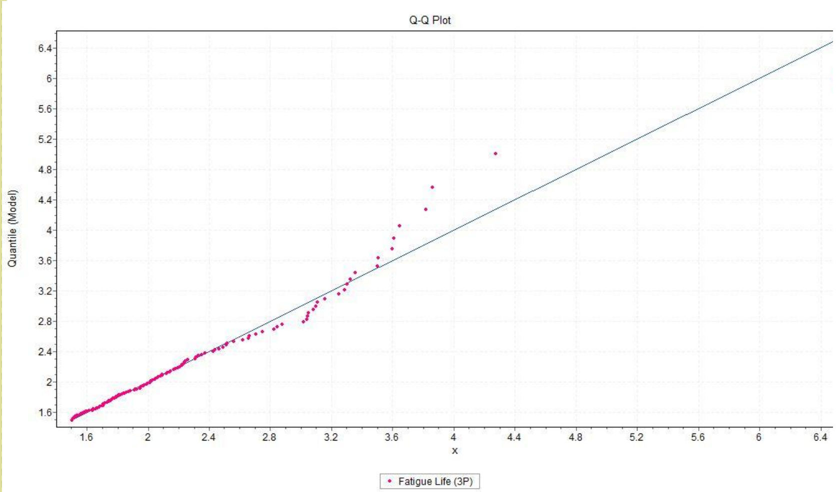
Data and results: QQ – Plots

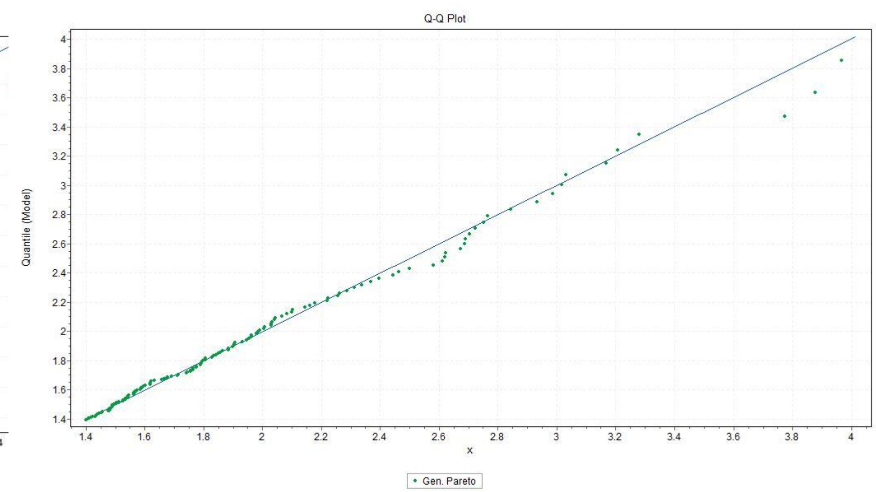
Left: PX lower



Right: PX upper







Left: BET – C lower

Right: BET – C upper

Data and results: VaR and ES

Value at Risk							
Model	**Level**	**Time**	**Lower / Upper**	**PX**	**WIG**	**BETC**	**BUX**
Historical Simulation	99%	1 day	Lower	0.044	0.038	0.048	0.042
	95%	1 day	Lower	0.022	0.020	0.023	0.025
	99%	10 days	Lower	0.139	0.119	0.151	0.132
	95%	10 days	Lower	0.068	0.064	0.074	0.078
	99%	1 day	Upper	0.037	0.035	0.042	0.045
	95%	1 day	Upper	0.021	0.021	0.023	0.026
	99%	10 days	Upper	0.118	0.111	0.134	0.142
	95%	10 days	Upper	0.066	0.065	0.071	0.082
EWMA RiskMetrics Approach	99%	1 day	Lower / Upper	0.020	0.020	0.015	0.028
	95%	1 day	Lower / Upper	0.014	0.014	0.011	0.020
	99%	10 days	Lower / Upper	0.063	0.063	0.049	0.088
	95%	10 days	Lower / Upper	0.045	0.045	0.034	0.062

