Volatility modeling in finance and banking

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Contents

Proposed objectives

A
- Identification of volatility features present on several data sets

B
- Estimation of volatility models such as GARCH, EGARCH and GJR-GARCh

C
- Volatility forecast

D
- Identification of best volatility forecasting model through performance indicators such as RMSE, MAE, MAPE and TIC

Data description

Variable	Data source	Start date	End date	Number of observations
BET	Bucharest Stock Exchange	04/01/2010	27/02/2015	1304
S&P500	Investing.com	01/02/2010	27/02/2015	1277
FTSE 100	Investing.com	01/02/2010	27/02/2015	1282
BUX	Budapest Stock Exchange	01/02/2010	27/02/2015	1266

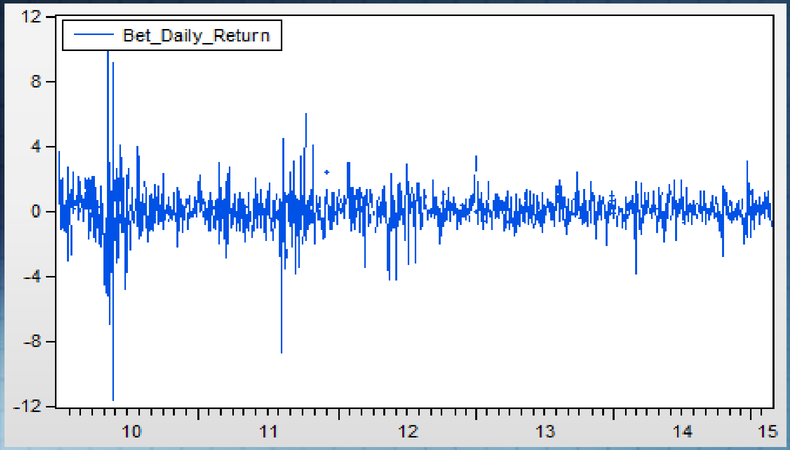
Stylized facts

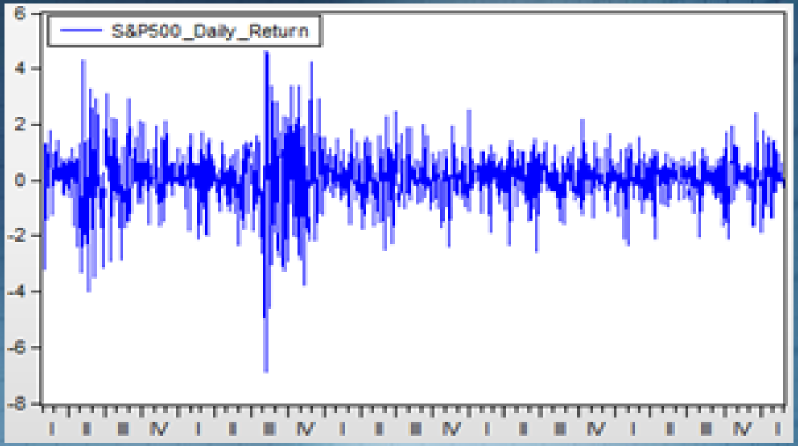
The distribution of financial time series has heavier tails than the normal distribution

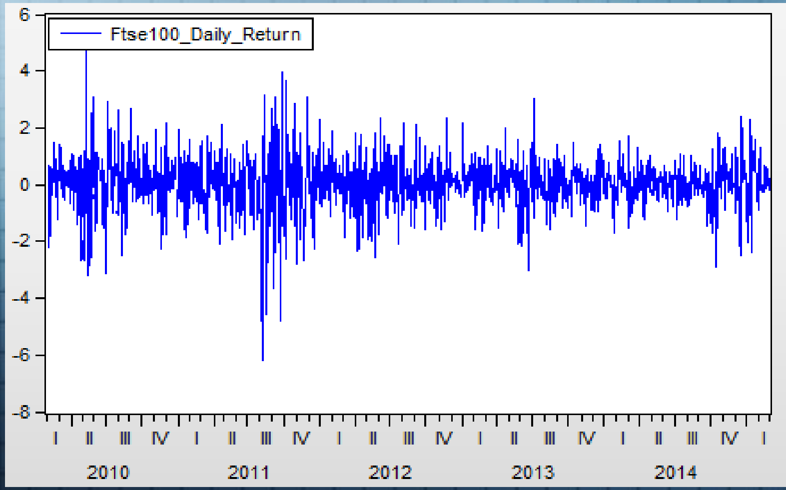
Highly correlated values for the squared returns

