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DISSERTATION PAPER

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Complex nonlinear dynamics of financial time series. An empirical analysis using chaos theory

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CONTENT

ABSTRACT

This paper presents an effort to implement metric and topological tools, to test for the presence of nonlinear dependence and deterministic chaos, in the returns series for eight stock market indices. Chaos theory might be useful in explaining the dynamics of financial markets, since chaotic models are capable of exhibiting behaviour similar to that observed in real financial data. In this context, the scope of this research is to provide an insight into the role that nonlinearities and, in particular, chaos theory may play in explaining the dynamics of financial markets.

Based on the following chaos tests: BDS test, Hurst exponent using R/S analysis, Recurrence Plots and Recurrence Quantification Analysis, the overall result of this study suggests that the returns series do not follow a random walk process. Rather it appears that the daily returns are serially correlated and the estimated Hurst exponents are indicative of marginal persistence. Result from the test of independence on filtered residuals suggests that the existence of nonlinear dependence, at least to some extent, can be attributed to the presence of conditional heteroskedasticity. It appears, therefore, that GARCH-type models can adequately explain some, but not all, of the observed nonlinear dependence in the data. Further, we find evidence to support the proposition that returns are generated by a chaotic system in five out of eight cases. Presence of chaos in market indices implies that profitable nonlinearity based trading rules may exist at least in the short-run. Finally, fairly contrary to the findings of previous studies, rejection of random walk hypothesis offers some possibility of returns predictability.

I. INTRODUCTION

The main aim of this study is to investigate the presence of nonlinear dependence and deterministic chaos in daily returns on eight stock market indices by contrasting the random walk hypothesis with chaotic dynamics. More specifically, I attempt to test for long-range dependence, nonlinear structure and chaos in both developed and emerging markets by investigating if daily returns series of stock market indices show any sign of biased-random walk and chaotic behavior.

Over the years, movements in stock prices have fascinated not only the speculative traders, but also the academicians and policy makers. For the last four decades, the efficient market hypothesis (EMH) has been the dominant theory in the financial markets. Many studies have been conducted to test the theory. Under the EMH, stock returns processes should be random. Market efficiency idea mentions that prices fully reflect all information and price movements do not follow any patterns or trends. That is, past price movements cannot be used to predict the future price movements but follow what is known as a random walk, an intrinsically unpredictable pattern. The idea that stock price variations are generated by a random process with no long-term memory has long been prominent in international and quantitative finance research. Under this approach, it was believed that stock returns are independent and identically distributed (IID) random variables. Presence of this traditional belief is also reflected in the assumptions of prominent asset pricing theories such as Sharpe–Lintner model of market equilibrium and the Black–Scholes theory of option pricing. The assertion of random walk seemed indisputable not only on empirical justifications but also for apparently strong theoretical reasons - namely, consistency with the efficient market paradigm (Abhyankar, Copeland, & Wong, 1997). Validity of the efficient market hypothesis in real world actually precludes the possibility that market players can generate higher returns from using trading rules. Interestingly, empirical findings of earlier studies have, by and large, confirmed the validity of random walk (Fama, 1970).

However, the pioneering work of Mandelbrot (1963) challenged this classical conviction by establishing that increments of stock prices or return variations did indeed possess a long-memory, which may be best described by fractional Brownian motion. Further, Rogers (1997) countered the traditional random walk hypothesis much strongly by establishing that under the condition of fractional Brownian motion (when Hurst exponent $H \neq 0.5$), arbitrage opportunities and monetary profits can be generated from financial markets without taking any substantial risk, which is certainly anathema to the prominent financial theory. Following this line of argument, an increasing number of studies using chaotic and nonlinear estimation techniques for modeling financial data have highlighted the nonlinear deterministic behaviour of stock prices. These findings strongly and collectively suggest that stock prices may be more predictable than it was previously thought under the random walk approach. In other words, adherents of biased-random walk approach believe that seemingly random stock price and returns sequences may not be random and there are reasons to believe that they may arise from deterministic nonlinear dynamical systems, instead.

The discovery of nonlinear dependence and deterministic chaos in financial data has altered our traditional view of the erratic behaviour of financial variables by providing an entirely different perspective to analyze financial data moving well beyond the realm of linear paradigm and random walk approach of stock price movements. Nonlinear deterministic systems with a few degrees of freedom can create output signals that appear complex and mimic stochastic signals from the point of view of conventional time series analysis but are chaotic. Chaotic systems are complex systems which belong to the class of deterministic dynamical systems. They are detected and used in a lot of fields for control or forecasting. Deterministic chaos has been rigorously and extensively studied by mathematicians and other scientists. It is almost impossible to give a precise mathematical definition of deterministic chaos that encapsulates everything in the diverse literature. Chaos is said to be an irregular oscillatory process broadly characterized by three conditions: nonlinearity, fractal attractor, and sensitive dependence on initial conditions (SDIC) (Faggini, 2011). A unique feature of chaotic system is that it can generate large and apparently random fluctuations, quite similar to the sudden ups and downs sometimes seen in the stock market. Interestingly, stochastic models explain that many of these sudden fluctuations are actually caused by external random shocks. However, in a chaotic system these abrupt fluctuations are considered to be internally generated as part of the deterministic process (Gilmore, 1996). This makes a strong case for the application of chaotic dynamic to model and explain nonlinearity in financial time series. Although chaos is highly unpredictable, its deterministic nature offers good opportunity for profitable forecast at least in the short-run. However, forecasting over long horizon is not possible mainly because of the SDIC property of a chaotic system.

Researchers in economics and finance have been interested in testing nonlinear dependence and chaos for almost three decades. A wide variety of reasons for this interest have been suggested, including an attempt to improve the forecasting accuracy of linear time series models and to better explain the dynamics of the underlying variables of interest using a richer class of models than that permitted by limiting the set to the linear case. The issue of whether a financial series is indeed chaotic may not be of great importance to a financial forecaster who is only interested in adjusting dynamic trading strategies according to apparent predictability in time series. During these three decades the search for chaos in economics has gradually became less enthusiastic, as little or no empirical support for the presence of chaotic behaviours in economics has been found. The literature did not provide a solid support for chaos as a consequence of the high noise level that exists in most economic time series, and the relatively small sample sizes of data.

Against this background, in the present study I attempt to investigate nonlinear and chaotic structure in daily returns series of market indices. The motivation for undertaking this study is not only the dearth of research in this domain but also the potential implications of such a study for players in these markets. Detection of a deterministic chaos would mean an opportunity for hedgers, speculators as well as arbitrageurs to play the markets better.

This paper offers several contributions to the existing literature. Although there are many studies on this issue, covering different sample periods and markets, but to my knowledge, this is one of the first attempts to investigate chaotic structure in both developed and emerging markets. The search for chaos in financial markets has been mostly restricted to stock markets and in too developed countries. However given the very different institutional features of financial markets in developing countries, it is important to explore the possibilities of such markets exhibiting chaotic behavior. Financial markets in developing countries are less mature as compared to those in developed countries, and the implications of complex nonlinear behavior could be significant for traders, institutional investors for devising suitable trading strategies. Second, instead of performing a direct test for chaos, I apply different techniques to investigate the underlying data generating process. These tests will help investigate the adequacy of generally applied linear or nonlinear econometric models for forecasting these financial time series. Finally, the study of chaotic dynamic will help determine the degree of predictability and efficiency in financial markets.

The rest of the paper is organized as follows. Section 2 presents a brief review of literature. Section 3 discusses empirical methodologies and provides a brief account of tests used in the study. Section 4 presents empirical results. The final section provides concluding observations based on the findings of the study.

II. LITERATURE REVIEW

The efficient market hypothesis and the random walk approach to explain the time series behaviour of stock prices have been in the centre of attention for years. There are many studies supporting the EMH in the literature (Kendall, 1953; Brealey, 1970; Cunningham, 1973; Brock, 1987). These studies on the United Kingdom and Canadian stock markets, based on the assumption that stock market price changes are i.i.d., detect the weak form market efficiency and find no evidence of chaos in macroeconomic time series.

However, the pioneering work of Mandelbrot (1963) challenged the random walk theory and initiated a new debate by bringing the concept of long-memory and biased random walk into perspective.

In 1965, Fama admitted that linear modeling techniques have limitations as they are not sophisticated enough to capture complicated “patterns” which chartists claim to see in stock prices.

The recent empirical literature has mainly focused on testing for the presence of long-memory, nonlinear dependence and chaos in financial data by using new techniques and models indicative of complex dynamics (Abhyankar, Copeland, and Wong, 1995; Abhyankar, 1997).

Although some studies have produced conflicting results, but now a broad consensus has emerged that nonlinear structure in financial time series is a somewhat realistic phenomenon (Brock, Hsieh, & LeBaron, 1992). The literature, especially after the earlier findings of nonlinear dependence in returns by Hinich and Patterson (1985) and Frank and Stengos (1989), has seen many such studies. In general, recent studies have consistently documented strong evidence of nonlinearity in the returns of various assets. Studies applying tests based on nonlinear dynamics have also concluded that residuals of filtered stock returns are not IID and, therefore, market returns do not follow random walk process. While considering the case of long-memory, contrary to the traditional belief, many recent studies have reported strong evidence of long-range dependence in the returns of various assets (Cajueiro and Tabak, 2009; Helms et al., 1984). Using the classical rescaled-range analysis, Howe et al. (1999) find strong evidence of long-range nonlinear deterministic structure in the returns of the Japanese, Singaporean, Korean, and Taiwanese indices with cycle length ranging from 3 to 4 years. However, contrary to these findings, Lo (1991), Cheung and Lai (1995) and Jacobsen (1996) failed to find any evidence of long-range dependence in stock returns for some European countries, the United States and Japan.

As far as the presence of nonlinear deterministic and chaotic structures in market returns are concerned, the published evidence is rather mixed. For example, studies such as Frank and Stengos (1989), Hsieh (1991), Blank (1991) and DeCoster, Labys, and Mitchell (1992), have found strong evidence of nonlinear dependence and chaotic structure in economic and financial time series whereas Kosfeld and Robe (2001) for German bank stock returns and Opong, Mulholland, Fox, and Farahmand (1999) for London Financial Times Stock Exchange found that low order GARCH models are sufficient to explain the existing nonlinearity in the data. Similarly, in his study Brooks (1998) reported strong evidence of nonlinearity but failed to find any significant evidence of deterministic chaos in the data. Scheinkman and LeBaron (1989) study U.S.A. weekly returns on the Center for Research in Security Prices (CRSP) value-weighted index, employing the BDS statistic, and find rather strong evidence of nonlinearity and some evidence of chaos. Brock, Hsieh and LeBaron (1991) concluded that the evidence for the presence of deterministic low-dimensional chaotic generators in economic and financial data is not very strong.

Nevertheless, some recent studies have documented encouraging evidence of chaos in exchange rate data. For example, in their study Serletis and Gogas (1997) and Scarlat, Stan, and Cristescu (2007) found consistent evidence of chaotic dynamics in various markets.

In a working paper, Wei and Leuthold (1998) looked at six agricultural futures markets—corn, soybeans, wheat, hogs, coffee and sugar—and found that five of them (all except sugar) were chaotic processes.

Andreou, Pavlides and Karytinos (2000) examined four major currencies against GRD and found evidence of chaos in two out of four.

Panas and Ninni (2000) found strong evidence of chaos in daily oil products for the Rotterdam and Mediterranean petroleum markets.

It is clear that while there is a broad consensus on the presence of nonlinear dependence in market returns, the issue is still unsettled for chaos in financial data. Furthermore, there is hardly any study on emerging markets to explain the time series behaviour of stock returns. Therefore, this study attempts to fill this gap by providing some additional evidence from the emerging countries.

III. EMPIRICAL METHODOLOGY FOR THE ANALYSIS

The fast development of computer resources available to the scientist community and the parallel growing bulk of theoretical knowledge about complex dynamics have allowed many researches to look for nonlinear dynamics in data whose evolution linear ARMA models are unable to explain in a satisfactory manner. The methods involved in Nonlinear Time Series Analysis can be classified into metric, dynamical, and topological tools. The metric approach depends on the computation of distances on the system's attractor, and it includes Grassberger-Procaccia correlation dimension and BDS test. The dynamical approach deals with computing the way nearby orbits diverge by means of estimating Lyapunov exponents. Topological methods are characterised by the study of the organisation of the strange attractor, and they include recurrence plots.

In practice, various criteria and methods are used to detect nonlinear structure and chaos in the data. Given that the study of chaos is relatively new in financial research, there is no single commonly accepted statistical test to determine precisely the nature of nonlinearity and chaotic structure in the data (Gilmore, 1996). A best alternative approach, therefore, would be to use all available criteria for analyzing the behaviour of stock price or return time series. The tests applied in this study are widely used in literature.

3.1. Metric Tools

3.1.1. The BDS test

Brock, Dechert, LeBaron, and Scheinkman (1996) developed a powerful test for independence and identical distribution based on correlation function developed by Grassbeger and Procaccia (1983). This test is also known as BDS test for nonlinear dependence between points on a reconstructed attractor. The BDS test tests the null hypothesis of whiteness (IID observations) against an unspecified alternative using a nonparametric technique.

In fact, BDS test is not considered to be a direct test for chaos. It is useful because it is a well defined, and easy to apply test which has power against any type of structure in a series. This feature can be viewed as both a cost and a benefit. On the one hand it can detect many types of nonlinear dependence that might be missed by other tests. On the other hand, a rejection using this test is not very informative. One extension of this test is to use it as a residual diagnostic, as a model selection tool to obtain some information about what kind of dependency exists after removing linear dependency from the data. However, it is possible to use the BDS test to indirectly search for nonlinear dependence which is necessary but not sufficient condition for chaos. The test is applied to residuals to check if the best-fit model for a given time series is a linear or nonlinear model.

Under the null hypothesis of whiteness, the BDS statistic is given by:

$$\mathrm{W(N,m,\varepsilon)} = \sqrt{N}\,\frac{C(N,m,\varepsilon) - C(N,1,\varepsilon)^m}{\hat{\sigma}\,(\mathrm{N,m,\varepsilon})}$$

where $\hat{\sigma}\,(\mathrm{N,m,\varepsilon})$ is an estimate of the standard deviation of $C(N,m,\varepsilon) - C(N,1,\varepsilon)^m$.

The correlation function asymptotically follows standard normal distribution N(0,1):

$$\lim_{N\to\infty} W(N,m,\varepsilon) \sim N(0,1), \qquad \forall\, m, \varepsilon$$

Moving from the hypothesis that a time series is IID, the BDS tests the null hypothesis that $\mathrm{C(N,m,\varepsilon)} = \mathrm{C(N,1,\varepsilon)^m}$, which is equivalent to the null hypothesis of whiteness against an unspecified alternative.

Both positive as well as negative values of the test statistic are taken as an indication of non-IID behaviour. BDS statistics takes a positive value if the probability of any two m-histories (x_t, x_{t+1},…, x_{t+m-1}) and (x_s, x_{s+1},……., x_{s+m-1}) of being “close” together is higher than that of mth power of the any two points x_t and x_s. In other words, a significant and positive BDS statistics indicates that certain patterns such as “clustering” are too frequent compared to a true random process whereas a significant and negative BDS test statistic indicates that certain patterns are too infrequent compared to a true random process.

If series are IID so that linear or even conditional heteroskedasticity can describe the relations between data, chaotic tests will not be required. However, if this is not the case, investigating the main properties of chaoticity should not be disregarded.

Because it is based on the correlation dimension, the BDS test suffers from the same limitations. In particular, its performance depends on the size of data sets (N) and ε, even though Brock (1991) showed how the statistics of this test are correctly approximated in finite samples if:

- the number of data N is greater than 500.

- ε lies between 0.5σ and 2σ, where σ is the standard deviation of the series.

- the embedding dimension m is lower than N/200.

3.1.2. Rescaled range analysis and Hurst exponent

The EMH assumes that all investors immediately react to the new information. Some recent studies, however, argue that this is not always true in the market. For example, Peters (1994) argues that most people do not react immediately on the arrival of new information. Instead they wait for confirming the information and do not react until a trend is clearly visible in the market. Therefore, there will be an uneven assimilation of information and this will cause stock price movements to follow a biased-random walk rather than pure random walk. If this is true, the possibility of biased-random walk implies that there is memory or temporal dependence in the underlying series.

In literature, a tool extensively used for testing long-term memory and fractality of a time series is the R/S analysis. In the present study, we broadly follow Peters (1994) to conduct R/S analysis.

In the first stage, the time period is divided into A contiguous sub-periods of length n such that A×n=N where N is the length of the series N_t. We then label each sub period I_a, a=1,2,3,4,...,A. Each element in I_a is labeled $N_{k,a}$ such that k=1,2,3,4,…,n. For each I_a of length n, the average value, e_a, is defined as:

$$e_a = \left(\frac{1}{n}\right) \times \sum_{k=1}^{n} N_{\mathrm{k,a}}$$

In the next stage, the range R_{Ia} is defined as the maximum less the minimumvalue, $X_{k,a}$,within each sub-period Ia given by $R_{Ia}= \max(X_{k,a}) - \min(X_{k,a})$, given that $1 \le k \le n$, and $1 \le a \le A$, where $X_{k,a}= \sum_{i=1}^{k}(N_{i,a} - e_a)$, k=1,2,…,n, is the time series of accumulated departures from mean value for each sub-period.

Further, each range R_{Ia} is normalized by dividing by the sample standard deviation S_{Ia} corresponding to it given by:

$$S_{Ia} = \left[\left(\frac{1}{n}\right) \times \sum_{k=1}^{n} \left(N_{\mathrm{k,a}} - e_a\right)^2\right]^{0.5}$$

the average R/S values for the length n is defined as

$$\left(\frac{R}{S}\right)_n = \left(\frac{1}{A}\right) \times \sum_{a=1}^{A} (R_{\mathrm{Ia}}/S_{Ia})$$

Now the final stage involves applying an ordinary least square (OLS) regression with log(n) as the independent variable and (R/S)n as the dependent variable. Hurst (1951) show that R/S could be estimated by the following empirical relationship, generally referred to as Hurst's Empirical Law:

$$(R/S) = a \times (N)^H$$

where a is a constant and H equals the Hurst exponent. Now after obtaining logs of both sides of the Hurst's equation, we obtain:

$$\log (R/S) = H \times \log (N) + \log (a).$$

The Hurst exponent, H, is the slope coefficient obtained from this regression. For the classification of time series, the Hurst exponent can be interpreted as follows:

- a Hurst exponent of 0.5 indicates that the series behaves in a manner consistent with the random walk or nondeterministic process;
- an H of greater than 0.5 indicates ‘persistence’ or trend-reinforcing series;
- an H of less than 0.5 indicates ‘antipersistence’ or ergodic series.

3.2. Topological tests: Recurrence Analysis

The failure to find convincing evidence for chaos in economic and financial time series redirected the interest to additional tests that work with small data sets and that are robust against noise. This goal seems to be reached by topological tools based on topological invariant testing procedure. Compared to the existing metric and dynamical classes of testing procedures, these tools could be better suited to testing for chaos in financial and economic time series and to provide information about the underlying system responsible for chaotic behaviour.

The topological approach to testing for chaos has origins as far back as Poincaré (1892) and attempts to determine how the unstable periodic orbits of the strage attractor are interwined. Topological tools are characterised by studying the organisation of the strange attractor because they exploit an essential property of a chaotic system, i.e. the tendency of the time series to nearly, although never exactly, repeat itself over time. This property is known as the recurrence property.

The processes of stretching and compression are responsible for organising the strange attractor in a unique way and if one can determine how the unsable periodic orbits are organised, we can identify the stretching and compressing mechanisms responsible for the creation of the strange attractor. Once these mechanisms have been identified, a geometric model can be constructed, which describes how to model the stretching and squeezing mechanisms responsible for generating the original time series. That is to say, topological tests may not only detect the presence of chaos (the only information provided by the metric class of tests), but can also provide information about the underlying system responsible for the chaotic behavior.

Unlike the metric approach, as the topological method preserves time ordering, that’s the temporal correlation in a time series in addition to the spatial structure of the data, where evidence of chaos is found, the researcher may proceed to characterise the underlying process in a quantitative way.

An example of these topological tests is Recurrence Analysis. Recurrence Analysis is composed by the Recurrence Plot (RP) developed by Ekmann (1987), the graphical tool that evaluates the temporal and phase space distance, designed to locate hidden recurring patterns, nonstationarity and structural changes, and Recurrence Quantification Analysis (RQA), the statistical quantification of RP.

3.2.1. Recurrence Plot

Recurrence plots are graphical devices specially suited to detect hidden dynamical patterns and nonlinearities in data. With recurrence plots, one can also graphically detect structural changes in data or see similarities in patterns across the time series under study. The fundamental assumption underlying the idea of the recurrence plots is that an observable time series (a sequence of observations) is the realization of some dynamical process, the interaction of the relevant variables over time.