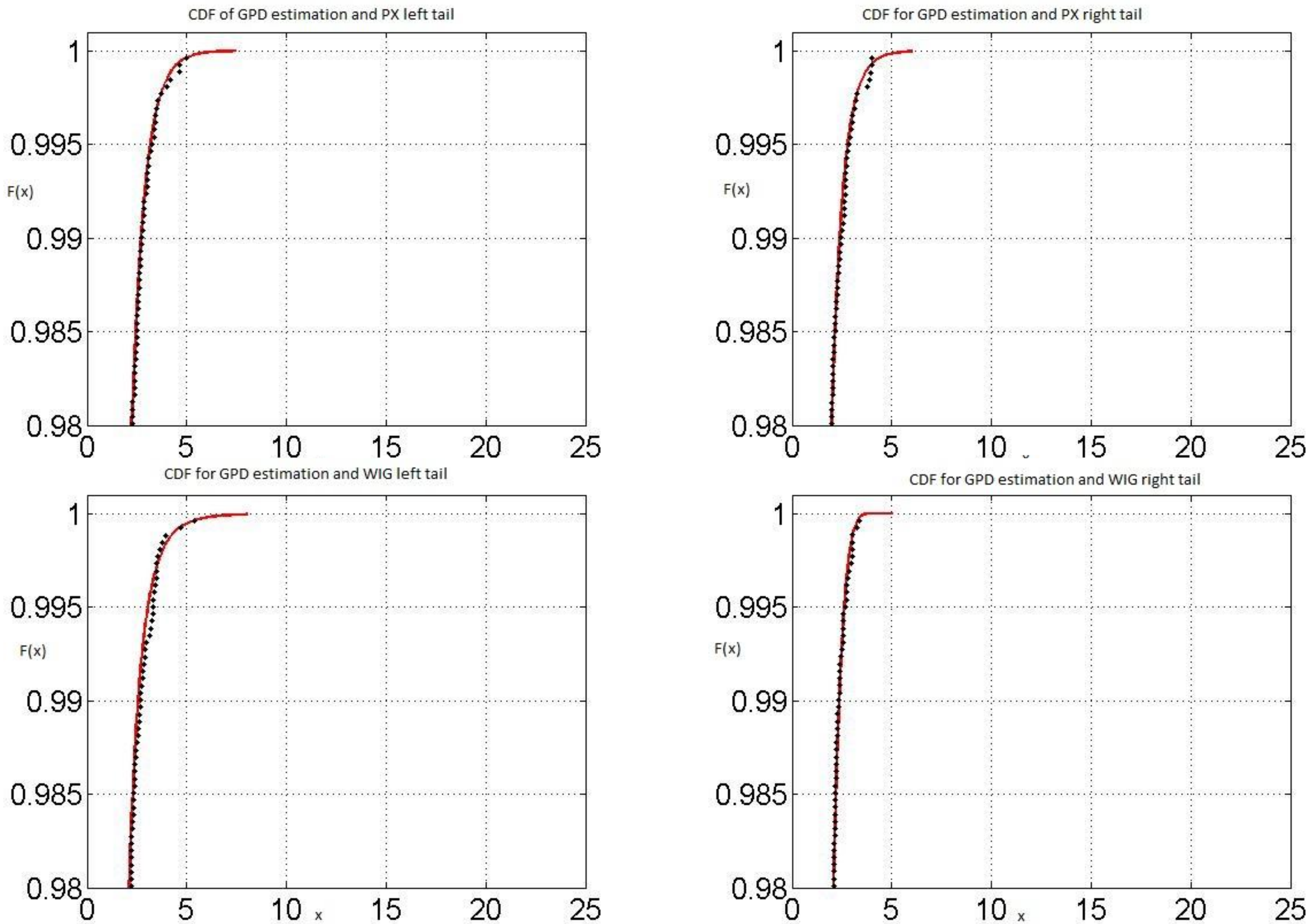
Expected ShortFall							
Model	**Level**	**Time**	**Lower / Upper**	**PX**	**WIG**	**BETC**	**BUX**
Historical Simulation	99%	1 day	Lower	0.0650	0.0498	0.0685	0.0608
	95%	1 day	Lower	0.0365	0.0317	0.0395	0.0368
	99%	10 days	Lower	0.2055	0.1575	0.2166	0.1923
	95%	10 days	Lower	0.1154	0.1002	0.1249	0.1164
	99%	1 day	Upper	0.0631	0.0438	0.0577	0.0626
	95%	1 day	Upper	0.0338	0.0289	0.0346	0.0387
	99%	10 days	Upper	0.1995	0.1385	0.1825	0.1980
	95%	10 days	Upper	0.1069	0.0914	0.1094	0.1224
EWMA RiskMetrics Approach	99%	1 day	Lower / Upper	0.0337	0.0315	0.0229	0.0383
	95%	1 day	Lower / Upper	0.0265	0.0248	0.0244	0.0311
	99%	10 days	Lower / Upper	0.1067	0.0995	0.0947	0.1212
	95%	10 days	Lower / Upper	0.0839	0.0785	0.0771	0.0982