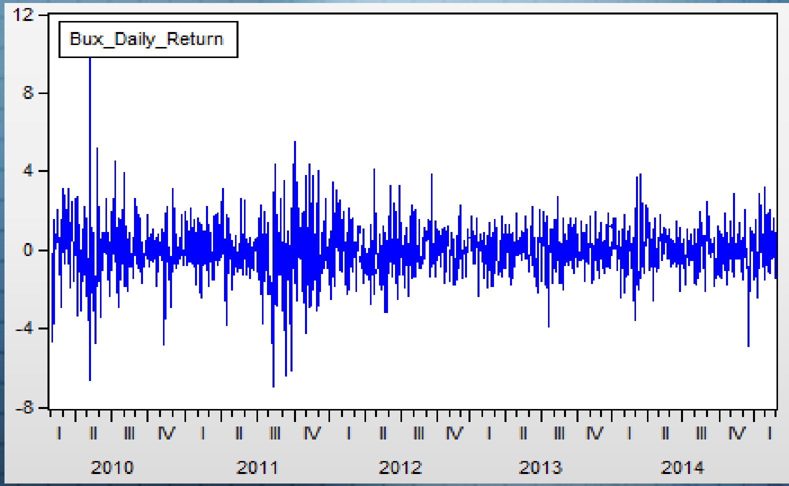
Changes in the returns tend to cluster

Graphical analysis









Descriptive Statistics

	BET	S&P500	FTSE 100	BUX
Observations no.	1303	1276	1281	1265
Mean	0.031320	0.051618	0.021899	-0.014692
Maximum	10.56451	4.631738	5.032269	10.67431
Minimum	-11.61165	-6.895833	-6.199408	-6.984201
Std. Dev	1.233848	1.005644	0.990514	1.388589
Skewness	-0.425505	-0.469451	-0.324257	0.112170
Kurtosis	19.22570	7.647679	6.330105	7.730152
Jarque-Bera	14332.87	1195.317	614.3552	1181.967

Time series stationarity

Augmented Dickey-Fuller Unit Root Test				
H_0: X are o rădăcină unitate				
	BET	S&P500	FTSE 100	BUX
Val. ADF	-35.86016	-38.15924	-34.47723	-35.58251
Val. Critică 1%	-3.435161	-3.435267	-3.435247	-3.435311
Val. Critică 5%	-2.863552	-2.863599	-2.863590	-2.863619
Val. Critică 10%	-2.567891	-2.567916	-2.567911	-2.567926
Kwiatkowski-Phillips-Schmidt-Shin Unit Root Test				
H_0: X este staţionar				
	BET	S&P500	FTSE 100	BUX
Val. KPSS	0.074747	0.042847	0.021935	0.039209
Val. Critică 1%	0.739000	0.739000	0.739000	0.739000
Val. Critică 5%	0.463000	0.463000	0.463000	0.463000
Val. Critică 10%	0.347000	0.347000	0.347000	0.347000

Volatility models

- ARCH/GARCH class of models
    - Engle (1982)
    - Bollerslev (1986)
    - Nelson (1991)
    - Glosten, Jaganathan, and Runkle (1993)
    - Engle and Bollerslev (1986)
    - Bollerslev and Mikkelsen (1996)

The GARCH model

- $mean\ \ eq.:\ r_t = \sqrt{h_t}\varepsilon_t$
- $\text{variance}\ \ eq.: h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i r_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}$

Parameter constraints:

✓ ensuring variance to be positive

$$\begin{cases} \alpha_0 > 0, \\ \alpha_\mathrm{i} \geq 0\ \forall\ \mathrm{i} \geq 1, \\ \beta_\mathrm{i} \geq 0\ \forall\ \mathrm{j} \geq 1 \end{cases}$$

✓ stationarity condition:

$$\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1$$

Error distribution

A. Normal

- *The density function:*

$$f\left(\varepsilon_t\right)=\frac{1}{\sqrt{2\pi h_t}}\cdot\exp\left(-\frac{1}{2}\cdot\frac{\varepsilon_t^{2}}{h_t}\right)$$

- *Implied kurtosis:*

  k=3

- *The log-likelihood function:*

$$L_{Normal}=-\frac{1}{2}\sum_{t=1}^{T}\left[\ln\left(2\pi\right)+\ln\left(h_t\right)+\frac{\varepsilon_t^{2}}{h_t}\right]$$