As remarkable as it seems, it has been proven mathematically that one can recreate a topologically equivalent picture of the original multidimensional system behavior by using the time series of a single observable variable (Takens, 1981). The basic idea is that the effect of all the other (unobserved) variables is already reflected in the series of the observed output. Furthermore, the rules that govern the behavior of the original system can be recovered from its output.

The starting point of the RPs is based on the time delay method through which the original series is transformed into a set of m-histories. The Recurrence Plot is a two dimensional representation of those m-histories whose coordinates are the present and lagged values of the series.

The original series is transformed into an m-dimensional system that, depending on the fulfilment of certain conditions, is topologically equivalent to the original system from which the series was supposedly determined. The one-dimensional signal is expanded into an m-dimensional phase space by substituting each observation with vector:

$$\mathrm{Yi} = \left\{x_i, x_{i-d}, x_{i-2d}, \dots, x_{i-(m-1)d}\right\}$$

As a result, we have a series of vectors:

$$\mathrm{Y} = \{\mathrm{y}(1), \mathrm{y}(2), \dots, \mathrm{y}(\mathrm{N}-(\mathrm{m}-1)\mathrm{d})\}$$

where N is the number of observations, m is the embedding dimension and d is the delay time.

Time delay determines the time separation or predictability of the components in the reconstructed vectors of the system state. It should be chosen so that the elements in the embedding vectors are no longer correlated, thus subsequent analysis would reveal spatial (or geometrical) structures.

The embedding dimension determines the number of the components in the reconstructed vector of the system state. It should be large enough to unfold the system trajectories from self-overlaps, but not too large as the noise will amplify.

If the unknown system that generated $\{x_t\}_{t=1}^{n}$ is N-dimensional, and provided that embedding dimension, if m ≥ 2n+1, the set of m-histories recreates the dynamics of the data-generating system and can be used to analyse its dynamics. However, the sequence of embedded vectors is useful only if parameters m and d are properly chosen by using appropriate methods.

Next, a symmetric matrix of distances (e.g., Euclidean distances) can be constructed by computing distances between all pairs of embedded vectors. By using an appropriate norm and fixing a threshold ε that determines if vectors x(i) and x(j) are sufficiently close together – distance between them below or equal to ε - we obtain a recurrence matrix formally expressed as following:

$$\mathrm{R}(\mathrm{i},\mathrm{j}) = \mathrm{H}(\varepsilon\|\mathrm{x}(\mathrm{i}) - \mathrm{x}(\mathrm{j})\|) \text{ for i, j = M}$$

where M = N-(m-1)d, H is the Heaviside function, and || || is a norm, generally Euclidian. The matrix R consists of values 0 (no recurrence) and 1 (recurrence). More formally:

$$\mathrm{R}(\mathrm{i},\mathrm{j}) = \begin{cases} 0, \text{if } \|\mathrm{x}(\mathrm{i}) - \mathrm{x}(\mathrm{j})\| > \varepsilon \\ 1, \text{if } \|\mathrm{x}(\mathrm{i}) - \mathrm{x}(\mathrm{j})\| \leq \varepsilon \end{cases}$$

The recurrence plot relates each distance of such a matrix to a colour (e.g., the larger is the distance, the “cooler" is the colour). Thus, the recurrence plot is a solid rectangular plot consisting of pixels whose colours correspond to the magnitude of data values in a two-dimensional array and whose coordinates correspond to the locations of the data values in the array. Generally dark colour marks nonzero values, that is, short distances, and a light colour zero values, that is, the long distance.

Both axes of the RP are time axes and show rightwards and upwards (convention). Vectors compared with themselves necessarily compute to distances of zero, which means that by definition the RP always has a black main diagonal line, the line of identity and it is symmetric with respect to the main diagonal, i.e. $R_{i,j} \equiv R_{j,i}$.

This graphic tool shows different structures depending on the nature of the series under study. In particular, it is capable of detecting the time recurrence patterns underlying deterministic systems (whether they are chaotic or not). Non-chaotic deterministic systems exhibit very simple regular structures, while the RPs of chaotic systems also show a certain regularity but with more complex and denser features. On the other hand, the RPs obtained from purely random systems do not show distinguishable patterns, appearing as a cloud of points with no apparent structure.

To illustrate the basic ideas behind RP some examples by Visual Recurrence Analysis (VRA) are used. In VRA, a one-dimensional time series from a data file is expanded into a higher-dimensional space, in which the dynamic of the underlying generator takes place. This is done using a technique called “delayed coordinate embedding”, which recreates a phase space portrait of the dynamical system under study from a single (scalar) time series. The idea of such reconstruction is to capture the original system states at each time we have an observation of that system output.

The first recurrence plot that VRA shows can be a beautiful picture, but absolutely uninformative. We must choose a suitable embedding dimension and an adequate time delay. To choose the appropriate time delay, we can compute the “average mutual information function”, as an alternative to the classical autocorrelation function; the latter detects linear correlations, but the former is useful to detect both linear and non-linear correlations. The time delay should be chosen such that the elements in embedding vectors are no longer correlated, thus subsequent analysis would reveal spatial or geometrical structures. We can also use a procedure called “false nearest neighbours method" to find the optimal embedding dimension.

Once the dynamical system is reconstructed in a manner outlined above, a recurrence plot can be used to show which vectors in the reconstructed space are close and far from each other. This way one can visualize and study the motion of the system trajectories and infer some characteristics of the dynamical system that generated the time series.

If the analysed time series is deterministic, then the recurrence plot shows short line segments parallel to the main diagonal.

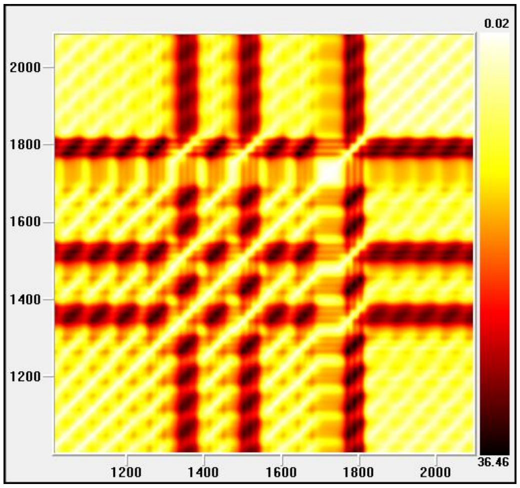

Figure 1. Lorenz attractor

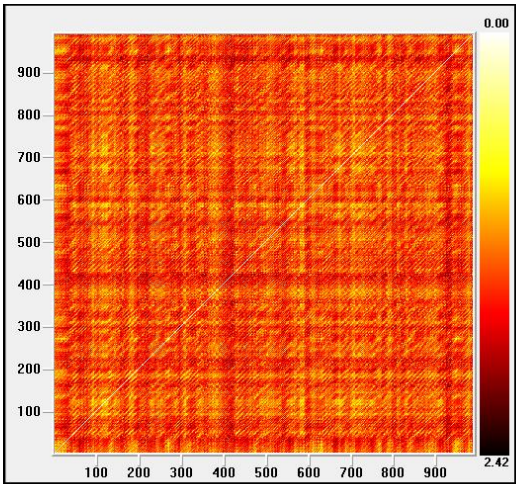

Figure 2. White noise

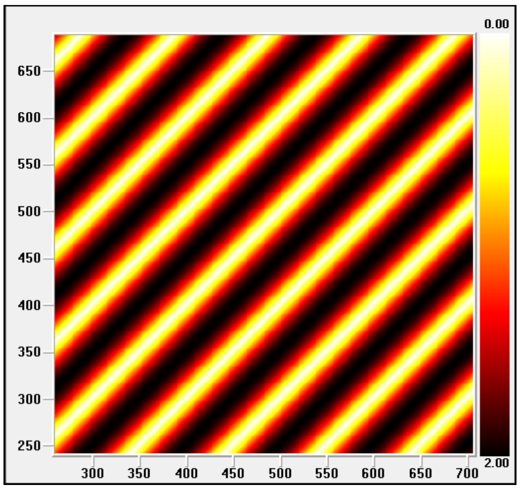

Figure 3. Sine wave

As an illustration, Figure 1 shows the recurrence plot from the (chaotic) Lorenz attractor, for a time delay d = 17 (selected through the method of average mutual information) and an embedding dimension m = 3 (selected through the false nearest neighbours method). Figure 2 shows the recurrence plot from a Gaussian white noise, for d = 1 and m = 12 and figure 3 shows the recurrence plot from a Sine wave, for d = 25 and m = 2. All figure have been attained using VRA.

Recurrent points in Figure 1 for the Lorenz attractor form distinct short diagonals parallel to the main diagonal. The upward diagonal lines result from strings of vector patterns repeating themselves multiple times down the dynamics. This type of recurrent structure indicates that the dynamics is visiting the same region of an attractor at different times; therefore, the presence of diagonal lines indicates that deterministic rules are present in the dynamics. The set of lines parallel to the main diagonal is the signature of determinism.

Alternatively, in Figure 2, recurrence points for the white noise are simply distributed in a homogeneous random pattern – a cloud of points, signifying that a random variable lacks of deterministic structures.

Diagonal structures show (Figure 3) the range in which a piece of the trajectory is rather close to another piece of the trajectory at different times. From the occurrence of lines parallel to the diagonal in the recurrence plot, it can be seen how fast neighboured trajectories diverge in phase space. These lines would not occur in a random as opposed to deterministic process. Thus, if the analysed time series is chaotic, then the recurrence plot shows short segments parallel to the main diagonal: chaotic behaviour causes very short diagonals, whereas deterministic behaviour causes longer diagonals (Figure 1 vs. Figure 3).

This procedure has some advantages such as simplicity of implementation, robustness to sample length, high dimensionality, noisy dynamics in the underlying equations of motion and fewer prior requirements of the database used. RP analysis is independent of limiting constraints such as data set size, noise, and stationarity; prewhitening of the data (linear filtering, detrending, or transforming the data to conform to any particular distribution) is not necessary as stationarity is not as essential like for the metric approach (Faggini, 2013).

Nevertheless some limitations are present. The first one is the construction of RPs and obtaining the Recurrence Matrix (RM). Because they are carried out on the basis of the time delay method, which requires previously fixing the values of the embedding dimension and the time delay, the results obtained from the RP application are sensitive to the values chosen for these parameters. The second one is the difficulty to interpret the graphical output of RP. Sometimes the signature of determinism, the set of lines parallel to the main diagonal might not be so clear (e.g., the size of the lines being relatively short among a field of scattered recurrent points), i.e., the recurrence plot could contain subtle patterns not easily ascertained by visual inspection; in this context, Zbilut and Webber (1992) propose the so called recurrence quantification analysis (RQA).

3.2.2. Recurrence Quantification Analysis

The RQA considers that it is possible to quantify the information supplied by RP and, using certain simple pattern recognition algorithms, to summarize the information in a set of indicators or statistics. In this way more objective information than that which could be derived from a purely visual analysis are obtained.

Considering that RP is symmetric, the set of indicators is obtained using the upper or lower triangular part of RP excluding the main diagonal. The main indicators are recurrence rate, determinism, averaged length of diagonal structures, entropy and trend.

Recurrence rate (REC): recurrence points percentage defined as:

$$\%REC = \frac{NREC}{NP} \times 100$$

where NREC is the number of recurrent points and NP is the total element of the recurrence matrix. This variable can range from 0% (no recurrent points) to 100% (all points recurrent). Roughly speaking REC is what is used to compute the correlation dimension of data.

Determinism rate (DET) is the ratio of recurrence points forming diagonal structures to all recurrence points. DET measures the percentage of recurrent points forming line segments that are parallel to the main diagonal and is calculated as

$$\%DET = \frac{NPD}{NREC} \times 100$$

where NPD is the number of points on lines parallel to the main diagonal caused by the existence of time correlation within the trajectory. Diagonal line segments must have a minimum length defined by the line parameter.

The presence of such diagonal structuring in RM is assumed to be a distinctive feature of deterministic structures, absence, instead, of randomness. DET is related with the determinism of the system: the greater the number of points is on line segments, the greater the general dependence of the series will be. Periodic signals (e.g. sine waves) will give very long diagonal lines, chaotic signals (e.g. Hénon attractor) will give very short diagonal lines, and stochastic signals (e.g. random numbers) will give no diagonal lines at all.

Maxline (MAXLINE) represents the averaged length of diagonal structures and indicates the longest line segments that are parallel to the main diagonal. Unlike the %DET counts all the points on the parallel lines equally regardless of their size, this indicator considers the length of the different lines. This is a very important recurrence variable because it inversely scales with the the largest positive Lyapunov exponent (Eckmann et al., 1987; Trulla et al., 1996). Positive Lyapunov exponents gauge the rate at which trajectories diverge, and are the hallmark for dynamic chaos. Thus, the shorter the linemax, the more chaotic (less stable) the signal.

Entropy (ENT) (Shannon entropy) measures the distribution of those line segments that are parallel to the main diagonal and reflects the complexity of the deterministic structure in the system. ENT is a measure of signal complexity and is calibrated in units of bits/bin. Individual histogram bin probabilities are computed for each non-zero bin and then summed according to Shannon's equation. A high ENT value indicates a large diversity in diagonal line lengths; low values indicate small diversity in diagonal line lengths. For simple periodic systems in which all diagonal lines are of equal length, the entropy would be expected to be 0.0 bins/bin.

The value trend (TREND) quantifies the degree of system stationarity. It measures the paling of the patterns of RPs away from the main diagonal used for detecting drift and non-stationarity in a time series. It is calculated as a slope of the %REC as a function of the displacement of the main diagonal. If recurrent points are homogeneously distributed across the recurrence plot, TND values will hover near zero units. If recurrent points are heterogeneously distributed across the recurrence plot, TND values will deviate from zero units.

Laminarity (LAM) is analogous to %DET except that it measures the percentage of recurrent points comprising vertical line structures rather than diagonal line structures. The line parameter still governs the minimum length of vertical lines to be included.

Trapping time (TT) is simply the average length of vertical line structures.

IV. AN EMPIRICAL ANALYSIS USING CHAOS THEORY

4.1. Data

The empirical application in this paper is based on eight data sets representing the closing prices of stock indices, selected from both developed countries and emerging countries, namely: **BET-C** (Bucharest Stock Exchange - Romania), **BUX** (Budapest Stock Exchange - Hungary), **DAX** (Frankfurt Stock Exchange - Germany), **FTSE 100** (London Stock Exchange - United Kingdom), **FTSE MIB** (Milan Stock Exchange - Italy), **Nikkei 225** (Tokyo Stock Exchange - Janponia), **SOFIX** (Sofia Stock Exchange - Bulgaria) and **S&P 500** (New York Stock Exchange - USA).

The data used are on a daily basis covering the period January 2nd, 2002 to May 22th, 2014. In the sample period are included 3232 observations. Missing data were replaced by the arithmetic mean of the last two values available.

Based on the eight data sets collected, the prices were converted in daily returns. We apply the following transformation to the raw data before conducting statistical tests:

$$R_{it} = \ln(P_{i,t}) - \ln(P_{i,t-1})$$

where: R_{it} is the rate of ruturn of stock index i at time t;

$P_{i,t}$ este is the close price of stock index i at time t.

This transformation implements an effective detrending of the series. This method also provides an effective way to measure the continuously compounded rates of returns. For each set of data we have a number of 3231 calculated returns.

In the analysis were used the following programs: Eviews7, Matlab R2013a and VRA 4.9.

4.2. Empirical results

First we analyzed the behavior of daily returns. In the table below we present the characteristics of the data series.

The highest, and the lowest yield was obtained for the BUX index. If we compare the standard deviations, the most risky is BUX index trading and the least risky is FTSE 100 index trading.

Table 1. Descriptive statistics for daily returns

	BET-C	BUX	DAX	FTSE 100	FTSE MIB	NIKKEI 225	SOFIX	S&P 500
Mean	0.000585	0.000305	0.000196	0.0000829	-0.000140	0.0000977	0.000499	0.000153
Median	0.000571	0.000449	0.000724	-0.0000441	0.000446	0.000312	0.000465	0.000663
Maximum	0.108906	0.131777	0.107975	0.093842	0.108742	0.094941	0.083878	0.109572
Minimum	-0.122582	-0.126489	-0.074335	-0.092646	-0.085991	-0.121110	-0.113600	-0.094695
Std. Dev.	0.015043	0.015918	0.015369	0.012297	0.015191	0.015194	0.013501	0.012772
Skewness	-0.675487	-0.138154	0.0048171	-0.134973	-0.041630	-0.642058	-0.506626	-0.202021
Kurtosis	12.85947	9.770674	8.034013	10.33820	8.012025	9.069276	11.87536	12.26718
Jarque-Bera	13332.5	6181.761	3412.821	7259.260	3382.767	5181.051	10742.91	11583.65
Probability	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

The coefficient of asymmetry (skewness) is negative for BET-C, BUX, FTSE 100, FTSE MIB, NIKKEI 225, SOFIX and S&P 500, which indicates that the distribution yields are asymmetrical to right and for DAX, it is positive, the distribution of returns is asymmetric to the left.

In all cases, it is observable that the daily returns have a high kurtosis, much greater than 3 (the kurtosis of the normal distribution) for all indices, reflecting the presence of a leptokurtotic distribution, sharper than a normal distibution with more values concentrated around the average values and more-tailed than a normal distibution.

Most financial assets have such a distribution. In a leptokurtotic distribution the probability of occurrence of an extreme event is higher then the probability involved in a normal distribution. So price valuation models can generate errors if it is assumed a normal distribution.

The distributions yield indexes are shown in **Appendix 3**. Analyzing the eight figures is immediately apparent that the empirical distribution of daily returns deviates from the normal distribution, being more elongated than that.

The null hypothesis of normality is strongly rejected by the Jarque-Bera test. The test confirms that the returns of market indices are not normally distributed. Test statistic is significant for a level of 1% confidence as the associated probability is 0% in all eight cases.

Using Quantile-Quantile chart (Q-Q Plot) to compare the empirical distribution of daily returns to a theoretical distribution (in this case the normal distribution) I have reached the same conclusion. If the empirical distribution is normal, the Q-Q graph result is the first bisector. For each series, the empirical quintiles chart indicates deviations from the normal line. (**Appendix 4**).

Also, it was necessary to analyze the evolution of daily returns. Figure 1 illustrates the variation of daily returns for BET-C in the sample period. Based on the figure below we can draw some conclusions.

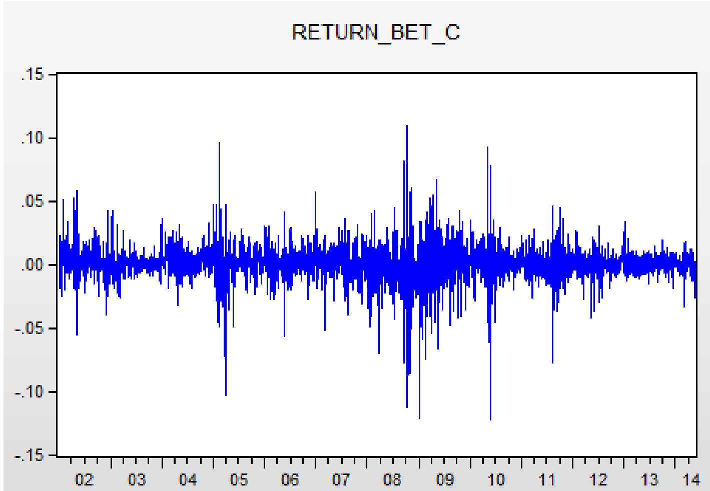

Figure 4. Evolution of BET-C returns

First, a simple visual inspection of the series indicates that yields presents two specific characteristics of nonlinear models (leverage and volatility clustering). Volatility is concentrated in short periods of time, indicating possible correlations between current and historical volatilities.

The series is heteroscedastic and volatility clustering phenomenon is present, alternating periods of low volatility followed by small variations with periods of high volatility followed by large variations in yields. The phenomenon is best observed after the global financial crisis occurrence, especially in the second half of 2008, when yields has the highest volatility.

The evolution of all indices are shown in **Appendix 5**. As can be seen from the graphs, all returns series show volatility clustering phenomenon (i.e. low values of volatility are followed by low values and high values are followed by other high values).

These features are consistent with other studies in literature on financial time series behavior. This manifestation of data is confirmed by autocorrelation function (ACF) and partial autocorrelation function (PACF) estimated up to lag 15. Since this phenomenon is specific to GARCH type models, the return series behavior could be captured by this type of models.

To check the hypothesis of stationarity of the return series we apply unit root tests to determine the order of integration. Stationarity tests used are: ADF (Augmented Dickey-Fuller) and PP (Phillips-Perron). The summary results of these tests are shown in Table 2.