Data and results: Extreme VaR

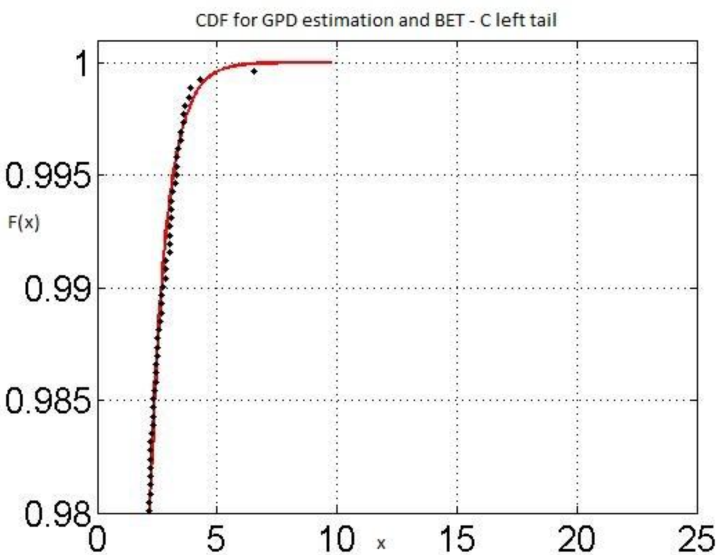
Model	Level	Time	Lower / Upper	PX	WIG	BETC	BUX
Extreme VaR	99%	1 day	Lower	2.709	2.564	2.681	2.387
	95%	1 day	Lower	1.632	1.585	1.544	1.559
	99%	10 days	Lower	8.567	8.108	8.480	7.548
	95%	10 days	Lower	5.161	5.011	4.883	4.930
	99%	1 day	Upper	2.389	2.374	2.532	2.500
	95%	1 day	Upper	1.484	1.635	1.591	1.682
	99%	10 days	Upper	7.553	7.508	8.006	7.905
	95%	10 days	Upper	4.692	5.169	5.031	5.319
Extreme ES	99%	1 day	Lower	3.389873	3.343808	3.399152	2.944407
	95%	1 day	Lower	2.302572	2.213984	2.251967	2.078145
	99%	10 days	Lower	10.71972	10.57405	10.74906	9.311031
	95%	10 days	Lower	7.281373	7.001233	7.121345	6.57167
	99%	1 day	Upper	2.983623	2.689691	3.027625	3.039276
	95%	1 day	Upper	2.049569	2.081987	2.166752	2.193538
	99%	10 days	Upper	9.435044	8.50555	9.574191	9.611036
	95%	10 days	Upper	6.481305	6.583821	6.851871	6.936577