Error distribution

B. T-student

- *The density function:*

$$f\left(\varepsilon_t\right)=\frac{\Gamma\left[\left(\nu+1\right)/2\right]\cdot s_t^{-1/2}}{\left(\pi\nu\right)^{1/2}\Gamma\left(\nu/2\right)\left[1+\varepsilon_t^2/\left(s_t\nu\right)\right]^{\left(\nu+1\right)/2}};\ \mathrm{var}\left(\varepsilon_t\right)=\frac{s_t\nu}{\nu-2},\nu>2$$

- *Implied kurtosis:*

$$k=\frac{3\left(\nu-2\right)}{\nu-4},\nu>4$$

- *The log-likelihood function:*

$$L_{Student}=T\cdot\left\{\ln\left[\Gamma\left(\frac{\nu+1}{2}\right)\right]-\ln\left[\Gamma\left(\frac{\nu}{2}\right)\right]-\frac{1}{2}\cdot\ln\left[\pi\left(\nu-2\right)\right]\right\}-\frac{1}{2}\sum_{t=1}^{T}\left[\ln\left(\sigma_t^2\right)+\left(1+\nu\right)\ln\left(1+\frac{\varepsilon_t^2}{\sigma_t^2\left(\nu-2\right)}\right)\right]$$

Error distribution

C. Generalized Error Distribution (GED)

- *The density function:*

$$f(\varepsilon_t) = \frac{\nu \cdot \exp\left[-\left(\frac{1}{2}\right)\left|\frac{\varepsilon_t}{\lambda}\right|^{\nu}\right]}{\lambda \cdot 2^{\frac{\nu+1}{\nu}} \Gamma\left(\frac{1}{\nu}\right)}; \lambda = \sqrt{\frac{\Gamma\left(\frac{1}{\nu}\right) \cdot 2^{-\frac{2}{\nu}}}{\Gamma\left(\frac{3}{\nu}\right)}}$$

- *Implied kurtosis:*

$$k = \Gamma\left(\frac{1}{\nu}\right) \cdot \Gamma\left(\frac{5}{\nu}\right) \Big/ \Gamma\left(\frac{3}{\nu}\right)^2$$

- *The log-likelihood function:*

$$L_{GED} = \sum_{t=1}^{T}\left[\ln\left(\frac{\nu}{\lambda}\right) - \frac{1}{2}\left|\frac{\varepsilon_t}{\lambda}\right|^{\nu} - \left(\frac{\nu+1}{\nu}\right)\ln(2) - \ln\Gamma\left(\frac{1}{\nu}\right)\right]$$

The EGARCH model

$$Variance\ \ eq.: \ln(h_t) = \alpha_0 + \sum_{i=1}^{q} \alpha_i g(z_{t-i}) + \sum_{j=1}^{p} \gamma_i \ln(h_{t-j})$$

$$where$$

$$g(z_t) = \theta z_t + \gamma\big[|z_t| - E|z_t|\big]$$

$$z_t = \varepsilon_t / \sqrt{h_t}$$

The properties of the EGARCH model are summarized as follows:

- The function $g(z_t)$ is linear in z_t with slope coefficient θ+1 if z_t is positive while $g(z_t)$ is linear in z_t with slope coefficient θ-1 if z_t is negative;
- Suppose that θ = 0. Large innovations increase the conditional variance if $|z_t| - E|z_t| > 0$ and decrease the conditional variance if $|z_t| - E|z_t| < 0$
- Suppose that θ<1. The innovation in variance, $g(z_t)$, is positive if the innovations z_t are less than $(2/\pi)^{1/2}/(\theta-1)$. Therefore, the negative innovations in returns, ε_t, cause the innovation to the conditional variance to be positive if θ is much less than 1

Other models

The GJR-GARCH model

$$Variance\ \ eq.: h^2{}_t = \alpha_0 + \sum_{i=1}^{q} (\alpha_i \varepsilon_{t-i}^2 + \gamma_i I_{t-i} \varepsilon_{t-i}^2) + \sum_{j=1}^{p} \beta_j h^2{}_{t-j}$$

$where$

$$I_{t-i} = \begin{cases} 0, r_{t-i} \geq \mu \\ 1, r_{t-i} < \mu \end{cases}$$

The IGARCH model

$$Variance\ \ eq.: (1 - \alpha(L) - \beta(L))\varepsilon_t^2 = \alpha_0 + (1 - \beta(L))\nu_t$$

$where$

$$\nu_t \equiv \varepsilon_t^2 - \sigma_t^2$$

$$If\ (1 - \alpha(L) - \beta(L)) = \varphi(L)(1 - L) then$$

$$\varphi(L)(1 - L)\varepsilon_t^2 = \alpha_0 + (1 - \beta(L))\nu_t$$

The FIEGARCH model

$$Variance\ \ eq.: \varphi(L)(1 - L)^d \varepsilon_t^2 = \alpha_0 + (1 - \beta(L))\nu_t$$