Table 2. ADF and PP test results

	BET-C	BUX	DAX	FTSE 100	FTSE MIB	NIKKEI 225	SOFIX	S&P 500
ADF test statistic	**-51.74986**	**-27.02303**	**-58.35544**	**-27.49092**	**-57.65096**	**-59.43818**	**-17.47361**	**-64.45599**
1%	-3.432186	-2.565679	-2.565678	-2.565679	-2.565678	-2.565678	-2.565680	-2.565678
5%	-2.862237	-1.940922	-1.940922	-1.940922	-1.940922	-1.940922	-1.940922	-1.940922
10%	-2.567185	-1.616634	-1.616634	-1.616634	-1.616634	-1.616634	-1.616633	-1.616634
PP test statistic	**-52.31744**	**-54.92960**	**-58.55270**	**-60.28837**	**-57.67984**	**-59.52710**	**-54.86382**	**-65.01303**
1%	-3.432186	-2.565678	-2.565678	-2.565678	-2.565678	-2.565678	-2.565678	-2.565678
5%	-2.862237	-1.940922	-1.940922	-1.940922	-1.940922	-1.940922	-1.940922	-1.940922
10%	-2.567185	-1.616634	-1.616634	-1.616634	-1.616634	-1.616634	-1.616634	-1.616634

Stationarity tests used have in the null hypothesis that the series analyzed contains a unit root, i.e. it is not stationary. As can be seen from the table above, for all the series the test statistic has a value lower than the critical value, at a level of significance of 5% and 1%, and the probabilities associated are less than 5% (**Appendices 6 and 7**), which means that the hypothesis of a unit root is rejected. Therefore return series used in the analyze are stationary. We can say that they are integrated of order 0.

An important problem posed by financial series is serial correlation of residuals. So we check if there is correlation in each of the eight rows of data. At first sight, analyzing autocorrelation functions (ACF) and partial autocorrelation functions (PACF) estimated up to lag 15 it can be seen that there is serial autocorrelation, but this is weak. Autocorrelation and partial autocorrelation coefficient vary within the range of -0,125 (S&P 500), and 0.107 (SOFIX). (**Appendix 8**).

Even if ACF and PACF graphs indicates some autocorrelation, autocorrelation is insufficiently argued at this point. To confirm the existence of autocorrelation is used Ljung-Box test. The test confirms the results of visual analysis. Q-test statistic is significantly different from zero for 6 of the 8 series returns analyzed up to lag 15 and the associated probability is 0% (except DAX and Nikkei 225), which means that at a level of relevance of 1% we can reject the null hypothesis of absence of serial correlation.

Autocorrelation analysis should be extended to square returns and absolute returns errors to check the presence of ARCH effects. Although the autocorrelation function for raw returns indicate a relatively low correlation, the ACF of square returns indicate significant correlation and persistence of second order moments. In **Annexes 9 and 10** the correlogram of these series are presented. It can be seen that the autocorrelation has increased for all series. In addition, all autocorrelation coefficients are statistically significant for the first 15 lags and the test null hypothesis is rejected at a significance level of 1% for all series analyzed, confirming the existence of serial correlation, so of heteroscedasticity.

There has been increasing evidence of time varying volatility and deviations from normality in financial time series, and therefore, it is important to conduct a test that is robust to heteroskedasticity. We perform the variance ratio test of random walk.

An important property of all the random walk hypothesis is that the variance of the residual variable has to be a linear function of the time.

Considering RW1 r_t = μ + ε_t, as returns are independent and follow the same distribution, we have that Var[r_t+r_{t-1}] = 2Var[r_t]. Therefore, we can determine whether the random walk hypothesis is plausible by checking variance ratio: VR(2)=$\frac{\text{Var}[r_t+r_{t-1}]}{2\text{Var}[r_t]}$. If RW1 hypothesis is true, then this report should be significantly equal to one.

As can be seen from the table below, the variance ratio for each of the selected times to carry out the test, is less than 1. Also the overall test probability is below the confidence level of 1%. Therefore, we reject the null hypothesis that the series would follow a random walk model.

Based on VR (Variance Ratio) test the null hypothesis of random walk is strongly rejected for all return series considered, this means that daily returns can follow some predictable patterns.

Table 3. The VR test results

	BET-C	BUX	DAX	FTSE 100	FTSE MIB	NIKKEI 225	SOFIX	S&P 500
Value	10.37315	13.78791	14.87617	13.90972	15.58898	16.01136	10.54322	12.46981
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
VR at period 2	0.537740	0.547839	0.494616	0.488516	0.500461	0.473389	0.492723	0.451375
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
VR at period 4	0.281595	0.23767	0.239734	0.219040	0.236411	0.236127	0.256709	0.224858
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
VR at period 8	0.132905	0.127081	0.120224	0.115065	0.120067	0.121883	0.130626	0.106031
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
VR at period 16	0.067472	0.063326	0.060632	0.058527	0.058225	0.060024	0.063665	0.054117
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)

This rejection of the null hypothesis of IID return observations indicates toward either an underlying chaotic process or a nonlinear stochastic process. In the next stage, we turn to discuss the results from the BDS test to determine the nature of dependence present in the data.

4.2.1. The BDS test results

We conduct the BDS test for the embedding dimensions from 2 to 5. Further, it is required to select a value for ε to conduct the BDS test. As pointed by Scheinkman and LeBaron (1989), the null hypothesis of IID will be accepted frequently irrespective of it being true or false, if the selected value of ε is too small. Therefore, it is recommended to conduct the test for a range of ε values. Following Brock et al. (1992), we conduct the test for a range of values of ε as 0.5, 1.0, 1.5 and 2.0 standard deviations of the data. A lower ε value represents a more strict criteria because points in the ε-dimensional space must be clustered closer together to meet the criteria of being 'close' in terms of the BDS statistic. Therefore, ε=0.5σ indicates the most stringent test and ε=2.0σ is the most relaxed criteria used for analysis purpose. The results are reported in table below.

Table 4. The BDS test results BDS

ε	m	BET-C	BUX	DAX	FTSE 100	FTSE MIB	NIKKEI 225	SOFIX	S&P 500
0.5	2	21.23900	9.40763	10.36483	12.43114	12.06333	5.39203	21.36927	11.22198
	3	27.04612	11.47585	15.99321	17.70461	18.45617	8.301017	27.73256	16.76949
	4	32.86570	13.40742	21.53942	22.75439	25.56062	11.16435	33.75821	21.49585
	5	39.86068	15.55492	27.02964	28.27511	33.90317	14.13351	40.81501	26.47854
1.0	2	21.68286	10.53119	11.42678	13.85043	11.72176	5.47092	23.34487	12.40405
	3	25.73022	12.94014	16.81059	18.86117	17.47996	8.572856	27.13938	17.86854
	4	28.80250	15.07931	21.21927	23.04570	22.49917	11.36092	29.82042	21.81638
	5	31.88194	17.19407	24.80167	27.13201	27.36474	13.98566	32.53488	25.97915
1.5	2	20.56651	11.63991	13.13240	14.82958	10.93500	7.34784	23.10543	13.83909
	3	23.35373	13.90931	18.73351	19.24399	15.78295	10.67708	26.33341	18.78193
	4	25.52706	16.14979	22.47758	22.42287	19.44707	13.15572	27.66137	22.15508
	5	27.01387	18.05562	25.14774	25.07492	22.59355	15.13433	28.75184	25.07561
2.0	2	19.82528	11.93198	12.92377	14.92024	10.42079	9.97416	21.16839	15.33010
	3	21.90607	14.03261	18.31170	19.27442	14.68862	13.43247	24.13756	19.67866
	4	23.42068	16.41710	21.66436	22.02076	17.75775	15.68964	25.13892	22.36626
	5	24.18678	18.06259	24.08048	24.10471	20.19292	17.30214	25.64310	24.23783

It is clearly observable that the BDS test conducted on raw returns strongly reject the null of IID in every case. It is noteworthy that the rejection of the null by the BDS statistic for raw returns does not necessarily indicate that the time series exhibits a low complexity chaotic behaviour. Rather, the rejection of IID null can be consistent with any types of non-IID behaviour such as linear dependence, nonlinear stochastic process (ARCH-type models), and chaos (nonlinear deterministic process).

In the first stage of analysis, we remove the linear dependence in the data by fitting a best linear model and then conduct the BDS test on linearly filtered residuals to test whether filtered residuals are IID or not. For this purpose, we estimated through the "least squares method" several ARMA(m,n) models.

Before estimating this models is necessary to establish the specifications of mean equation. Based on the autocorrelation coefficients (autocorrelation function) and partial correlation coefficients (partial autocorrelation function) I have determined the autoregressive starting models. To choose the right model, namely for the choice of orders m and n, I used the information criterias Log likelihood, Akaike (Akaike Information Criterion, AIC) and Schwarz (Schwartz Bayesian Criterion, SBC). These indicators are used when you have to choose an equation from severals. According to the information criterion is selected the specification for which the log likelihood is maximum, and AIC and SBC have the lowest values.

ARMA model estimation results are presented in **Appendix 12**. Considering all the above mentioned criteria, I considered that best fit models for the daily returns series are: AR(1) – BET-C, ARMA(2,2) – BUX, ARMA(3,5) – DAX, AR(4)MA(1)MA(3)MA(5) – FTSE 100, ARMA(3,5) – FTSE MIB, ARMA(3,1) –NIKKEI 225, AR(1)AR(2)MA(2)MA(5) – SOFIX, ARMA(1,8) – S&P 500.

The results of the best fit ARMA models for each series analyzed were summarized in the table below.

Table 5. Estimated parameters of ARMA models

	BET-C		BUX		DAX		FTSE 100			
Variable	**C**	**AR(1)**	**AR(2)**	**MA(2)**	**AR(3)**	**MA(5)**	**AR(4)**	**MA(1)**	**MA(3)**	**MA(5)**
Coefficient	0.000526	0.093221	-0.821603	0.759593	-0.041036	-0.052012	0.077791	-0.055223	-0.094939	-0.05658
Std. Error	0.000264	0.017522	0.058591	0.066906	0.017585	0.017585	0.017665	0.017566	0.017521	0.017488
t-Statistic	1.99493	5.3201	-14.02265	11.35309	-2.333569	-2.95775	4.40369	-3.143675	-5.418468	-3.235391
Prob.	0.0461	0.0000	0.0000	0.0000	0.0197	0.0031	0.0000	0.0017	0.0000	0.0012
R-squared	0.008692		0.011914		0.004146		0.019691			
Adjusted R-squared	0.008385		0.011608		0.003837		0.018778			
S.E. of regression	0.01498		0.015825		0.015339		0.012182			
Sum squared resid	0.724383		0.80813		0.759039		0.47833			
Log likelihood	8987.145		8807.231		8905.153		9646.933			
Mean dependent var	0.000581		0.000296		0.000192		0.0000811			
S.D. dependent var	0.015043		0.015918		0.015369		0.012298			
Akaike info criterion	-5.563557		-5.453844		-5.516204		-5.976407			
Schwarz criterion	-5.559792		-5.450078		-5.512437		-5.968872			
Durbin-Watson stat	2.003313		1.92891		2.052431		1.993533			

	FTSE MIB		NIKKEI 225		S&P 500		SOFIX			
Variable	**AR(3)**	**MA(5)**	**AR(3)**	**MA(1)**	**AR(1)**	**MA(8)**	**AR(1)**	**AR(2)**	**MA(2)**	**MA(5)**
Coefficient	-0.04834	-0.07273	-0.050663	-0.04398	-0.123455	0.040574	0.08823	0.87808	-0.805026	-0.10685
Std. Error	0.017584	0.01757	0.017577	0.017595	0.01747	0.017596	0.012913	0.019701	0.025074	0.015035
t-Statistic	-2.74943	-4.13941	-2.88234	-2.49986	-7.066685	2.305947	6.832638	44.56972	-32.10617	-7.10691
Prob.	0.006	0.0000	0.004	0.0125	0.0000	0.0212	0.0000	0.0000	0.0000	0.0000
R-squared	0.007264		0.004585		0.017298		0.045062			
Adjusted R-squared	0.006956		0.004277		0.016994		0.044174			
S.E. of regression	0.015139		0.015158		0.012664		0.013175			
Sum squared resid	0.739396		0.741179		0.517724		0.559829			
Log likelihood	8947.47		8943.583		9529.589		9399.9			
Mean dependent var	-0.000143		0.0000837		0.00015		0.000514			
S.D. dependent var	0.015192		0.01519		0.012773		0.013476			
Akaike info criterion	-5.542423		-5.540014		-5.899436		-5.819696			
Schwarz criterion	-5.538656		-5.536248		-5.895671		-5.812165			
Durbin-Watson stat	2.018552		1.99949		2.007995		1.994088			

Since the probabilities attached to t-statistic test are below the 5% level of relevance for both autoregressive processes AR and moving average MA, the coefficients are considered significant in statistical terms. Instead, with the exception of the AR (1) for BET-C the constant is not significantly different from 0. Which is why I reestimated these models without the constant this time.

If the model is well specified, then the residuals from the estimated model are generated by a white noise process type (sequence of independent random variables, identically distributed) with zero mean and normally distributed. To detect some dependencies in the residue series ACF and PACF functions are examined.

In **Appendix 14** it can be observed that up to lag 15 autocorrelation and partial correlation coefficients are not significantly different from 0, which leads to the conclusion that the residues are not correlated.

The correlogram of squared errors tests the autocorrelation of squared residues of the regression equation by the same principles as the autocorrelation of the errors. If there is autocorrelation of squared errors, this is an indication of the existence of heteroscedasticity. According to the econometric results, for the estimated equations above, in **Appendix 15** it can be observed that there is serial correlation of squared errors, so we may have ARCH terms.

To verify the hypothesis of heteroscedasticity of errors I used the ARCH LM and the White test.

ARCH-LM tests for ARCH effects. The test has the null hypothesis of no ARCH terms. Since the probabilities attached to F-statistic are in all eight cases below the level of significance of 5%, the null hypothesis is rejected and we accept the presence of these effects.

Table 6. ARCH – LM test results

		BET-C	BUX	DAX	FTSE 100	FTSE MIB	NIKKEI 225	SOFIX	S&P 500
F-statistic		409.5185	425.4928	114.9481	172.0325	106.49	171.4602	452.5793	127.3903
		(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Obs*R-squared		363.6267	376.145	111.0609	163.4191	103.15	162.9055	397.1441	122.6283
		(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)

White test relates to the equal spreading of the error in relation to all factors, so that calls to a regression analysis of the error in relation to the factors. This test has in the null hypothesis that each coefficient of the regression is significantly different from 0. Since the probability associated with the test is below the level of significance chosen of 5%, the null hypothesis is rejected. Thus, we reject the existence of a constant residual variances, so of the homoscedasticity.

Table 7. White test results

	BET-C	BUX	DAX	FTSE 100	FTSE MIB	NIKKEI 225	SOFIX	S&P 500
F-statistic	221.5869	93.35216	161.9008	97.07056	1413.315	86.84533	107.6564	88.35302
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Obs*R-squared	390.0228	257.9994	422.6342	748.1937	748.1937	241.355	809.4476	245.2381
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Scaled explained SS	2324.938	1017.242	1429.228	3188.237	3188.237	981.9791	3729.383	1343.214
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)

The lack of serial correlation shown by the correlogram of errors is confirmed by the test Serial Correlation LM test. The null hypothesis of the test is that there is no serial correlation of the errors of the regression equation. The probability associated with the test is greater than 0.05 (except BUX and FTSE 100), it is higher than the level of relevance. The null hypothesis is accepted, so we accept the absence of serial correlation.

Table 8. BG test results

	BET-C	BUX	DAX	FTSE 100	FTSE MIB	NIKKEI 225	SOFIX	S&P 500
F-statistic	1.089652	4.06943	2.219754	4.986172	0.287987	0.218754	0.056391	3.454432
	(0.2966)	(0.0437)	(0.1364)	(0.0256)	(0.5916)	(0.6400)	(0.8123)	(0.0632)
Obs*R-squared	1.090297	2.833148	1.611008	4.79288	0.00000	0.099708	0.00000	2.923412
	(0.2964)	(0.0923)	(0.2044)	(0.0286)	(1.00000)	(0.7522)	(1.00000)	(0.0873)

To test the normality of errors it is used Jarque-Bera test. The normal distribution of errors is especially important when we want to make predictions based on the econometric equation estimated. Since the probability associated Jarque-Bera test is 0%, we can say that the errors of ARMA models are not normally distributed.

Table 9. Descriptive statistics for ARMA models

	BET-C	BUX	DAX	FTSE 100	FTSE MIB	NIKKEI 225	SOFIX	S&P 500
Medie	-7.36E-19	0.00031	0.000211	0.0000943	-0.000161	0.0000921	0.000209	0.000162
Mediană	0.0000456	0.000464	0.000769	0.000475	0.000633	0.000403	0.000213	0.000842
Maxim	0.10125	0.119075	0.1058	0.085409	0.106367	0.09478	0.08635	0.108108
Minim	-0.12488	-0.118569	-0.075804	-0.086543	-0.089539	-0.118087	-0.090797	-0.094631
Deviaţia standard	0.014978	0.015819	0.015335	0.012176	0.015136	0.015155	0.013168	0.012661
Skewness	-0.574155	-0.115486	-0.021898	-0.318741	-0.139341	-0.693595	-0.050305	-0.290047
Kurtotica	12.93684	8.908951	7.774825	9.554318	7.658367	9.164627	10.24436	11.98577
Jarque-Bera	13466.33	4704.79	3066.717	5830.849	2929.147	5370.171	7062.21	10912.1
Probabilitate	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

We now conduct the BDS test on ARMA residuals. The results are reported in Table 10. Again it is clearly observable that the the null of IID is strongly reject in every case. The rejection of null hypothesis at this stage suggests that some kind of dependence is still left in the data. Since linear structures have already been removed using the best fit autoregressive–moving-average model, the rejection of null hypothesis is indicative of some nonlinear dependencies in the returns series.

Table 10. BDS test results

ε	m	BET-C	BUX	DAX	FTSE 100	FTSE MIB	NIKKEI 225	SOFIX	S&P 500
0.5	**2**	20.85404	9.05669	10.72699	11.98833	11.62502	4.61095	20.48468	9.84113
	3	26.73114	11.21028	16.43940	17.82753	18.27849	7.55941	27.32732	15.59580
	4	32.44339	12.96338	22.12497	23.45954	25.71516	10.62564	33.83649	20.31128
	5	39.48533	14.87797	27.80116	29.76924	34.38904	14.46141	41.82862	25.05381
1.0	**2**	21.47715	10.06301	11.47863	13.42828	11.48518	4.76526	22.78616	11.28315
	3	25.78233	12.30262	16.97194	18.67017	17.27493	7.89080	27.29603	16.99293
	4	28.89316	14.33610	21.43380	23.18735	22.40226	10.68752	30.39679	21.01966
	5	32.08189	16.29264	25.07085	27.62683	27.29821	13.32575	33.44802	25.13252
1.5	**2**	20.85124	11.21513	12.99526	14.07523	10.99497	6.67766	23.04555	13.31352
	3	24.05517	13.07780	18.68524	18.78007	15.78002	9.84248	26.64263	18.31499
	4	26.23713	15.19643	22.43234	22.24422	19.50648	12.32883	28.19900	21.70267
	5	27.82834	16.98912	25.12748	25.13560	22.66971	14.32270	29.57038	24.69741
2.0	**2**	20.07422	11.78482	12.94515	14.34879	10.52895	9.32508	22.26887	14.63865
	3	22.67976	13.40236	18.30898	19.00731	14.69481	12.62530	25.24498	19.07058
	4	24.20914	15.67073	21.67818	22.01718	17.77636	14.88138	26.16900	21.78561
	5	25.01601	17.17728	24.11509	24.21664	20.24158	16.50055	26.72446	23.74991

After the confirmation that some type of nonlinearity is present in the data, we next move to investigate the nature of this nonlinearity, i.e. stochastic or deterministic, by using the BDS test of independence. We conduct the BDS test on the data after removing the nonlinear dependence caused by heteroskedasticity. We use different ARCH type models for daily returns series.

In this step we will try to trace the equation that best describes the volatility of daily returns series. Before estimating a GARCH model we have selected the best ARMA models for the return series analyzed and shown that they have significant statistic coefficients. According to the Jarque-Bera test the error distribution is not normal. White test for heteroscedasticity confirmed the presence of ARCH effects. And with ACF and PACF functions we analyzed the autocorrelation and we concluded that residues are not correlated, but instead we have significant serial correlation of squared errors. Therefore a GARCH type model may be considered an appropriate change to the initial model.

For the choice of orders p and q, and the type of ARCH model (GARCH / TGARCH, GARCH-M, EGARCH, PARCH) were made successive attempts to find the desired equation and were analyzed all possible combinations seeking to maximize criterion Log likelihood, and AIC and SBC criteria minimization.

I also compared the results obtained for the three possible distributions: normal distribution, Student-t and GED ("Generalized Error Distribution").