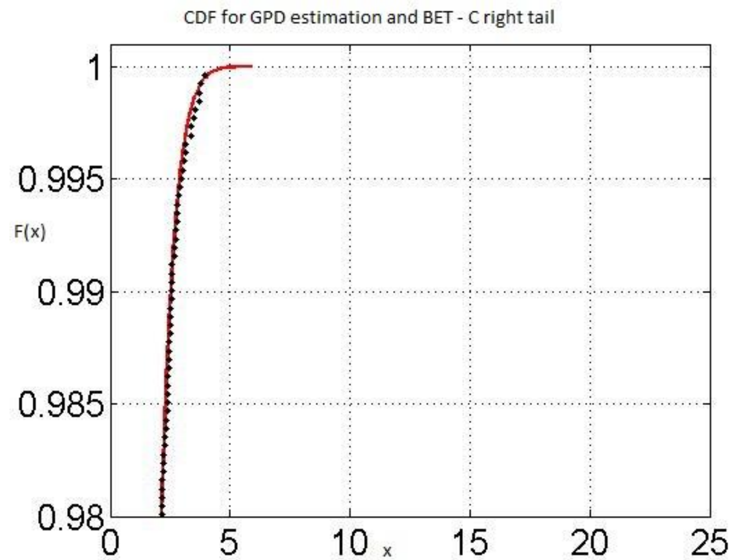
- ES > VaR
- Extreme VaR > VaR

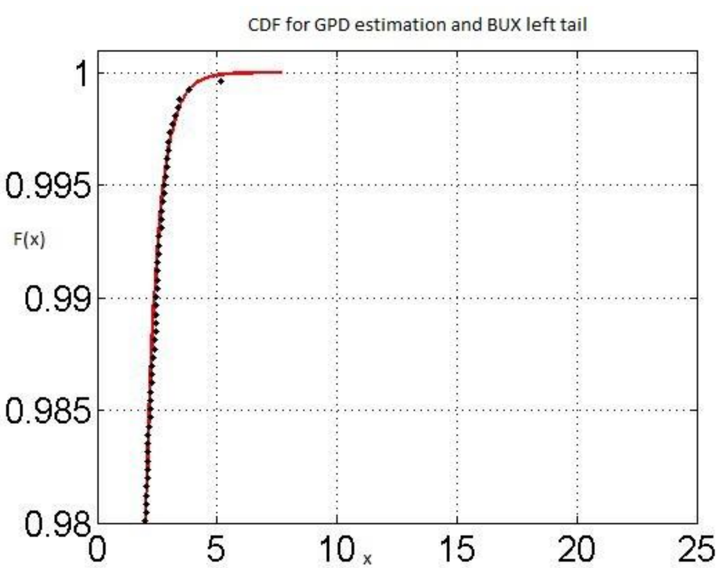
Data and results: GPD Tail fit with ML ζ (I)

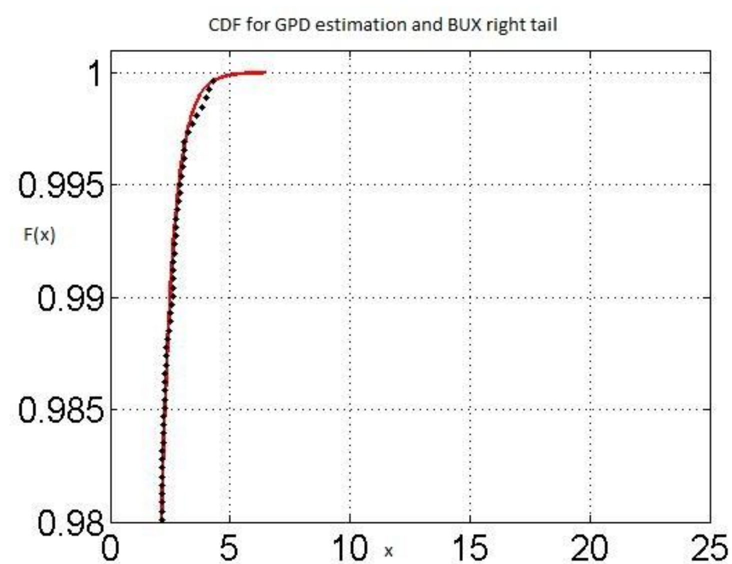


Data and results: GPD Tail fit with ML ζ (II)









Backtesting – Percentage of failures

VaR - EWMA	99%		95%		99%		95%		99%		95%		99%		95%	
	BET C LONG	BET C SHORT	BET C LONG	BET C SHORT	PX LONG	PX SHORT	PX LONG	PX SHORT	WIG LONG	WIG SHORT	WIG LONG	WIG SHORT	BUX LONG	BUX SHORT	BUX LONG	BUX SHORT
The percentage of failures	1.58%	1.71%	4.61%	5.00%	1.71%	1.71%	5.56%	3.60%	1.71%	1.84%	5.13%	5.66%	1.32%	0.92%	4.47%	6.45%
Number of violation	12	**13**	35	**38**	13	13	42	**27**	13	14	39	43	10	**7**	34	49
The first violation is observed after	3	87	3	54	37	59	36	31	41	73	30	10	42	172	40	2
EVT VaR - EWMA																
The percentage of failures	1.50%	2.63%	4.21%	5.66%	1.45%	1.60%	4.34%	6.32%	1.32%	1.77%	4.74%	5.30%	1.30%	2.11%	4.08%	6.05%
Number of violation	**11**	20	**32**	43	**11**	**12**	**33**	48	**10**	**13**	**36**	**40**	**10**	16	**31**	**46**
The first violation is observed after	3	2	3	1	42	2	42	2	41	10	17	10	40	2	40	2

- EVT VaR performs better for LONG positions (negative tails) in every case, for 99% and 95%;
- Three Zones: Green, Yellow and Red;
- The percentage and the numbers of failures must be at low levels