$where$

$$\nu_t \equiv \varepsilon_t^2 - \sigma_t^2, 0 < d < 1, \varphi(z) = 0 and \beta(z) = 1$$

Forecast criteria

Forecast evaluation measures

$RMSE = [\Sigma(e^2_t/n)]^{1/2}$

$MAE = \Sigma(|e_t|/n)$

$MAPE = \Sigma(|e_t|/Y_t)$

$TIC = [\Sigma(Y_t – f_t)^2]^{1/2}/[(\Sigma\ f_t^2)^{1/2} + (\Sigma\ Y_t^2)^{1/2}]$

Where:

Y_t = actual values

f_t = forecast values

$e_t = Y_t - f_t$ = forecast errors

n = number of forecasts

Results BET

	GARCH (1,1)		EGARCH (1,1)		GJR-GARCH (1,1)	
	Normal	t-Student	Normal	t-Student	Normal	t-Student
ω	0.0499	0.0472	-0.2692	-0.2266	0.0585	0.0561
(s.e.)	(0.0079)	(0.0135)	(0.0197)	(0.0320)	(0.0081)	(0.0146)
(p-value)	(0.0000)	(0.0005)	(0.0000)	(0.0000)	(0.0000)	(0.0001)
α_1	0.1902	0.1381	0.3592	0.3079	0.1058	0.0944
(s.e.)	(0.0149)	(0.0251)	(0.0253)	(0.0423)	(0.0206)	(0.0292)
(p-value)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0012)
γ_1			-0.0839	-0.0671	0.1375	0.1124
(s.e.)			(0.0156)	(0.0254)	(0.0268)	(0.0432)
(p-value)			(0.0000)	(0.0084)	(0.0000)	(0.0092)
β_1	0.7893	0.8278	0.9352	0.9455	0.7863	0.8095
(s.e.)	(0.0148)	(0.0270)	(0.0082)	(0.0138)	(0.0163)	(0.0284)
(p-value)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
υ		5.0556		5.1316		5.3639
(s.e.)		(0.8133)		(0.8202)		(0.8997)
(p-value)		(0.0000)		(0.0000)		(0.0000)

	Forecast evaluation measures			
Model	RMSE	MAE	MAPE	TIC
Garch Normal	0.771543	0.559901	100.0044	0.998366
Garch T-student	0.771550	0.559885	99.99607	0.999570
Egarch Normal	0.771509	0.559864	100.0032	0.996697
Egarch T-student	0.771558	0.559905	100.0015	0.999765
Gjr-Garch Normal	0.771500	0.559850	100.0027	0.996366
Gjr-Garch T-student	0.771519	0.559870	100.0020	0.997549

Results S&P 500

	GARCH (1,1)		EGARCH (1,1)		GJR-GARCH (1,1)	
	Normal	t-Student	Normal	t-Student	Normal	GED
ω	0.0352	0.0364	-0.1339	-0.1041	0.0461	89011.98
(s.e.)	(0.0075)	(0.0118)	(0.0214)	(0.0262)	(0.0053)	(40140.16)
(p-value)	(0.0000)	(0.0020)	(0.0000)	(0.0001)	(0.0000)	(0.0266)
α_1	0.1423	0.1545	0.1525	0.1280	-0.0899	-3.5997
(s.e.)	(0.0180)	(0.0323)	(0.0255)	(0.0323)	(0.0120)	(4.3465)
(p-value)	(0.0000)	(0.0000)	(0.0000)	(0.0001)	(0.0000)	(0.4076)
γ_1			-0.2299	-0.2810	0.3404	1.5629
(s.e.)			(0.0191)	(0.0291)	(0.0334)	(3.3111)
(p-value)			(0.0000)	(0.0000)	(0.0000)	(0.6369)
β_1	0.08308	0.8171	0.9431	0.9392	0.8704	-0.9855
(s.e.)	(0.0210)	(0.0325)	(0.0061)	(0.0082)	(0.0143)	(0.0152)
(p-value)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
υ		5.2254		6.2058		0.2280
(s.e.)		(1.0342)		(1.2860)		(0.0151)
(p-value)		(0.0000)		(0.0000)		(0.0000)