After this comparison between GARCH models for volatility modeling, we decided to fit the data with the following models: BET-C – GARCH(1,1), BUX – EGARCH(1,1,1), DAX – GARCH(1,1), FTSE 100 – GARCH(2,1), FTSE MIB – GARCH(1,1), NIKKEI – GARCH(1,1), SOFIX – EGARCH(1,1,1), S&P 500– EGARCH(2,1,1).

The results of the best fit GARCH models for each series analyzed were summarized in table 11 below.

Square error and conditional variance coefficients of the variance equation are statistically significant (significance level of 1% and 5%).

Table 11. Estimated parameters of GARCH models

BET-C		BUX		DAX		FTSE 100	
AR(1) - GARCH(1,1) S		**ARMA(2,2) - EGARCH(1,1,1) G**		**ARMA(3,5) - GARCH(1,1) N**		**AR(4)MA(1)MA(3)MA(5)-GARCH(2,1) N**	
Variable	**Coefficient**	**Variable**	**Coefficient**	**Variable**	**Coefficient**	**Variable**	**Coefficient**
C	0.000672	AR(2)	-0.718548	AR(3)	-0.030194	AR(4)	0.019962
	(0.0000)		(0.0000)		(0.0916)		(0.2791)
AR(1)	0.08477	MA(2)	0.699326	MA(5)	-0.034794	MA(1)	-0.062726
	(0.0000)		(0.0000)		(0.0549)		(0.0004)
-	-	-	-	-	-	MA(3)	-0.036008
							(0.0492)
-	-	-	-	-	-	MA(5)	-0.015422
							(0.3882)
Variance Equation		**Variance Equation**		**Variance Equation**		**Variance Equation**	
Variable	**Coefficient**	**Variable**	**Coefficient**	**Variable**	**Coefficient**	**Variable**	**Coefficient**
C	5.11E-06	C	-0.306997	C	2.02E-06	C	1.50E-06
	(0.0000)		(0.0000)		(0.0000)		(0.0000)
ARCH(1)	0.197431	ARCH(1)	0.163172	ARCH(1)	0.080019	ARCH(1)	0.059448
	(0.0000)		(0.0000)		(0.0000)		(0.0000)
GARCH(1)	0.801247	Asymmetric coefficient	-0.046085	GARCH(1)	0.910248	ARCH(2)	0.049735
	(0.0000)		(0.0000)		(0.0000)		(0.0040)
-		GARCH(1)	0.978582	-		GARCH(1)	0.880604
			(0.0000)				(0.0000)
R-squared	0.008533	**R-squared**	0.005701	**R-squared**	0.003741	**R-squared**	0.01192
Adjusted R-squared	0.008225	**Adjusted R-squared**	0.005393	**Adjusted R-squared**	0.003433	**Adjusted R-squared**	0.011001
S.E. of regression	0.014981	**S.E. of regression**	0.015875	**S.E. of regression**	0.015342	**S.E. of regression**	0.012231
Sum squared resid	0.724499	**Sum squared resid**	0.813212	**Sum squared resid**	0.759347	**Sum squared resid**	0.482121
Log likelihood	9880.67	**Log likelihood**	9269.64	**Log likelihood**	9569.992	**Log likelihood**	10398.46
Mean dependent var	0.000581	**Mean dependent var**	0.000296	**Mean dependent var**	0.000192	**Mean dependent var**	8.11E-05
S.D. dependent var	0.015043	**S.D. dependent var**	0.015918	**S.D. dependent var**	0.015369	**S.D. dependent var**	0.012298
Akaike info criterion	-6.114347	**Akaike info criterion**	-5.737157	**Akaike info criterion**	-5.926265	**Akaike info criterion**	-6.439699
Schwarz criterion	-6.103052	**Schwarz criterion**	-5.723976	**Schwarz criterion**	-5.916848	**Schwarz criterion**	-6.424628
Durbin-Watson stat	1.985939	**Durbin-Watson stat**	1.931968	**Durbin-Watson stat**	2.052525	**Durbin-Watson stat**	1.98296

FTSE MIB		NIKKEI 225		S&P 500		SOFIX	
ARMA(3,5)-GARCH(1,1) N		ARMA(3,1)-GARCH(1,1) N		ARMA(1,8)-EGARCH(2,1,1) G		AR(1)AR(2)MA(2)MA(5)-EGARCH(1,1,1) G	
Variable	**Coefficient**	**Variable**	**Coefficient**	**Variable**	**Coefficient**	**Variable**	**Coefficient**
C	-0.026186	AR(3)	0.00917	AR(1)	-0.076468	AR(1)	0.060901
	(0.1430)		(0.5906)		(0.0000)		(0.0000)
AR(1)	-0.029998	MA(1)	-0.035908	MA(8)	0.009361	AR(2)	0.840472
	(0.0886)		(0.0781)		(0.5731)		(0.0000)
-	-	-	-	-	-	MA(2)	-0.810569
							(0.0000)
-	-	-	-	-	-	MA(5)	-0.024328
							(0.0417)
Variance Equation		**Variance Equation**		**Variance Equation**		**Variance Equation**	
Variable	**Coefficient**	**Variable**	**Coefficient**	**Variable**	**Coefficient**	**Variable**	**Coefficient**
C	9.82E-07	C	3.41E-06	C	-2.94E-01	C	-0.943369
	(0.0000)		(0.0000)		(0.0000)		(0.0000)
ARCH(1)	0.074271	ARCH(1)	0.09109	ARCH(1)	-0.125145	ARCH(1)	0.464986
	(0.0000)		(0.0000)		(0.0052)		(0.0000)
GARCH(1)	0.923379	GARCH(1)	0.895046	ARCH(2)	0.252789	Asymmetric coefficient	-0.044994
	(0.0000)		(0.0000)		(0.0000)		(0.0308)
-		-		Asymmetric coefficient	-0.139159	GARCH(1)	0.933315
					(0.0000)		(0.0000)
-		-		GARCH(1)	0.978603	-	
					(0.0000)		
R-squared	0.005046	**R-squared**	0.000923	**R-squared**	0.013929	**R-squared**	0.029677
Adjusted R-squared	0.004737	**Adjusted R-squared**	0.000613	**Adjusted R-squared**	0.013623	**Adjusted R-squared**	0.028774
S.E. of regression	0.015156	**S.E. of regression**	0.015185	**S.E. of regression**	0.012686	**S.E. of regression**	0.013281
Sum squared resid	0.741048	**Sum squared resid**	0.743907	**Sum squared resid**	0.519499	**Sum squared resid**	0.568849
Log likelihood	9598.664	**Log likelihood**	9361.708	**Log likelihood**	10497.75	**Log likelihood**	10378.38
Mean dependent var	-0.000143	**Mean dependent var**	8.37E-05	**Mean dependent var**	0.00015	**Mean dependent var**	0.000514
S.D. dependent var	0.015192	**S.D. dependent var**	0.01519	**S.D. dependent var**	0.012773	**S.D. dependent var**	0.013476
Akaike info criterion	-5.94403	**Akaike info criterion**	-5.797217	**Akaike info criterion**	-6.495199	**Akaike info criterion**	-6.422657
Schwarz criterion	-5.934613	**Schwarz criterion**	-5.7878	**Schwarz criterion**	-6.48014	**Schwarz criterion**	-6.405711
Durbin-Watson stat	2.023722	**Durbin-Watson stat**	2.01805	**Durbin-Watson stat**	2.102132	**Durbin-Watson stat**	1.95012

In EGARCH models for BUX, S&P 500 and SOFIX the asymmetric coefficient seems to be statistically significant because the probabilities attached are less than 5%. To confirm this, we applied the Wald test. This test has the null hypothesis that these coefficients are not significantly different from zero.

Table 12. The Wald test results

	t-statistic	F-statistic	Chi-square
BUX	**-4.73663 (0.0000)**	**22.43568 (0.0000)**	**22.43568 (0.0000)**
S&P 500	**-12.0982 (0.0000)**	**146.3661 (0.0000)**	**146.3661 (0.0000)**
SOFIX	**-2.16009 (0.0308)**	**4.665999 (0.0308)**	**4.665999 (0.0308)**

Since in all three cases the probabilities attached are less than the critical value of 0.05, the null hypothesis of the test is rejected, which means that in these cases the impact of the information will be asymmetrical.

As expected, when estimating a GARCH model for financial data series, the sum of coefficients is very close to 1. The constant term is very small, and the conditional variance coefficient is greater than 0.8 in all cases, this means that shocks in the conditional variance are persistent and significant changes in the conditional variance are followed by other large changes, and small changes by other small changes.

Having established suitable models for the return series considered, now they must be assessed by a number of statistical tests and graphs. If the models are correctly specified, then the standardized residues must not longer poses serial correlation, heteroscedasticity or any other type of non-linear dependence.

Therefore, for the beginning we estimate the ACF and PACF functions of the squared standardized residuals of the models and use Q-statistics (Ljung-Box) test to investigate the existence of serial correlation up to lag 15.

According to correlogram of squared errors (shown in **Appendix 22**), there is no additional ARCH terms. Since the coefficients of autocorrelation and partial correlation functions are very close to the value 0 it can be stated that residues are not correlated.

Also, the probability of more than 5% of the Q-statistic test, corresponding to the null hypothesis that there is no autocorrelation in residues up to lag 15, confirms the absence of autocorrelation for the squared errors.

Further, the existence of other possible ARCH effects remaining in the residue is tested using ARCH-LM test. If the variance equation proposed by our model is correctly specified, then we should not have more ARCH effects.

LM test investigates the null hypothesis of absence of ARCH effects and it is necessary to have an estimated value that is not statistically significant to not have the power to reject H0.

Table 13. The ARCH – LM test results

	BET-C	BUX	DAX	FTSE 100	FTSE MIB	NIKKEI 225	SOFIX	S&P 500
F-statistic	3.241016	0.589139	2.972783	0.497674	2.118539	1.026956	0.062340	0.080758
	(0.0719)	(0.4428)	(0.0848)	(0.4806)	(0.1456)	(0.3110)	(0.8029)	(0.7763)
Obs*R-squared	3.239771	0.589397	2.971887	0.497906	2.118461	1.027265	0.062378	0.080806
	(0.0719)	(0.4428)	(0.0847)	(0.4804)	(0.1455)	(0.3108)	(0.8028)	(0.7762)

ARCH LM test shows that we can accept the null hypothesis of absence of ARCH effects as attached test probabilities are greater than 0.05, the results not having statistical significance.

Jarque-Bera test also confirms that the residues are still not normally distributed.(Prob = 0%)

Table 14. Descriptive statistics for GARCH models

	BET-C	BUX	DAX	FTSE 100	FTSE MIB	NIKKEI 225	SOFIX	S&P 500
Medie	0.000933	0.029533	0.024918	0.011698	-0.006387	0.006999	0.038238	0.018818
Mediană	-0.011723	0.032606	0.063245	0.037652	0.044786	0.023157	0.027858	0.091280
Maxim	7.949975	4.112991	3.395123	4.434716	4.680145	3.267584	6.566573	3.625933
Minim	-6.844023	-5.369757	-6.817613	-4.407328	-5.169177	-5.404181	-5.900159	-6.482276
Deviaţia standard	0.992793	1.000605	1.000360	1.000246	1.000198	1.000147	1.006995	1.000690
Skewness	0.128544	-0.064769	-0.353129	-0.309079	-0.378137	-0.383344	0.192677	-0.450817
Kurtotica	7.142378	4.010356	4.251021	3.708267	4.230684	4.024651	6.642557	4.210318
Jarque-Bera	2318.25	139.6003	277.5886	118.8290	280.6386	220.2734	1805.108	306.5558
Probabilitate	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

We now apply the BDS test of independence to standardized residuals of GARCH and EGARCH models. In doing so, we use the BDS test to test the null hypothesis of independent random variables against the alternative hypothesis of non-independent random variables. The results are presented in Table 15.

The results suggest that the ARCH-type models have removed considerable serial dependence from the raw and filtered data and also the values of BDS statistic are noticeably reduced at all dimensions. It is also observable that the null hypothesis is not rejected in six cases, i.e. BUX, DAX, FTSE 100, FTSE MIB, NIKKEI 225 and S&P 500.

The non-rejection of the null hypothesis at this stage of the analysis indicates that conditional heteroskedasticity is the main cause for the initial rejection of the null and the nature of dependence in the data is best described as a nonlinear stochastic system. It appears, therefore, that the behaviour of these return series is adequately explained by ARCH-type models. However, the null hypothesis is again rejected for BET-C and SOFIX. In these cases, the significant BDS statistics for the standardized residuals suggest that the returns series are non-IID and the ARCH type models are not sufficient to capture all the information present in the data. The rejection of the null hypothesis, at this stage, is consistent with deterministic chaos as there remains some further dependence in the data that cannot be explained with reference to GARCH and EGARCH models.

Table 15. The BDS test results

ε	m	BET-C	BUX	DAX	FTSE 100	FTSE MIB	NIKKEI 225	SOFIX	S&P 500
0.5	2	4.825549	-0.081942*	-1.653390*	-0.881034*	-1.682837*	-2.423919	3.913801	-1.033641*
	3	5.305013	-0.377128*	-0.024019*	0.338339*	-1.001259*	-1.717043*	4.460895	-0.425269*
	4	5.492119	-0.378757*	1.452941*	1.055317*	0.023915*	-0.641525*	4.013648	-0.352227*
	5	5.172980	0.000480*	2.482784	1.668677*	0.570817*	0.234327*	3.452289	-0.266141*
1.0	2	3.985862	-0.694551*	-2.645580	-0.650178*	-1.990842	-3.992148	3.175269	-0.975392*
	3	3.690628	-1.081481*	-1.296131*	0.146274*	-1.241394*	-3.532882	3.509793	-0.858417*
	4	3.367446	-1.057247*	-0.017167*	0.718921*	-0.184242*	-2.591520	3.280873	-0.852200*
	5	2.705422	-0.813497*	0.790931*	1.165214*	0.442030*	-1.823461*	2.660342	-0.773877*
1.5	2	3.584677	-0.542837*	-2.968780	-0.295083*	-2.372692	-4.056215	2.300337	-0.577788*
	3	3.188778	-0.86931*	-1.969512	0.099768*	-1.953353*	-3.464264	2.610313	-0.547093*
	4	2.845470	-0.708258*	-0.883220*	0.393346*	-0.956526*	-2.786503	2.561622	-0.670607*
	5	2.221953	-0.459095*	-0.267057*	0.708122*	-0.372994*	-2.218547	2.157721	-0.676532*
2.0	2	3.212322	-0.056239*	-2.641131	0.531851*	-2.178616	-3.290453	1.364244*	-0.275873*
	3	2.806847	-0.180037*	-1.630612*	0.539393*	-1.953942*	-2.700124	1.510578*	-0.282918*
	4	2.498565	0.144142*	-0.681718*	0.588599*	-1.092264*	-2.147277	1.644802*	-0.558696*
	5	1.958490*	0.394994*	-0.193644*	0.794627*	-0.529896*	-1.749933*	1.394414*	-0.662614*

(*) Indicates BDS statistics that are not significant at 5% critical level

4.2.2. The Rescaled Range analysis results

Now in order to ascertain whether the data series of returns are, indeed, a result of chaotic process, we further conduct a highly popular test, namely R/S analysis.

The R/S analysis is a more powerful indicator of the persistence of a time series where the influence of a set of past observations on a set of future observation is effectively captured. As a matter of fact, presence of some dependence between observations widely separated in time (i.e. long-memory) suggests that realizations from the remote past can help predict future returns. Therefore, it is possible to make consistent speculative profits. A precise summary of the estimated Hurst exponent for the raw residuals are presented in Table 16.

It is noteworthy that when an estimated value of H is different from 0.5, the observations are no longer independent and they carry some memory of all preceding events which can be described as ‘long-term memory’ process. Although marginally, but the value of H for all return series are significantly different from 0.5. This indicates for the presence of marginal persistence and temporal dependence in the data, and therefore, provide further confirmation to the rejection of random walk hypothesis.

Table 16. Hurst exponent

BET-C	BUX	DAX	FTSE 100	FTSE MIB	NIKKEI 225	SOFIX	S&P 500
0.7201	0.6080	0.6141	0.5240	0.6161	0.5332	0.7626	0.5907

In summary, the R/S analysis reveals the presence of a weak nonlinear temporal dependency (persistence) for developed markets such as FTSE 100 and NIKKEI and stronger persistence for emerging markets, i.e. SOFIX, BET-C.

An important implication of this is that if asset returns do not follow random walk, the process of annualizing risk by square root of time will lead to either overestimation or underestimation of the actual level of risk associated with an investment. Moreover, while considering the capital asset pricing model and Black–Scholes models, the misestimation of risk will result in highly incorrect valuations.

4.2.3. The Rcurrence Analysis results

The eight series data of indices were analysed with Visual Recurrence Analysis. The data were implemented and analysed without prewhitening.

The Recurrence Plots are shown below. There were built using a delay-time equal to 1 and embedding dimension determined by the method of false nearest neighbours.

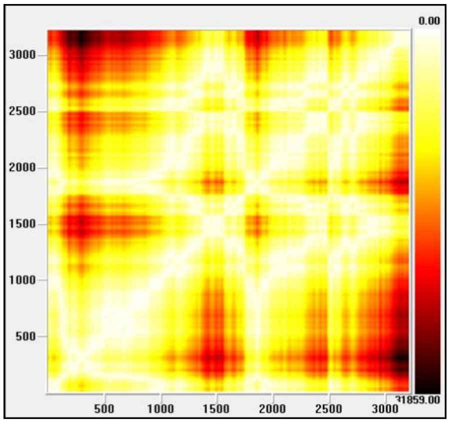

Figure 5. RP – DAX

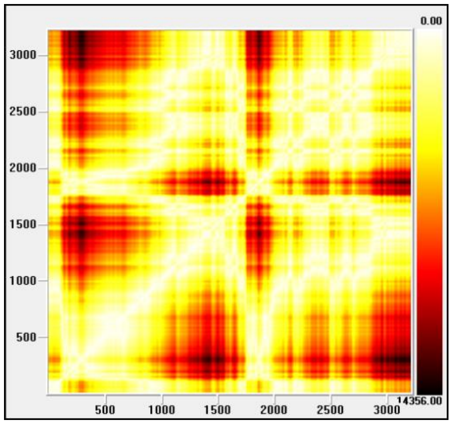

Figure 6. RP – FTSE 100

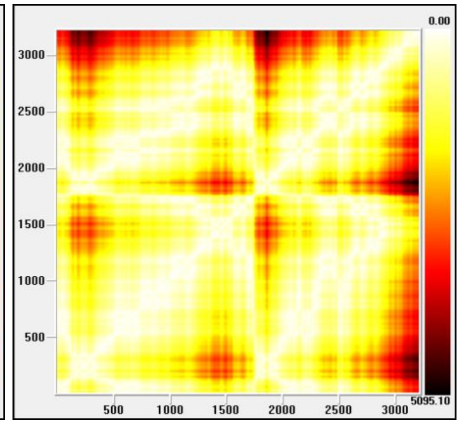

Figure 7. RP – S&P 500

Figures 5, 6 and 7 represent distance plots for the three largest Western financial markets: Germany, the United Kingdom and the United States. These plots exhibit many common features, possibly reflecting the high level of integration of these markets. Light shaded regions are always found in the vicinity of the main diagonal line. The light shading fades as the distance to the LoI increases, reflecting the non-stationarity of the series. However, interesting light shaded structures can be found far from the LoI. A ''butterfly'' shaped structure can be observed in the three plots.

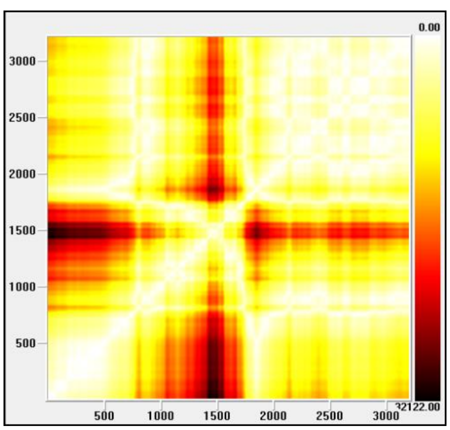

Figure 8. RP – BET-C

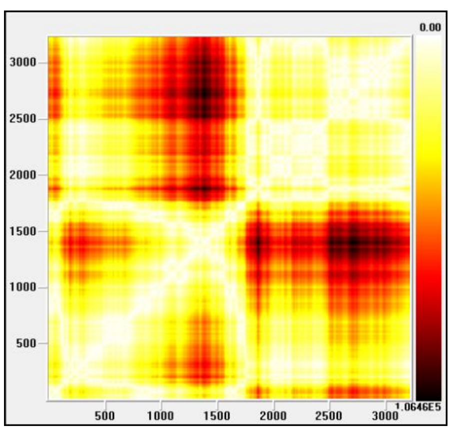

Figure 9. RP – FTSE MIB

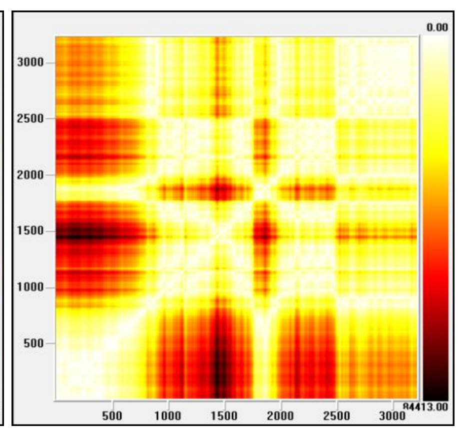

Figure 10. RP – BUX

Figures 8, 9 and 10 presents distance plots for: Romania, Italy and Hungary. These neighboring economies also share many common features. However, the patterns are structurally different from those exhibited by the western markets. These examples suggest that stock markets in countries with strong economic interdependence tend to display similar features in recurrence plots. The plots for BET-C, FTSE MIB and BUX also more structured than those in Figures 5,6,7. Instead of a ‘‘butterfly’’ shaped structure, these plots display an ‘‘arrow’’ shaped structure.