Backtesting – Tests (Kupiec with Unconditional Coverage)

EVT VaR - EWMA	Statistic	99%		95%		99%		95%		99%		95%		99%		95%	
		BET - C LONG	BET C - SHORT	BET - C LONG	BET C - SHORT	PX LONG	PX SHORT	PX LONG	PX SHORT	WIG LONG	WIG SHORT	WIG LONG	WIG SHORT	BUX LONG	BUX SHORT	BUX LONG	BUX SHORT
TUFF - Time Until First Failure LRTUFF: Likelihood Ratio of Time Until First	Value	5.4315	6.7549	2.3776	5.1026	0.5831	6.4579	0.7462	4.3215	0.6118	6.412	0.0264	2.924	0.6417	6.4579	0.6398	3.3215
	Chi - square (99% / 95%)	6.635	6.635	3.841	3.841	6.6350	6.635	3.841	3.841	6.635	6.635	3.841	3.841	6.635	6.635	3.841	3.841
	p - value	0.2	0.0110	0.1230	0.0749	0.4450	0.0110	0.3880	0.0680	0.4340	0.1200	0.8710	0.1323	0.4230	0.0110	0.424	0.068
Uncond. - Unconditional Coverage LRUC: Likelihood Ratio Unconditional Coverage	Value	3.1957	Inf	1.0513	Inf	1.3498	Inf	0.7234	Inf	0.6964	Inf	0.127	Inf	2.188	Inf	1.445	Inf
	Chi - square (99% / 95%)	6.635	6.635	3.841	3.841	6.6350	6.635	3.841	3.841	6.635	6.635	3.841	3.841	6.635	6.635	3.841	3.841
	p - value	0.074	0.000	0.305	0.000	0.2450	0.000	0.395	0.000	0.404	0.000	0.737	0.000	0.139	0.000	0.229	0.000
VaR - EWMA	Statistic																
TUFF - Time Until First Failure LRTUFF: Likelihood Ratio of Time Until First Failure	Value	5.4315	781.19	2.3776	307.69	0.7394	524.08	0.443	171.01	0.6118	652.6	0.1978	2.924	0.5831	Inf	0.6398	3.3215
	Chi - square (99% / 95%)	6.635	6.635	3.841	3.841	6.635	6.635	3.841	3.841	6.635	6.635	3.841	3.841	6.635	6.635	3.841	3.841
	p - value	0.2	0.0000	0.1230	0.0000	0.39	0.0000	0.5060	0.0000	0.4340	0.0000	0.6570	0.1323	0.4450	0.0000	0.424	0.068
Uncond. - Unconditional Coverage LRUC: Likelihood Ratio Unconditional Coverage	Value	2.188	Inf	0.2558	Inf	3.1957	Inf	0.6655	Inf	3.1957	Inf	0.0275	Inf	0.6964	Inf	0.4588	Inf
	Chi - square (99% / 95%)	3.841	3.841	3.841	3.841	3.841	3.841	3.841	3.841	3.841	3.841	8.97	3.841	3.841	3.841	3.841	3.841
	p - value	0.139	0.000	0.613	0.000	0.074	0.000	0.415	0.000	0.074	0.000	0.868	0.000	0.404	0.000	0.494	0.000

- EVT VaR is accepted for LONG positions, ad rejected for Short BUX (99%), Short BET-C (99%, 95%), Short PX (95%)
- p – value > 0.05 or 0.01;
- Statistics < Critical Value for acceptance of the model;

Conclusions

- The data used for this analysis presents the specific facts for financial data (stocks, indices, returns of this types of data);
- VaR models underestimate the risk. A better measure is ES for returns of our indices;
- EVT – VaR performs very well for LONG positions (left tail / negative returns);
- ARMA – EGARCH models performed good in order to compensate for autocorrelation and heteroscedasticity;
- GPD approach is a good fit distribution for indices returns; it provides relevant parameters for every distribution (see the goodness of fit tests and QQ Plots);
- With EVT we can predicted losses situated far (4 -5 sigmas);

Further research and remarks

- Remarks:
  - Use EVT for stress testing;
  - Use VaR for usual and medium size losses;
  - Determine the capital requirements with VaR models;

- Further research:
  - Use the models for portfolios, assets;
  - Applied Copula Models for large portfolio;
  - Determine the market risk and how to reduce this risk with our models;
  - Use more tests for backtesting;

Thank you!