	Forecast evaluation measures			
Model	RMSE	MAE	MAPE	TIC
Garch Normal	0.734270	0.542266	221.1603	0.905040
Garch T-student	0.733656	0.543624	280.2487	0.875427
Egarch Normal	0.735422	0.544425	143.5889	0.960137
Egarch T-student	0.737147	0.546931	100.0005	0.999830
Gjr-Garch Normal	0.736638	0.546814	145.4783	0.962627

Results FTSE100

	GARCH (1,1)		EGARCH (1,1)		GJR-GARCH (1,1)	
	Normal	t-Student	Normal	t-Student	Normal	t-Student
ω	0.0308	0.0248	-0.1615	-0.1445	0.0355	0.0355
(s.e.)	(0.0073)	(0.0092)	(0.0238)	(0.0284)	(0.0050)	(0.0064)
(p-value)	(0.0000)	(0.0071)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
α_1	0.1326	0.1164	0.1916	0.1710	-0.0181	-0.0345
(s.e.)	(0.0165)	(0.0217)	(0.0294)	(0.0347)	(0.0102)	(0.0146)
(p-value)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0754)	(0.0181)
γ_1			-0.1644	-0.1909	0.2450	0.2712
(s.e.)			(0.0139)	(0.0233)	(0.0258)	(0.0390)
(p-value)			(0.0000)	(0.0000)	(0.0000)	(0.0000)
β_1	0.8374	0.8604	0.9505	0.9527	0.8593	0.8651
(s.e.)	(0.0195)	(0.0253)	(0.0075)	(0.0091)	(0.0186)	(0.0212)
(p-value)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
υ		8.0834		9.6546		9.7999
(s.e.)		(2.1925)		(2.6744)		(2.9629)
(p-value)		(0.0002)		(0.0003)		(0.0009)

	Forecast evaluation measures			
Model	RMSE	MAE	MAPE	TIC
Garch Normal	0.746266	0.543646	99.95544	0.998268
Garch T-student	0.746027	0.543326	100.3560	0.993325
Egarch Normal	0.746274	0.543639	100.0283	0.999212
Egarch T-student	0.746209	0.543610	100.2535	0.998462
Gjr-Garch Normal	0.746272	0.543639	99.98233	0.999909
Gjr-Garch T-student	0.746189	0.543602	100.3313	0.997917

Results BUX

	GARCH (1,1)		EGARCH (1,1)		GJR-GARCH (1,1)	
	Normal	t-Student	Normal	t-Student	Normal	t-Student
ω	0.0563	0.0380	-0.0744	-0.0506	0.0328	0.0044
(s.e.)	(0.0132)	(0.0151)	(0.0130)	(0.0158)	(0.0075)	(0.0010)
(p-value)	(0.0000)	(0.0117)	(0.0000)	(0.0013)	(0.0000)	(0.0000)
α_1	0.0837	0.0578	0.1060	0.0695	-0.0102	-0.0428
(s.e.)	(0.0106)	(0.0139)	(0.0182)	(0.0215)	(0.0098)	(0.0029)
(p-value)	(0.0000)	(0.0000)	(0.0000)	(0.0012)	(0.2985)	(0.0000)
γ_1			-0.0784	-0.0731	0.1008	0.0772
(s.e.)			(0.0111)	(0.0142)	(0.0128)	(0.0032)
(p-value)			(0.0000)	(0.0004)	(0.0000)	(0.0000)
β_1	0.8853	0.9199	0.9800	0.9884	0.9390	0.9991
(s.e.)	(0.0159)	(0.0129)	(0.0051)	(0.0045)	(0.0104)	(0.0031)
(p-value)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
υ		7.4474		8.4307		8.6041
(s.e.)		(1.4674)		(1.7648)		(1.8712)
(p-value)		(0.0000)		(0.0000)		(0.0000)

	Forecast evaluation measures			
Model	RMSE	MAE	MAPE	TIC
Garch Normal	1.143775	0.872492	100.0001	0.999128
Garch T-student	1.143827	0.872498	100.0000	1.000000
Egarch Normal	1.143711	0.872472	99.98842	0.998122
Egarch T-student	1.143750	0.872488	99.99701	0.998659

Conclusions

- **BET**
  - GJR-GARCH(1,1) normal
  - EGARCH(1,1) normal
- **S&P500**
  - GARCH(1,1) t-student
  - GARCH(1,1) normal
- **FTSE 100**
  - GARCH(1,1) t-student
  - GJR-GARCH(1,1) t-student
- **BUX**
  - EGARCH(1,1) normal
  - GJR-GARCH(1,2) normal

Further research

Better forecast accuracy measures

Sample construction

A better proxy for realized volatility

Comparability of results produced by different econometrics packages

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