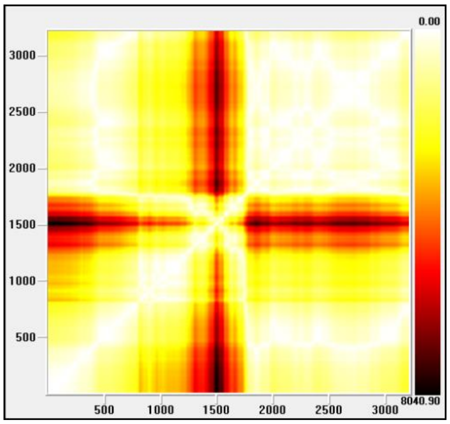

Figure 11. RP – SOFIX

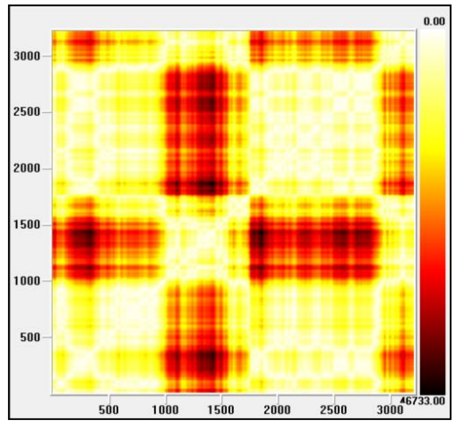

Figure 12. RP – NIKKEI 225

In Figures 11 and 12 are presented the recurrence plots for Bulgaria and Japan. From these RPs we can see clearly the difference between an emerging and a developed country in terms of recurrence point. The plot for SOFIX is definitely more structured than the one for NIKKEI indicating the deterministic nature of this series in contrast with the other one.

While the visual inspection of recurrence or distance plots provides interesting insights, their interpretation if often difficult and subjective. Recurrence quantification analysis introduces numerical measures that allow for the quantification of the structure and complexity of RPs.

The RQA results are displayed in table 17. Since the system is unknown, optimal time delay was estimated as the one where average mutual information reaches its first minimum. For a rough selection of the embedding dimension for our one-dimensional time series, the false nearest neighbour method was used.

REC: It is positive meaning that the data are correlated. The highest values for percent recurrence are obtained for SOFIX, FTSE MIB, BUX and BET-C and the lowest for more developed conutries, i.e. the western markets S&P 500, FTSE 100, DAX and NIKKEI 225.

DET: It is also positive (except for NIKKEI 225 and S&P 500) indicating that recurrent points are consecutive in time, that is, form segments parallel to the main diagonal. DET values indicate deterministic nature of the embedded series.

Table 17. RQA results

	BET-C	BUX	DAX	FTSE 100	FTSE MIB	NIKKEI 225	SOFIX	S&P 500
Mean	3108.297	17530.949	5246.146	5171.527	26219.318	11759.728	660.458	1157.442
Standard deviation	1788.443	6000.125	1400.887	797.902	8275.176	2860.041	432.708	200.121
Mean rescaled dist	53.567	36.323	43.390	49.421	35.049	52.293	52.699	55.588
Percent recurrence	0.090	0.159	0.036	0.007	0.195	0.009	0.372	0.001
Percent determinism	0.36174	0.55257	0.41858	0.04310	0.39701	0.000	0.72921	0.000
Percent laminarity	0.000	26.974	0.000	0.000	3.032	0.000	50.340	0.000
Trapping Time	-1.000	13.259	-1.000	-1.000	10.769	-1.000	12.674	-1.000
Ratio	400.878	347.301	1158.080	620.988	203.468	0.000	195.807	0.000
Entropy (bits)	3.038	4.261	2.502	0.000	3.923	-1.000	4.643	-1.000
Maxline	25	562	45	10	600	-1	762	-1
Trend	-0.153	-0.198	-0.056	-0.010	-0.234	-0.013	-0.566	-0.002

LAM: The LAM values indicate intermittency or laminarity in the process. In dynamical systems intermittency is the alternation of phases of apparently periodic and chaotic dynamics. The LAM values are high for SOFIX and BUX indicating extent of intermittency. The periodicity is less than as depicted from REC values. So, it means it is most of the time in chaotic phase than in periodic phase.

TT: The TT values are positive only for SOFIX, BUX and FTSE MIB. TT values indicate the average time the system is trapped in specific state.

Ratio: Ratio is the indicator of transition between non chaotic to chaotic states. High ratio indicates presence of transition to chaotic state, and low ratio represents quasi steady state. It is zero only for NIKKEI 225 and S&P 500, thus indicating the non deterministic nature of these series.

Entropy: It is an indicator of the amount of information required to identify the system. The values of -1 indicates the series are non chaotic or periodic.

Maxline: Indicates length of the segment in terms of recurrent points of the longer segment and also the periodicity of the process. High values of Maxline indicate process is periodic and low values indicate the process is chaotic. Maxline value is very high for SOFIX, BUX and FTSE MIB due to positive Trapping Time and Percent laminarity showing periodicity.

Trend: Trend indicates the drift or stationarity of the signal. A high value of trend indicates drift in the signal and low value of trend indicates stationarity.

The analysis led with VRA induces us to refuse the hypothesis of IID and to emphasize the presence of structure. The data are non-linear deterministic in six out of eight cases and this nonlinearity can be interpreted as chaos. The most evident results are obtained for the emerging countries.

Predicting with VRA

Even if we are not interested in time series forecasting as a subject matter, we can think of it as the ultimate test of your analysis of a particular dynamical system and the time series generated by that system. If our embedding parameters are optimal, than the corresponding predictive model should minimizes the prediction error.

VRA provides an important module on non-parametric forecasting, using local models by fitting a low order polynomial which maps k nearest neighbours of onto their next values, to use this map to predict future values.After prediction, a plot shows the actual and predicted values, jointly with the normalized prediction error and the magnitudes of RMSE (root of the mean squared error) and normalized error (mean squared error normalized by the mean squared error of the trivial predictor: the unconditional mean in multi-step forecasting, or the random walk predictor in the one-step ahead predictor). In **Appendix 14** are presented in sample predictions for all series analyzed.

From Figures 13 and 14 once again we can notice the difference between an emerging and a developed country in terms of in sample prediction. The RMSE is very low for SOFIX (i.e. 5.24), while for NIKKEI 225 is 514.40, a very high value. This suggests that in a country with well developed financial market it is not easy to make predictions. Intertemporal smoothing operations, such as arbitrage, tend to squash cycles and chaos in economic systems with rich enough variety of market instruments. However, given the very different institutional features of financial markets in developing countries, it is important to explore the possibilities of such markets exhibiting chaotic behavior. Financial markets in developing countries are less mature as compared to those in developed countries, and the implications of complex nonlinear behavior could be significant for traders, institutional investors for devising suitable trading strategies.

Common fallacies about markets claim financial markets are unpredictable. However, chaos theory together with powerful algorithms proves such statements are wrong. Markets are chaotic systems with complex dynamics, yet to a certain extent we can make valid stock market forecasts. Using these forecasts together with a careful risk management strategy may give a trader a significant competitive advantage.

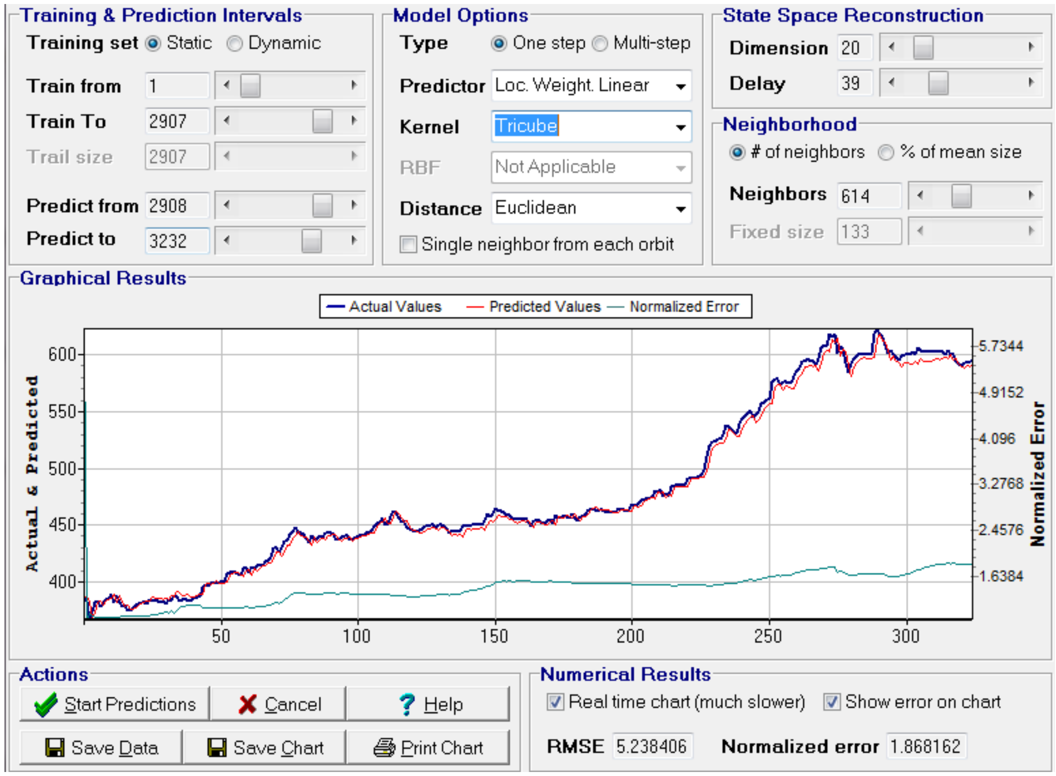

Figure 13. In sample prediction for SOFIX

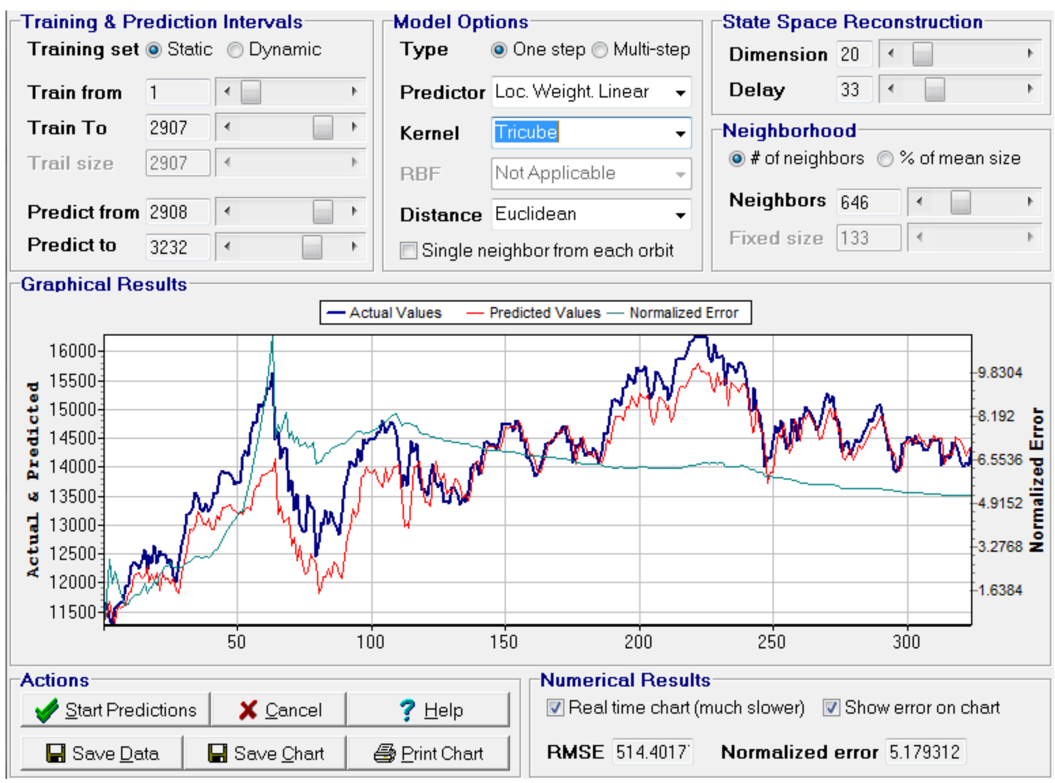

Figure 14. In sample prediction for NIKKEI 225

CONCLUSIONS

This study has examined the time series behaviour of close price based daily returns of equity indices for different markets by using recently developed tests of independence, nonlinearity and chaos.

Sometimes the conclusions both for and against chaos are reached by applying only one type of chaos test. To produce convincing results, we have to employ all tests for chaos to exploit their different potentials and limits. Few published papers have jointly applied the BDS test, R/S analysis, and topological tests.

Until recently, financial market researchers were ill equipped to detect the presence of chaos. The most commonly used nonlinear testing procedure was the BDS test, which is poorly suited for application to the small, noisy data sets common in finance. The introduction of recurrence plots and RQA for chaotic behaviour, however, has provided researchers with an exciting new tool for detecting chaos in financial data.

There are few existing studies of complex nonlinear dynamics which utilize this methodology, and so the application in this paper serves to illustrate the potential of this tool in the study of financial data, but more important to support the conclusion that the data analysed could be chaotic

In short, consistent with the findings of many previous studies results of this study reveal that there is a strong evidence of nonlinear dependence in daily increments of all equity indices analyzed. However, the nature of this nonlinear dependence appears to be deterministic only in five out of six cases.

More precisely, the results of variance ratio test suggest that the null hypothesis of random walk is strongly rejected for all the return series. It appears that daily increment in stock returns are highly autocorrelated. Further, the results based on Rescaled Range analysis also reveal that there is evidence of persistence or temporal dependencies in daily increments of market returns. The BDS test of independence produced mixed results when conducted on standardized residuals from GARCH and EGARCH models. In six out of eight cases the null of IID was not rejected. Non-rejection of the null hypothesis of IID observations suggests that low order GARCH or EGARCH type models are adequate to capture all potential nonlinear dependence in the data.

The findings of this study have some interesting implications. First, the existence of chaos in market indices could be exploitable and helpful for market players in the emerging countries such as Romania and Bulgaria. In other words, the presence of chaotic structure in return series implies that profitable nonlinearity based trading rules may exist at least in the short-run.

Second, presence of nonlinearity in the data suggests that asset pricing models and forecasting models should account for the existing nonlinearities in the data, otherwise their results may be biased and highly misleading. Finally, the presence of temporal dependence in market returns, as confirmed by the estimates of Hurst exponent, suggests that the process of annualizing risk by square root of time may lead to either overestimation or underestimation of the actual level of risk associated with an investment.

The VRA analysis, which can be applied and gives reliable results also with short data sets, shows presence of chaotic behaviour in five out of eight cases.

There are important reasons to understand the impact of nonlinearities and chaos in financial markets. Even if the future is unknowable, nonetheless Chaos Theory allows for the possibility of a range of future states represented by attractor on which orbits chaotic trajectories evolve. In the long run, a chaotic system moves into, and remains in it, though in principle determinate, resembles a random walk, repeatedly visiting each point in the attractor. The global behaviour of chaotic systems is bounded on the attractor: is not explosive. While economic fluctuations are unpredictable they will always lie within certain bounds. Thus, if we are able to know in which space the attractor lies, by determining the phase space using the embedding dimension for instance, and if we are able to re-build the orbits, then we can make predictions.

Although we cannot forecast the precise state of a chaotic system in the longer term, chaotic systems trace repetitive patterns which often provide useful information because they are the same at different scale of time. What is observed at a more global level is reproduced at a smaller scale because the chaotic attractor is a fractal. So, having knowledge of such patterns would make it possible to, on the average, make better predictions in short term.

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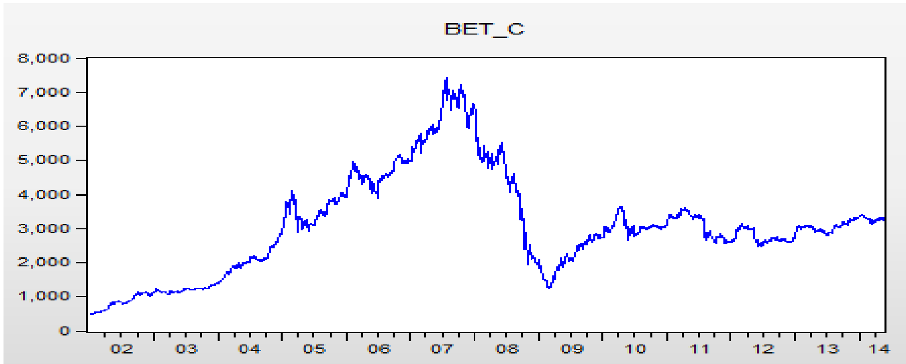
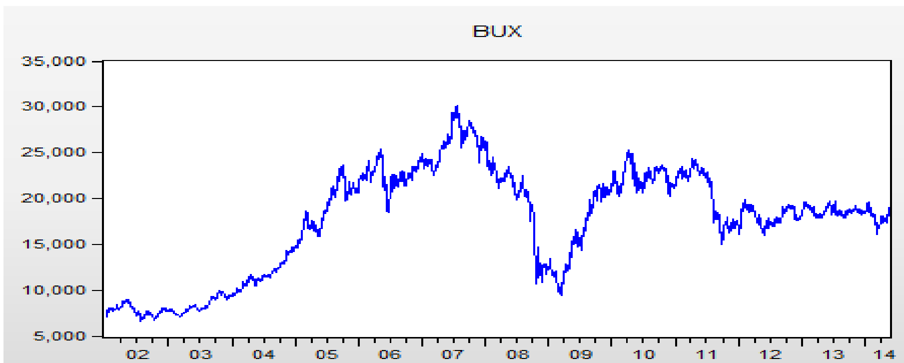
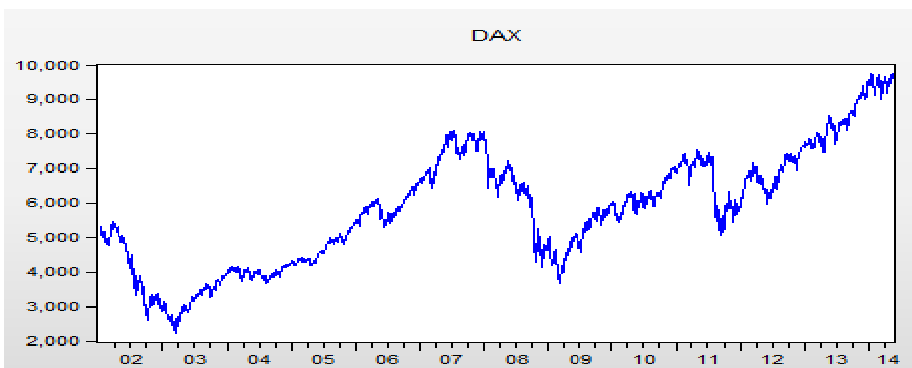
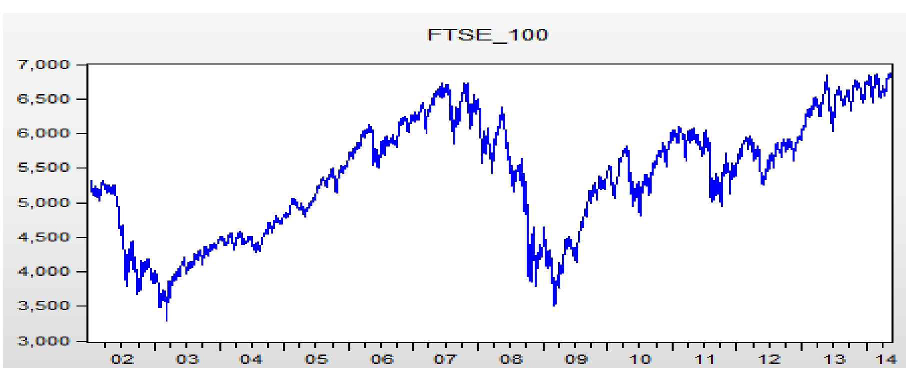
APPENDIX

APPENDIX 1









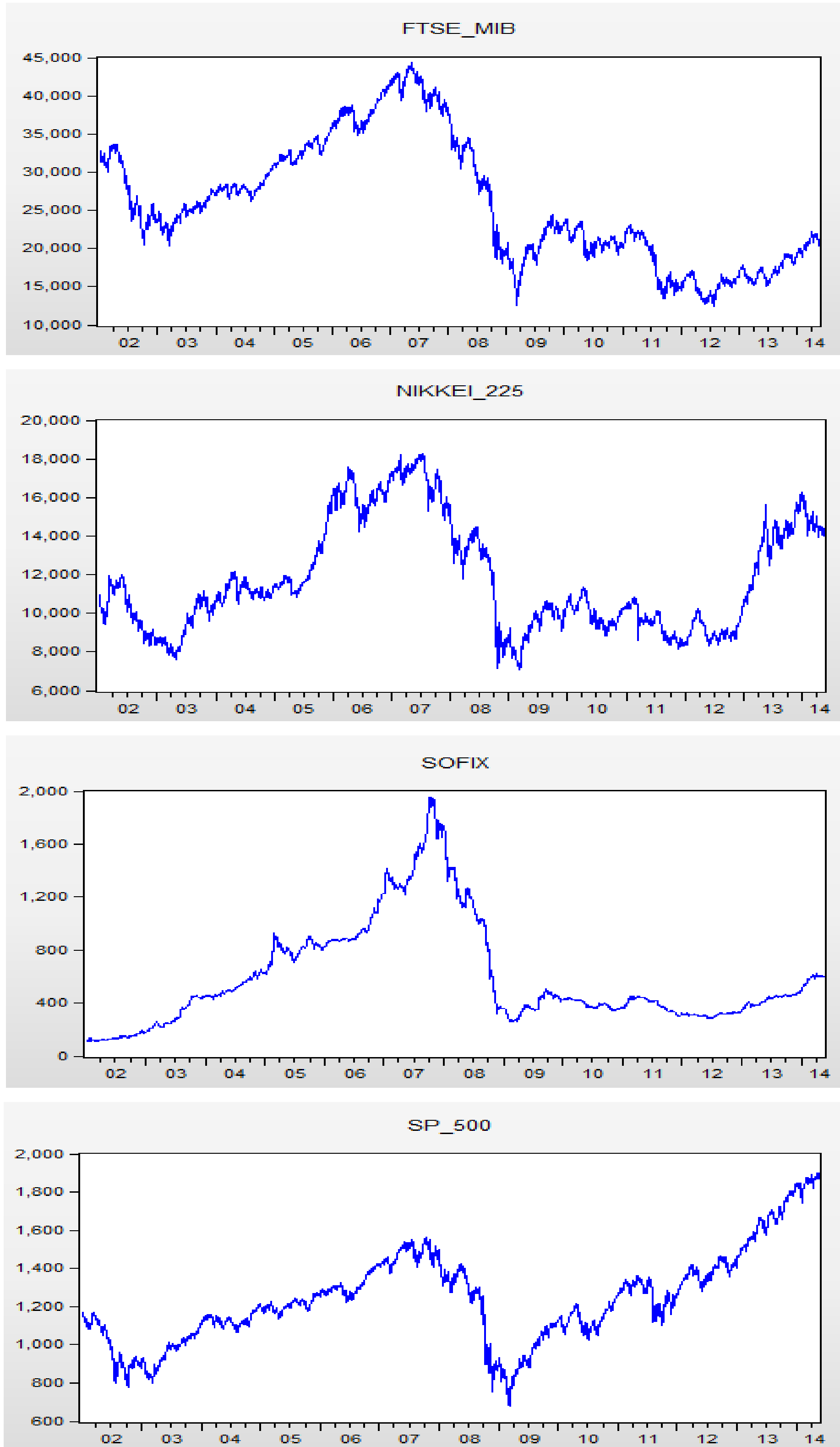
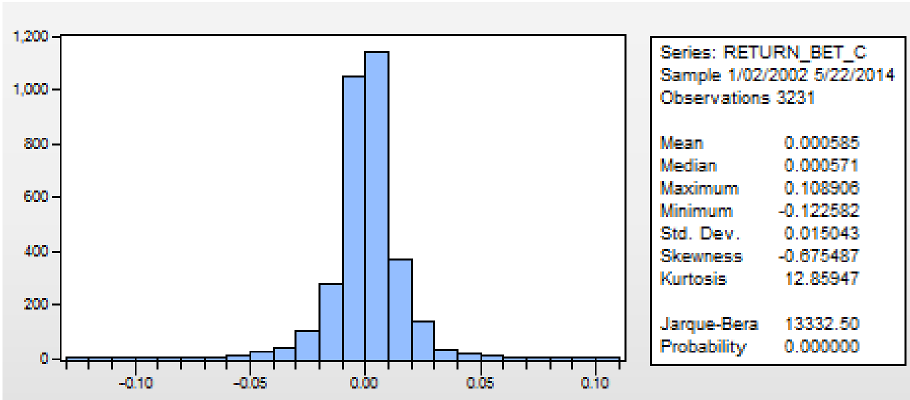


Figure 1. Daily close price evolution

APPENDIX 2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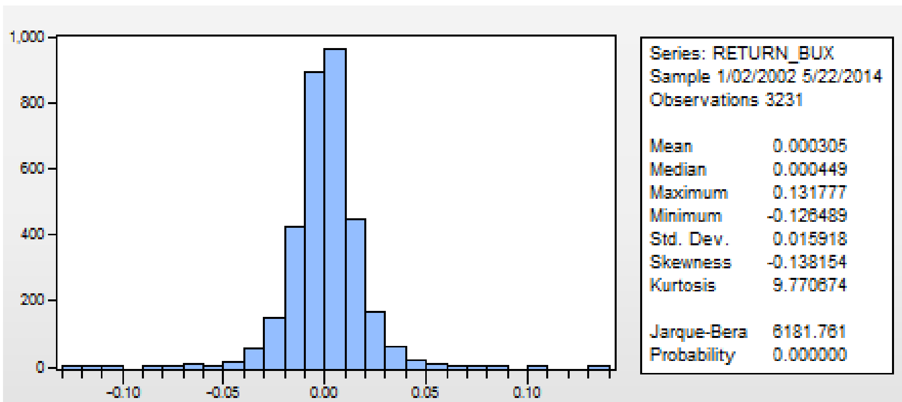
Sample 1/02/2002 5/22/2014
Observations 3231
Mean 0.000585
Median 0.000571
Maximum 0.108906
Minimum -0.122582
Std. Dev. 0.015043
Skewness -0.675487
Kurtosis 12.85947
Jarque-Bera 13332.50
Probability 0.000000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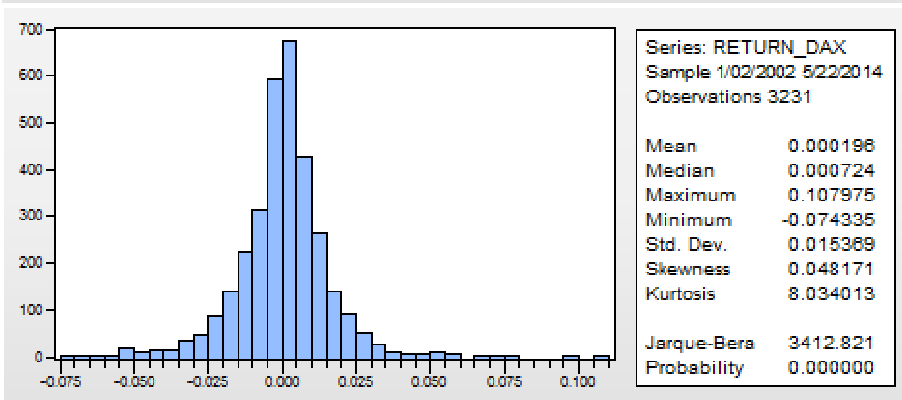
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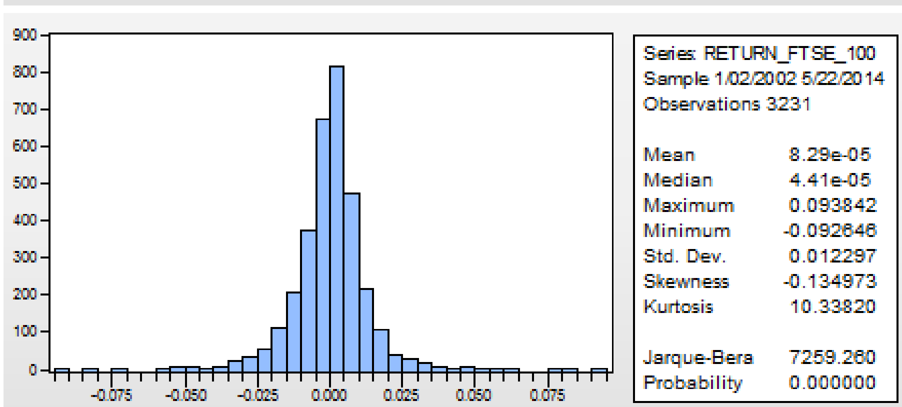
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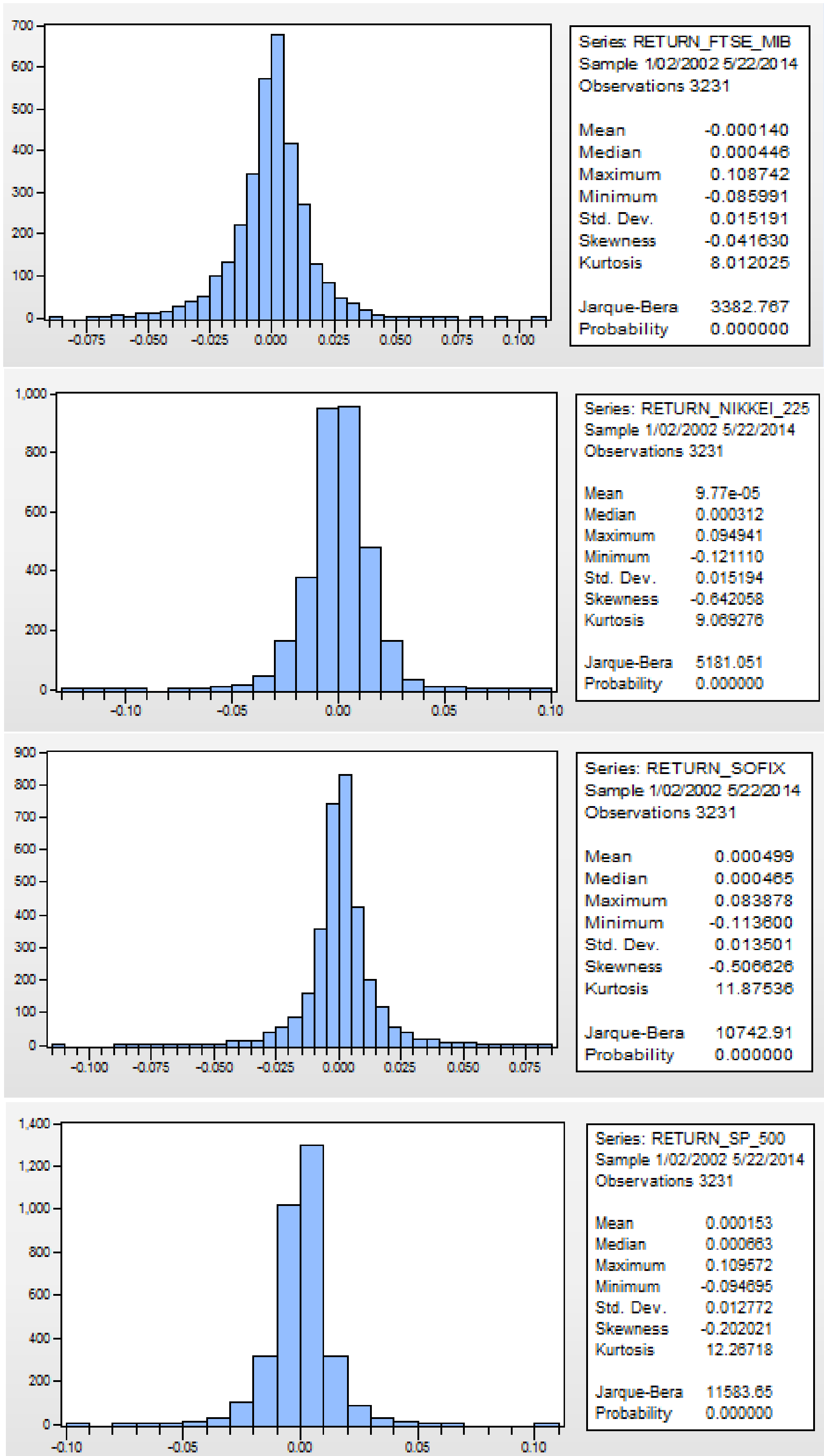
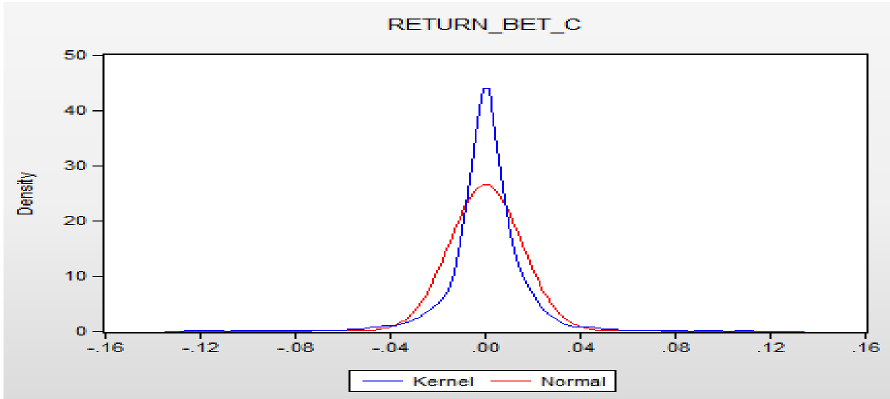


Figure 2. Histogram of daily returns

APPENDIX 3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20
10
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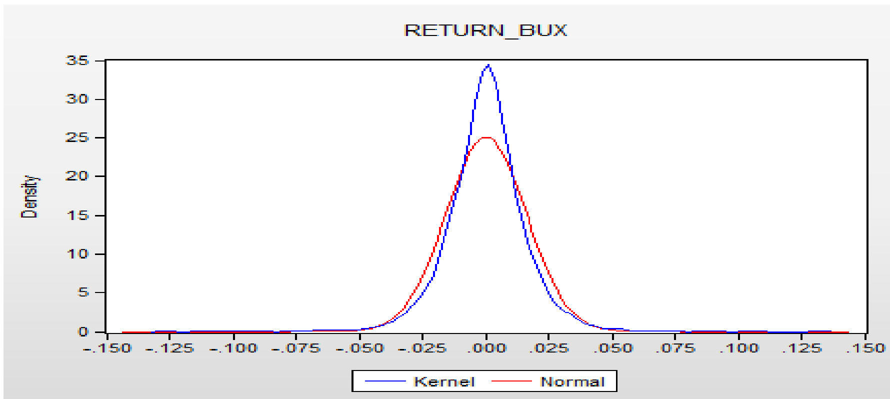
Kernel Normal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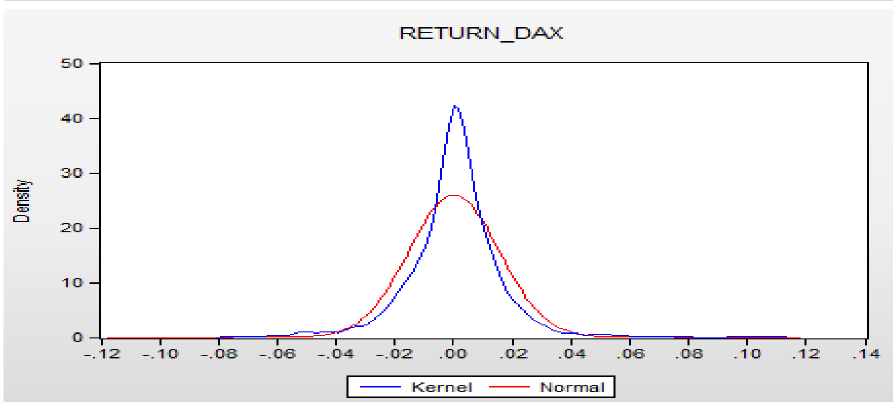
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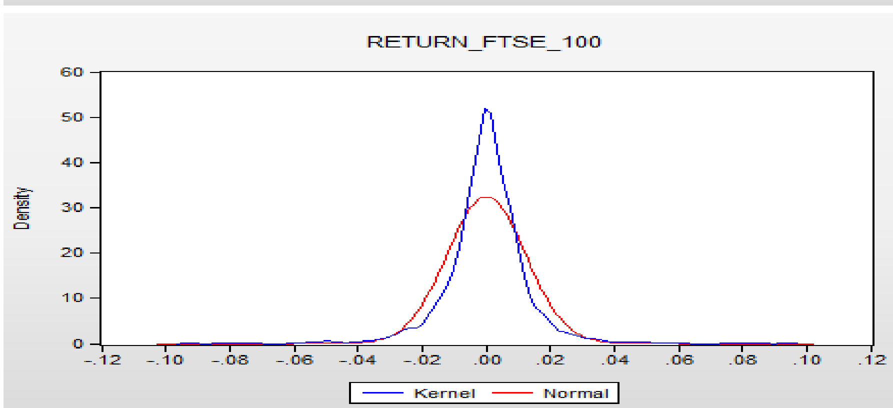
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Kernel Normal

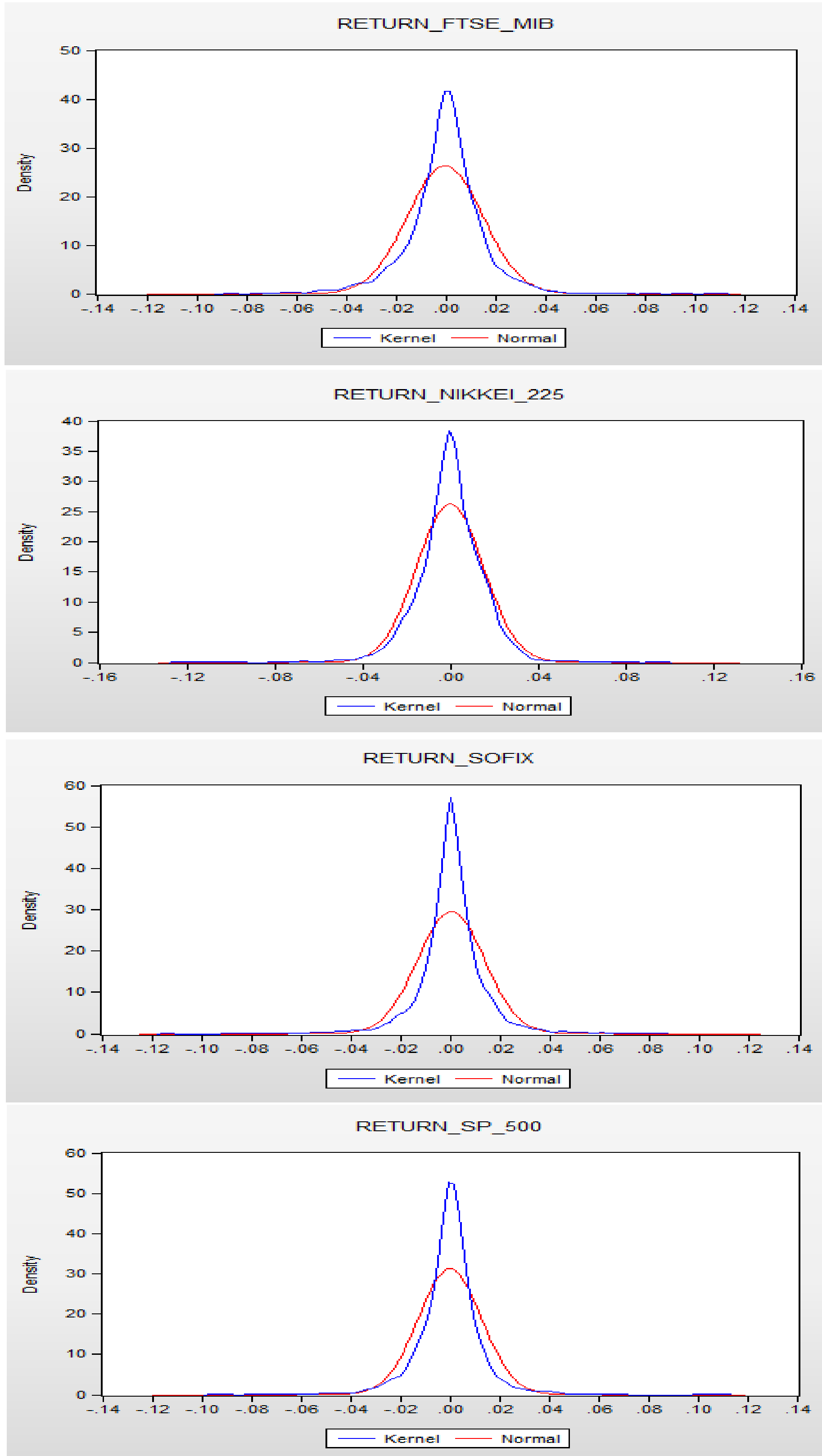


Figure 3. Normal vs. Empirical distribution

APPENDIX 4

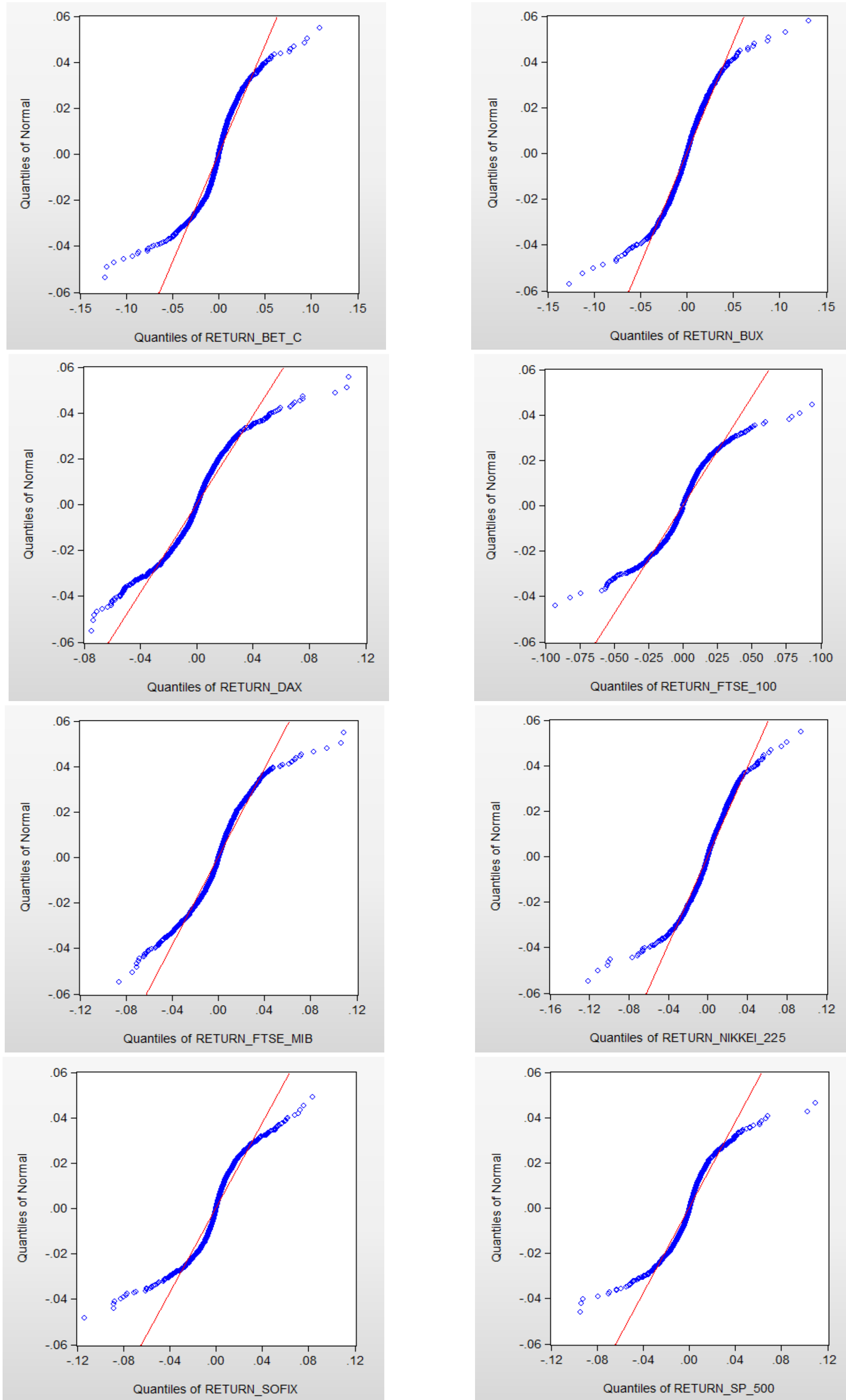


Figure 4. Q-Q Plot

APPENDIX 5

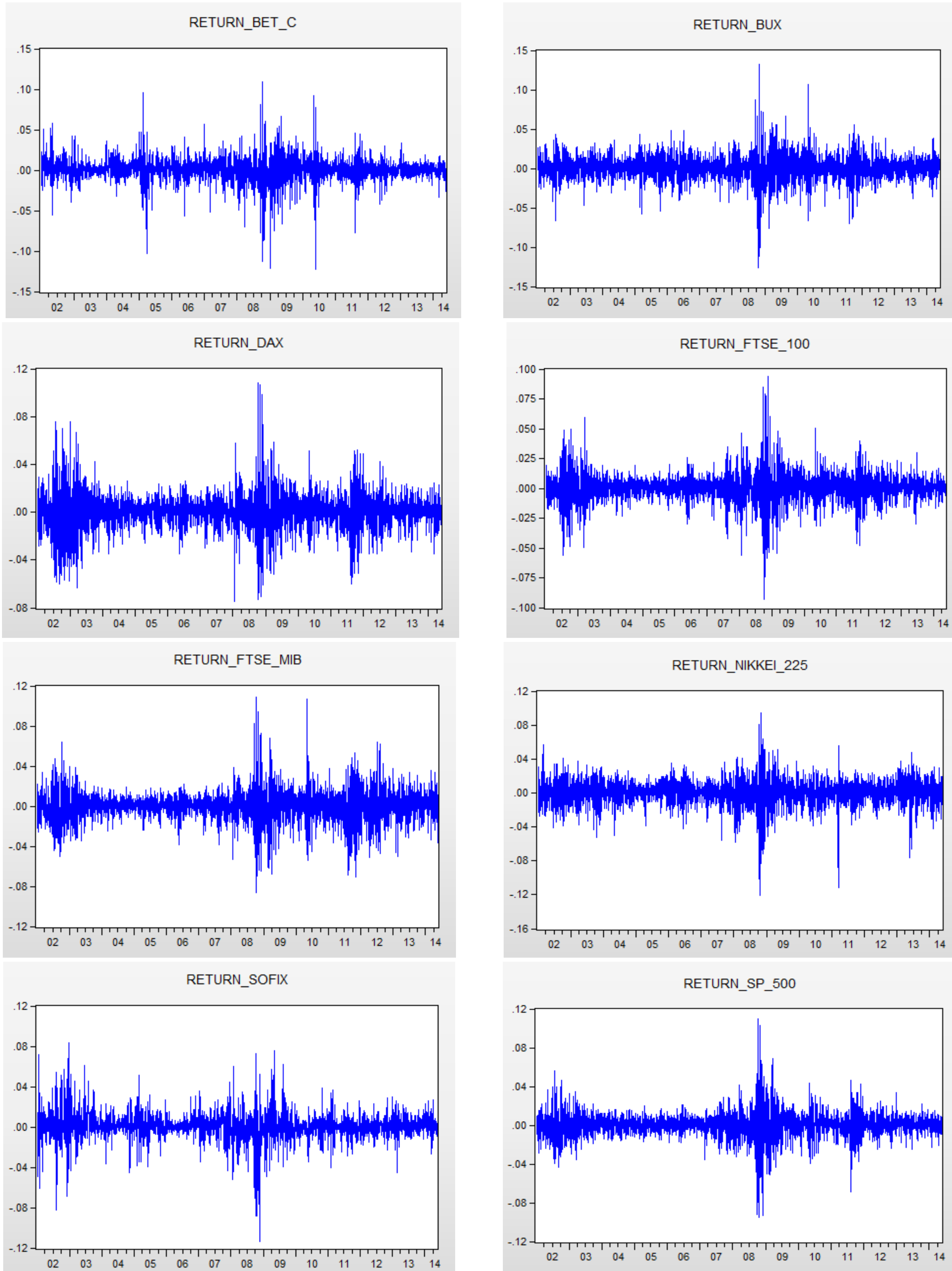


Figure 5. Evolution of daily returns

APPENDIX 6

Augmented Dickey-Fuller Unit Root Test on RETURN_BET_C

Null Hypothesis: RETURN_BET_C has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=28)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-51.74986	0.0001
Test critical values:	1% level	-3.432186	
	5% level	-2.862237	
	10% level	-2.567185	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(RETURN_BET_C)
Method: Least Squares
Date: 05/24/14 Time: 11:38
Sample (adjusted): 1/04/2002 5/22/2014
Included observations: 3230 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RETURN_BET_C(-1)	-0.906779	0.017522	-51.74986	0.0000
C	0.000526	0.000264	1.994930	0.0461

Augmented Dickey-Fuller Unit Root Test on RETURN_BUX

Null Hypothesis: RETURN_BUX has a unit root
Exogenous: None
Lag Length: 3 (Automatic - based on SIC, maxlag=28)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-27.02303	0.0000
Test critical values:	1% level	-2.565679	
	5% level	-1.940922	
	10% level	-1.616634	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(RETURN_BUX)
Method: Least Squares
Date: 05/24/14 Time: 11:39
Sample (adjusted): 1/09/2002 5/22/2014
Included observations: 3227 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RETURN_BUX(-1)	-0.955631	0.035364	-27.02303	0.0000
D(RETURN_BUX(-1))	-0.008821	0.030297	-0.291157	0.7710
D(RETURN_BUX(-2))	-0.063476	0.024402	-2.601236	0.0093
D(RETURN_BUX(-3))	-0.080321	0.017555	-4.575358	0.0000

Augmented Dickey-Fuller Unit Root Test on RETURN_DAX

Null Hypothesis: RETURN_DAX has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=28)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-58.35544	0.0001
Test critical values:	1% level	-2.565678	
	5% level	-1.940922	
	10% level	-1.616634	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(RETURN_DAX)
Method: Least Squares
Date: 05/24/14 Time: 11:52
Sample (adjusted): 1/04/2002 5/22/2014
Included observations: 3230 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RETURN_DAX(-1)	-1.026334	0.017588	-58.35544	0.0000

Augmented Dickey-Fuller Unit Root Test on RETURN_FTSE_100

Null Hypothesis: RETURN_FTSE_100 has a unit root
Exogenous: None
Lag Length: 4 (Automatic - based on SIC, maxlag=28)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-27.49092	0.0000
Test critical values:	1% level	-2.565679	
	5% level	-1.940922	
	10% level	-1.616634	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(RETURN_FTSE_100)
Method: Least Squares
Date: 05/24/14 Time: 11:52
Sample (adjusted): 1/10/2002 5/22/2014
Included observations: 3226 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RETURN_FTSE_100(-1)	-1.174833	0.042735	-27.49092	0.0000
D(RETURN_FTSE_100(-1))	0.124628	0.038046	3.275699	0.0011
D(RETURN_FTSE_100(-2))	0.083735	0.031917	2.623506	0.0087
D(RETURN_FTSE_100(-3))	-0.007077	0.025522	-0.277277	0.7816
D(RETURN_FTSE_100(-4))	0.056521	0.017584	3.214290	0.0013

Augmented Dickey-Fuller Unit Root Test on RETURN_FTSE_MIB

Null Hypothesis: RETURN_FTSE_MIB has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=28)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-57.65096	0.0001
Test critical values:	1% level	-2.565678	
	5% level	-1.940922	
	10% level	-1.616634	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(RETURN_FTSE_MIB)
Method: Least Squares
Date: 05/24/14 Time: 12:00
Sample (adjusted): 1/04/2002 5/22/2014
Included observations: 3230 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RETURN_FTSE_MIB(-1)	-1.014191	0.017592	-57.65096	0.0000

Augmented Dickey-Fuller Unit Root Test on RETURN_NIKKEI_225

Null Hypothesis: RETURN_NIKKEI_225 has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=28)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-59.43818	0.0001
Test critical values:	1% level	-2.565678	
	5% level	-1.940922	
	10% level	-1.616634	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(RETURN_NIKKEI_225)
Method: Least Squares
Date: 05/24/14 Time: 12:00
Sample (adjusted): 1/04/2002 5/22/2014
Included observations: 3230 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RETURN_NIKKEI_225(-1)	-1.045190	0.017584	-59.43818	0.0000

Augmented Dickey-Fuller Unit Root Test on RETURN_SOFIX

Null Hypothesis: RETURN_SOFIX has a unit root
Exogenous: None
Lag Length: 7 (Automatic - based on SIC, maxlag=28)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-17.47361	0.0000
Test critical values:	1% level	-2.565680	
	5% level	-1.940922	
	10% level	-1.616633	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(RETURN_SOFIX)
Method: Least Squares
Date: 05/24/14 Time: 12:09
Sample (adjusted): 1/15/2002 5/22/2014
Included observations: 3223 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RETURN_SOFIX(-1)	-0.685214	0.039214	-17.47361	0.0000
D(RETURN_SOFIX(-1))	-0.214395	0.037340	-5.741641	0.0000
D(RETURN_SOFIX(-2))	-0.125238	0.035530	-3.524809	0.0004
D(RETURN_SOFIX(-3))	-0.075533	0.033089	-2.282713	0.0225
D(RETURN_SOFIX(-4))	-0.017970	0.030694	-0.585470	0.5583
D(RETURN_SOFIX(-5))	-0.057431	0.027599	-2.080912	0.0375
D(RETURN_SOFIX(-6))	-0.001773	0.023633	-0.075022	0.9402
D(RETURN_SOFIX(-7))	-0.064985	0.017547	-3.703558	0.0002

Augmented Dickey-Fuller Unit Root Test on RETURN_SP_500

Null Hypothesis: RETURN_SP_500 has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=28)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-64.45599	0.0001
Test critical values:	1% level	-2.565678	
	5% level	-1.940922	
	10% level	-1.616634	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(RETURN_SP_500)
Method: Least Squares
Date: 05/24/14 Time: 12:09
Sample (adjusted): 1/04/2002 5/22/2014
Included observations: 3230 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RETURN_SP_500(-1)	-1.125282	0.017458	-64.45599	0.0000

Figure 6. ADF test results

APPENDIX 7

Phillips-Perron Unit Root Test on RETURN_BET_C

Null Hypothesis: RETURN_BET_C has a unit root
Exogenous: Constant
Bandwidth: 15 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-52.31744	0.0001
Test critical values:	1% level	-3.432186	
	5% level	-2.862237	
	10% level	-2.567185	

*MacKinnon (1996) one-sided p-values.

Residual variance (no correction)	0.000224
HAC corrected variance (Bartlett kernel)	0.000264

Phillips-Perron Test Equation
Dependent Variable: D(RETURN_BET_C)
Method: Least Squares
Date: 05/24/14 Time: 11:45
Sample (adjusted): 1/04/2002 5/22/2014
Included observations: 3230 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RETURN_BET_C(-1)	-0.906779	0.017522	-51.74986	0.0000
C	0.000526	0.000264	1.994930	0.0461

Phillips-Perron Unit Root Test on RETURN_BUX

Null Hypothesis: RETURN_BUX has a unit root
Exogenous: None
Bandwidth: 7 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-54.92960	0.0001
Test critical values:	1% level	-2.565678	
	5% level	-1.940922	
	10% level	-1.616634	

*MacKinnon (1996) one-sided p-values.

Residual variance (no correction)	0.000253
HAC corrected variance (Bartlett kernel)	0.000244

Phillips-Perron Test Equation
Dependent Variable: D(RETURN_BUX)
Method: Least Squares
Date: 05/24/14 Time: 11:45
Sample (adjusted): 1/04/2002 5/22/2014
Included observations: 3230 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RETURN_BUX(-1)	-0.966594	0.017589	-54.95419	0.0000

Phillips-Perron Unit Root Test on RETURN_DAX

Null Hypothesis: RETURN_DAX has a unit root
Exogenous: None
Bandwidth: 9 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-58.55270	0.0001
Test critical values:	1% level	-2.565678	
	5% level	-1.940922	
	10% level	-1.616634	

*MacKinnon (1996) one-sided p-values.

Residual variance (no correction)	0.000236
HAC corrected variance (Bartlett kernel)	0.000209

Phillips-Perron Test Equation
Dependent Variable: D(RETURN_DAX)
Method: Least Squares
Date: 05/24/14 Time: 11:55
Sample (adjusted): 1/04/2002 5/22/2014
Included observations: 3230 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RETURN_DAX(-1)	-1.026334	0.017588	-58.35544	0.0000

Phillips-Perron Unit Root Test on RETURN_FTSE_100

Null Hypothesis: RETURN_FTSE_100 has a unit root
Exogenous: None
Bandwidth: 4 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-60.28837	0.0001
Test critical values:	1% level	-2.565678	
	5% level	-1.940922	
	10% level	-1.616634	

*MacKinnon (1996) one-sided p-values.

Residual variance (no correction)	0.000151
HAC corrected variance (Bartlett kernel)	0.000137

Phillips-Perron Test Equation
Dependent Variable: D(RETURN_FTSE_100)
Method: Least Squares
Date: 05/24/14 Time: 11:55
Sample (adjusted): 1/04/2002 5/22/2014
Included observations: 3230 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RETURN_FTSE_100(-1)	-1.055004	0.017565	-60.06317	0.0000

Phillips-Perron Unit Root Test on RETURN_FTSE_MIB

Null Hypothesis: RETURN_FTSE_MIB has a unit root
Exogenous: None
Bandwidth: 3 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-57.67984	0.0001
Test critical values:	1% level	-2.565678	
	5% level	-1.940922	
	10% level	-1.616634	

*MacKinnon (1996) one-sided p-values.

Residual variance (no correction)	0.000231
HAC corrected variance (Bartlett kernel)	0.000221

Phillips-Perron Test Equation
Dependent Variable: D(RETURN_FTSE_MIB)
Method: Least Squares
Date: 05/24/14 Time: 12:02
Sample (adjusted): 1/04/2002 5/22/2014
Included observations: 3230 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RETURN_FTSE_MIB(-1)	-1.014191	0.017592	-57.65096	0.0000

Phillips-Perron Unit Root Test on RETURN_NIKKEI_225

Null Hypothesis: RETURN_NIKKEI_225 has a unit root
Exogenous: None
Bandwidth: 9 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-59.52710	0.0001
Test critical values:	1% level	-2.565678	
	5% level	-1.940922	
	10% level	-1.616634	

*MacKinnon (1996) one-sided p-values.

Residual variance (no correction)	0.000230
HAC corrected variance (Bartlett kernel)	0.000219

Phillips-Perron Test Equation
Dependent Variable: D(RETURN_NIKKEI_225)
Method: Least Squares
Date: 05/24/14 Time: 12:02
Sample (adjusted): 1/04/2002 5/22/2014
Included observations: 3230 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RETURN_NIKKEI_225(-1)	-1.045190	0.017584	-59.43818	0.0000

Phillips-Perron Unit Root Test on RETURN_SOFIX

Null Hypothesis: RETURN_SOFIX has a unit root
Exogenous: None
Bandwidth: 22 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-54.86382	0.0001
Test critical values:	1% level	-2.565678	
	5% level	-1.940922	
	10% level	-1.616634	

*MacKinnon (1996) one-sided p-values.

Residual variance (no correction)	0.000180
HAC corrected variance (Bartlett kernel)	0.000325

Phillips-Perron Test Equation
Dependent Variable: D(RETURN_SOFIX)
Method: Least Squares
Date: 05/24/14 Time: 12:12
Sample (adjusted): 1/04/2002 5/22/2014
Included observations: 3230 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RETURN_SOFIX(-1)	-0.892060	0.017495	-50.98863	0.0000

Phillips-Perron Unit Root Test on RETURN_SP_500

Null Hypothesis: RETURN_SP_500 has a unit root
Exogenous: None
Bandwidth: 9 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-65.01303	0.0001
Test critical values:	1% level	-2.565678	
	5% level	-1.940922	
	10% level	-1.616634	

*MacKinnon (1996) one-sided p-values.

Residual variance (no correction)	0.000161
HAC corrected variance (Bartlett kernel)	0.000143

Phillips-Perron Test Equation
Dependent Variable: D(RETURN_SP_500)
Method: Least Squares
Date: 05/24/14 Time: 12:12
Sample (adjusted): 1/04/2002 5/22/2014
Included observations: 3230 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RETURN_SP_500(-1)	-1.125282	0.017458	-64.45599	0.0000

Figure 7. PP test results

APPENDIX 8

Correlogram of RETURN_BET_C

Date: 05/24/14 Time: 11:46
Sample: 1/02/2002 5/22/2014
Included observations: 3231

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.093	0.093	28.104	0.000
		2	0.025	0.017	30.165	0.000
		3	-0.003	-0.007	30.202	0.000
		4	-0.020	-0.019	31.450	0.000
		5	0.026	0.030	33.612	0.000
		6	0.029	0.025	36.318	0.000
		7	0.007	0.001	36.491	0.000
		8	0.039	0.037	41.371	0.000
		9	0.026	0.020	43.503	0.000
		10	0.032	0.027	46.727	0.000
		11	0.015	0.008	47.417	0.000
		12	0.032	0.031	50.837	0.000
		13	0.027	0.020	53.119	0.000
		14	0.059	0.053	64.394	0.000
		15	0.057	0.046	75.086	0.000

Correlogram of RETURN_BUX

Date: 05/24/14 Time: 11:46
Sample: 1/02/2002 5/22/2014
Included observations: 3231

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.033	0.033	3.5331	0.060
		2	-0.059	-0.060	14.846	0.001
		3	-0.018	-0.014	15.946	0.001
		4	0.082	0.080	37.705	0.000
		5	0.021	0.014	39.156	0.000
		6	-0.050	-0.043	47.331	0.000
		7	0.006	0.014	47.431	0.000
		8	0.020	0.008	48.666	0.000
		9	-0.020	-0.025	49.925	0.000
		10	-0.041	-0.031	55.378	0.000
		11	0.017	0.019	56.367	0.000
		12	0.030	0.019	59.221	0.000
		13	-0.005	-0.003	59.300	0.000
		14	-0.031	-0.020	62.392	0.000
		15	-0.009	-0.010	62.639	0.000

Correlogram of RETURN_DAX

Date: 05/24/14 Time: 11:49
Sample: 1/02/2002 5/22/2014
Included observations: 3231

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.026	-0.026	2.2701	0.132
		2	-0.015	-0.016	2.9873	0.225
		3	-0.041	-0.042	8.4583	0.037
		4	0.017	0.015	9.4170	0.051
		5	-0.050	-0.051	17.607	0.003
		6	-0.012	-0.016	18.061	0.006
		7	0.013	0.012	18.616	0.009
		8	0.016	0.012	19.442	0.013
		9	-0.007	-0.005	19.580	0.021
		10	-0.010	-0.011	19.929	0.030
		11	0.036	0.035	24.140	0.012
		12	0.002	0.004	24.153	0.019
		13	0.001	0.003	24.155	0.030
		14	0.008	0.011	24.361	0.041
		15	-0.010	-0.012	24.664	0.055

Correlogram of RETURN_FTSE_100

Date: 05/24/14 Time: 11:49
Sample: 1/02/2002 5/22/2014
Included observations: 3231

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.055	-0.055	9.8000	0.002
		2	-0.030	-0.033	12.750	0.002
		3	-0.089	-0.093	38.473	0.000
		4	0.077	0.066	57.727	0.000
		5	-0.058	-0.057	68.453	0.000
		6	-0.030	-0.040	71.374	0.000
		7	0.028	0.034	73.936	0.000
		8	0.032	0.018	77.345	0.000
		9	-0.008	-0.001	77.547	0.000
		10	-0.018	-0.010	78.608	0.000
		11	-0.008	-0.014	78.838	0.000
		12	-0.014	-0.019	79.512	0.000
		13	0.012	0.012	79.946	0.000
		14	-0.011	-0.011	80.305	0.000
		15	-0.009	-0.015	80.570	0.000

Correlogram of RETURN_FTSE_MIB

Date: 05/24/14 Time: 11:58
Sample: 1/02/2002 5/22/2014
Included observations: 3231

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.014	-0.014	0.6593	0.417
		2	-0.015	-0.015	1.3557	0.508
		3	-0.049	-0.050	9.1668	0.027
		4	0.042	0.041	14.952	0.005
		5	-0.069	-0.070	30.338	0.000
		6	0.006	0.004	30.467	0.000
		7	0.020	0.022	31.790	0.000
		8	0.028	0.020	34.284	0.000
		9	-0.014	-0.007	34.923	0.000
		10	-0.027	-0.030	37.324	0.000
		11	0.018	0.019	38.403	0.000
		12	0.022	0.022	39.958	0.000
		13	0.019	0.022	41.145	0.000
		14	0.007	0.010	41.288	0.000
		15	-0.006	-0.010	41.411	0.000

Correlogram of RETURN_NIKKEI_225

Date: 05/24/14 Time: 11:58
Sample: 1/02/2002 5/22/2014
Included observations: 3231

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.045	-0.045	6.6081	0.010
		2	0.010	0.008	6.9571	0.031
		3	-0.052	-0.051	15.550	0.001
		4	0.014	0.009	16.145	0.003
		5	0.011	0.013	16.525	0.005
		6	-0.013	-0.015	17.096	0.009
		7	0.009	0.009	17.383	0.015
		8	-0.016	-0.014	18.254	0.019
		9	-0.017	-0.020	19.196	0.024
		10	0.000	-0.000	19.196	0.038
		11	0.016	0.015	19.992	0.045
		12	0.009	0.009	20.272	0.062
		13	0.023	0.024	21.937	0.056
		14	-0.013	-0.009	22.454	0.070
		15	0.003	0.002	22.493	0.096

Correlogram of RETURN_SOFIX

Date: 05/24/14 Time: 12:05
Sample: 1/02/2002 5/22/2014
Included observations: 3231

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.107	0.107	36.834	0.000
		2	0.118	0.108	81.972	0.000
		3	0.068	0.046	96.959	0.000
		4	0.082	0.060	118.86	0.000
		5	-0.012	-0.038	119.33	0.000
		6	0.070	0.058	135.26	0.000
		7	-0.042	-0.058	140.98	0.000
		8	0.067	0.064	155.66	0.000
		9	-0.001	-0.006	155.66	0.000
		10	0.053	0.038	164.61	0.000
		11	0.056	0.053	174.63	0.000
		12	0.076	0.044	193.21	0.000
		13	0.040	0.026	198.42	0.000
		14	0.063	0.025	211.46	0.000
		15	0.045	0.029	217.97	0.000

Correlogram of RETURN_SP_500

Date: 05/24/14 Time: 12:05
Sample: 1/02/2002 5/22/2014
Included observations: 3231

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.125	-0.125	50.887	0.000
		2	-0.015	-0.032	51.660	0.000
		3	0.008	0.002	51.858	0.000
		4	-0.011	-0.010	52.221	0.000
		5	-0.026	-0.029	54.378	0.000
		6	-0.006	-0.014	54.498	0.000
		7	-0.032	-0.037	57.857	0.000
		8	0.049	0.041	65.637	0.000
		9	-0.024	-0.015	67.572	0.000
		10	0.018	0.014	68.582	0.000
		11	0.018	0.019	69.598	0.000
		12	-0.018	-0.013	70.606	0.000
		13	0.026	0.024	72.746	0.000
		14	-0.025	-0.021	74.828	0.000
		15	-0.029	-0.031	77.600	0.000

Figura nr.8. Daily returns corellogram

APPENDIX 9

Correlogram of SQUARE_RETURN_BET_C

Date: 05/24/14 Time: 12:16
Sample: 1/02/2002 5/22/2014
Included observations: 3231

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.313	0.313	316.64	0.000
		2	0.253	0.172	523.54	0.000
		3	0.193	0.084	643.82	0.000
		4	0.168	0.065	735.19	0.000
		5	0.143	0.044	801.36	0.000
		6	0.124	0.032	851.43	0.000
		7	0.139	0.059	913.71	0.000
		8	0.144	0.058	980.62	0.000
		9	0.158	0.067	1061.5	0.000
		10	0.179	0.080	1164.9	0.000
		11	0.226	0.120	1330.5	0.000
		12	0.186	0.042	1442.9	0.000
		13	0.172	0.033	1539.4	0.000
		14	0.118	-0.020	1584.7	0.000
		15	0.159	0.058	1667.2	0.000

Correlogram of SQUARE_RETURN_BUX

Date: 05/24/14 Time: 12:16
Sample: 1/02/2002 5/22/2014
Included observations: 3231

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.324	0.324	338.52	0.000
		2	0.240	0.151	524.53	0.000
		3	0.172	0.065	620.25	0.000
		4	0.156	0.066	698.74	0.000
		5	0.159	0.075	780.99	0.000
		6	0.208	0.123	920.45	0.000
		7	0.180	0.059	1025.4	0.000
		8	0.260	0.157	1244.8	0.000
		9	0.262	0.120	1467.8	0.000
		10	0.276	0.123	1715.3	0.000
		11	0.152	-0.038	1790.0	0.000
		12	0.165	0.033	1878.0	0.000
		13	0.182	0.065	1985.6	0.000
		14	0.173	0.024	2082.7	0.000
		15	0.175	0.026	2182.2	0.000

Correlogram of SQUARE_RETURN_DAX

Date: 05/24/14 Time: 12:18
Sample: 1/02/2002 5/22/2014
Included observations: 3231

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.186	0.186	112.42	0.000
		2	0.268	0.242	344.83	0.000
		3	0.268	0.204	577.56	0.000
		4	0.228	0.124	745.28	0.000
		5	0.280	0.163	998.24	0.000
		6	0.174	0.025	1095.9	0.000
		7	0.247	0.100	1293.5	0.000
		8	0.210	0.062	1437.1	0.000
		9	0.257	0.116	1650.7	0.000
		10	0.224	0.062	1812.7	0.000
		11	0.279	0.133	2064.6	0.000
		12	0.244	0.068	2257.7	0.000
		13	0.142	-0.060	2323.0	0.000
		14	0.214	0.010	2471.1	0.000
		15	0.125	-0.060	2522.0	0.000

Correlogram of SQUARE_RETURN_FTSE_100

Date: 05/24/14 Time: 12:18
Sample: 1/02/2002 5/22/2014
Included observations: 3231

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.238	0.238	183.12	0.000
		2	0.302	0.260	477.63	0.000
		3	0.317	0.230	803.77	0.000
		4	0.290	0.158	1075.4	0.000
		5	0.359	0.216	1491.6	0.000
		6	0.229	0.031	1661.5	0.000
		7	0.218	-0.004	1815.1	0.000
		8	0.200	-0.021	1944.7	0.000
		9	0.263	0.087	2168.3	0.000
		10	0.281	0.118	2424.0	0.000
		11	0.210	0.041	2566.8	0.000
		12	0.239	0.061	2752.4	0.000
		13	0.244	0.063	2946.1	0.000
		14	0.176	-0.055	3046.6	0.000
		15	0.246	0.034	3243.3	0.000

Correlogram of SQUARE_RETURN_FTSE_MIB

Date: 05/24/14 Time: 12:21
Sample: 1/02/2002 5/22/2014
Included observations: 3231

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.179	0.179	104.00	0.000
		2	0.214	0.188	252.37	0.000
		3	0.249	0.197	452.55	0.000
		4	0.248	0.168	650.97	0.000
		5	0.268	0.170	884.08	0.000
		6	0.209	0.081	1025.2	0.000
		7	0.150	-0.003	1098.4	0.000
		8	0.148	-0.011	1169.1	0.000
		9	0.258	0.131	1385.4	0.000
		10	0.180	0.047	1490.6	0.000
		11	0.168	0.036	1582.5	0.000
		12	0.225	0.103	1746.6	0.000
		13	0.190	0.053	1863.3	0.000
		14	0.175	0.011	1963.1	0.000
		15	0.186	0.025	2075.6	0.000

Correlogram of SQUARE_RETURN_NIKKEI_225

Date: 05/24/14 Time: 12:21
Sample: 1/02/2002 5/22/2014
Included observations: 3231

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.241	0.241	187.45	0.000
		2	0.385	0.347	667.01	0.000
		3	0.241	0.117	854.38	0.000
		4	0.389	0.253	1345.3	0.000
		5	0.207	0.025	1483.7	0.000
		6	0.380	0.191	1951.3	0.000
		7	0.184	-0.013	2061.2	0.000
		8	0.284	0.038	2323.1	0.000
		9	0.203	0.039	2456.9	0.000
		10	0.320	0.104	2789.3	0.000
		11	0.184	0.013	2899.4	0.000
		12	0.244	-0.003	3092.4	0.000
		13	0.190	0.029	3209.9	0.000
		14	0.195	-0.039	3333.0	0.000
		15	0.193	0.043	3454.6	0.000

Correlogram of SQUARE_RETURN_SOFIX

Date: 05/24/14 Time: 12:22
Sample: 1/02/2002 5/22/2014
Included observations: 3231

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.356	0.356	408.91	0.000
		2	0.346	0.252	797.10	0.000
		3	0.309	0.155	1105.6	0.000
		4	0.272	0.092	1344.6	0.000
		5	0.171	-0.031	1439.7	0.000
		6	0.219	0.079	1595.4	0.000
		7	0.128	-0.030	1648.9	0.000
		8	0.248	0.161	1848.0	0.000
		9	0.112	-0.057	1888.6	0.000
		10	0.166	0.045	1977.5	0.000
		11	0.140	0.017	2040.8	0.000
		12	0.145	0.019	2109.3	0.000
		13	0.096	-0.004	2139.0	0.000
		14	0.159	0.051	2221.6	0.000
		15	0.091	-0.004	2248.6	0.000

Correlogram of SQUARE_RETURN_SP_500

Date: 05/24/14 Time: 12:22
Sample: 1/02/2002 5/22/2014
Included observations: 3231

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.214	0.214	148.37	0.000
		2	0.388	0.359	636.23	0.000
		3	0.212	0.099	781.82	0.000
		4	0.281	0.126	1036.6	0.000
		5	0.346	0.243	1423.9	0.000
		6	0.308	0.149	1730.4	0.000
		7	0.337	0.133	2098.6	0.000
		8	0.264	0.062	2323.9	0.000
		9	0.308	0.094	2632.2	0.000
		10	0.277	0.067	2881.4	0.000
		11	0.367	0.155	3317.4	0.000
		12	0.306	0.084	3620.6	0.000
		13	0.260	-0.021	3839.6	0.000
		14	0.150	-0.153	3912.6	0.000
		15	0.219	-0.043	4067.9	0.000

Figure 9. Square returns corellogram

APPENDIX 10

Correlogram of ABS_RETURN_BET_C

Date: 05/24/14 Time: 12:25
Sample: 1/02/2002 5/22/2014
Included observations: 3231

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.390	0.390	491.51	0.000
		2	0.305	0.180	791.47	0.000
		3	0.281	0.138	1046.3	0.000
		4	0.229	0.063	1215.4	0.000
		5	0.233	0.089	1391.3	0.000
		6	0.205	0.046	1527.2	0.000
		7	0.203	0.058	1661.2	0.000
		8	0.215	0.070	1810.8	0.000
		9	0.206	0.052	1948.6	0.000
		10	0.239	0.092	2133.6	0.000
		11	0.256	0.092	2346.4	0.000
		12	0.222	0.030	2506.0	0.000
		13	0.205	0.020	2642.7	0.000
		14	0.191	0.015	2761.0	0.000
		15	0.217	0.062	2913.7	0.000

Correlogram of ABS_RETURN_BUX

Date: 05/24/14 Time: 12:25
Sample: 1/02/2002 5/22/2014
Included observations: 3231

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.244	0.244	192.46	0.000
		2	0.206	0.156	330.23	0.000
		3	0.223	0.155	490.54	0.000
		4	0.204	0.112	625.01	0.000
		5	0.195	0.093	748.19	0.000
		6	0.201	0.092	879.38	0.000
		7	0.179	0.058	982.77	0.000
		8	0.186	0.068	1095.3	0.000
		9	0.204	0.083	1230.3	0.000
		10	0.200	0.072	1360.6	0.000
		11	0.165	0.025	1448.5	0.000
		12	0.177	0.043	1549.7	0.000
		13	0.192	0.060	1669.7	0.000
		14	0.179	0.040	1773.8	0.000
		15	0.176	0.036	1874.3	0.000

Correlogram of ABS_RETURN_DAX

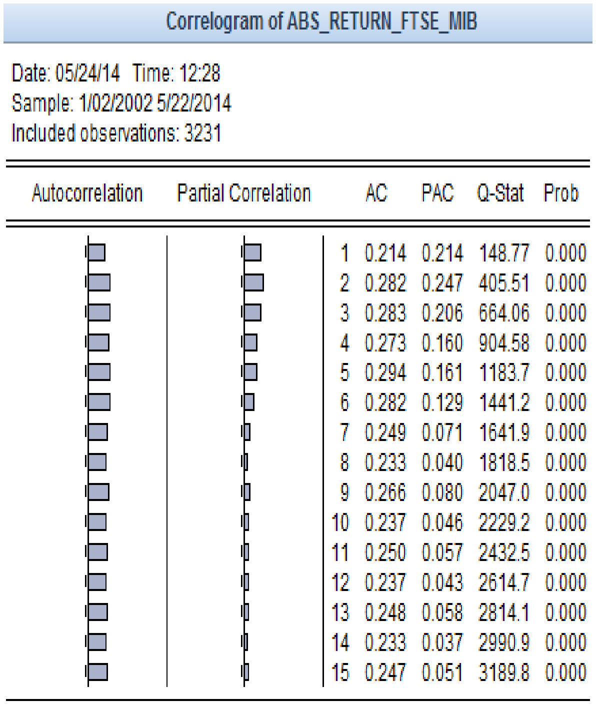
Date: 05/24/14 Time: 12:27
Sample: 1/02/2002 5/22/2014
Included observations: 3231

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.217	0.217	151.78	0.000
		2	0.305	0.270	451.99	0.000
		3	0.307	0.227	757.68	0.000
		4	0.281	0.156	1014.1	0.000
		5	0.300	0.152	1306.4	0.000
		6	0.261	0.087	1526.9	0.000
		7	0.294	0.114	1807.9	0.000
		8	0.273	0.083	2048.5	0.000
		9	0.296	0.104	2332.9	0.000
		10	0.259	0.050	2550.3	0.000
		11	0.282	0.071	2807.8	0.000
		12	0.257	0.037	3022.5	0.000
		13	0.245	0.019	3216.7	0.000
		14	0.253	0.027	3424.3	0.000
		15	0.219	-0.008	3579.9	0.000

Correlogram of ABS_RETURN_FTSE_100

Date: 05/24/14 Time: 12:27
Sample: 1/02/2002 5/22/2014
Included observations: 3231

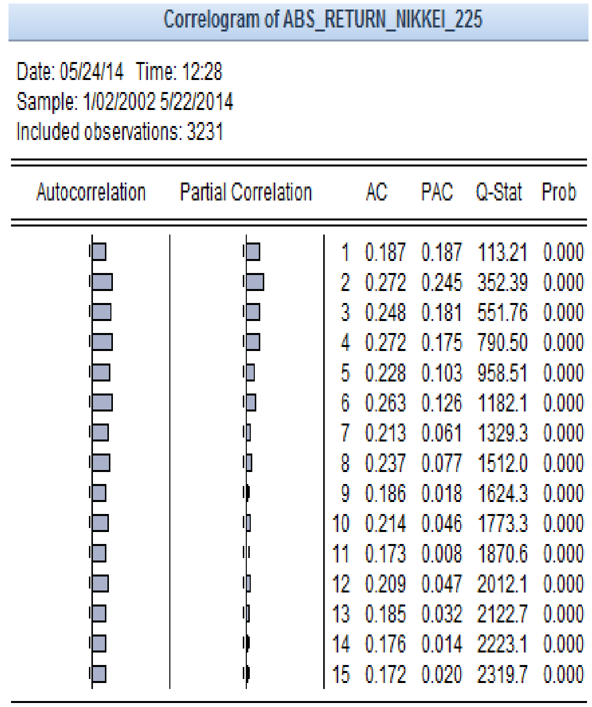
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.270	0.270	235.12	0.000
		2	0.326	0.273	578.55	0.000
		3	0.327	0.220	923.77	0.000
		4	0.295	0.142	1204.9	0.000
		5	0.336	0.171	1570.7	0.000
		6	0.292	0.097	1846.4	0.000
		7	0.263	0.045	2071.1	0.000
		8	0.272	0.055	2311.6	0.000
		9	0.277	0.068	2560.2	0.000
		10	0.284	0.075	2821.9	0.000
		11	0.277	0.062	3070.3	0.000
		12	0.263	0.043	3295.2	0.000
		13	0.277	0.058	3543.4	0.000
		14	0.236	0.001	3724.6	0.000
		15	0.250	0.019	3928.1	0.000



Correlogram of ABS_RETURN_FTSE_MIB

Date: 05/24/14 Time: 12:28
Sample: 1/02/2002 5/22/2014
Included observations: 3231

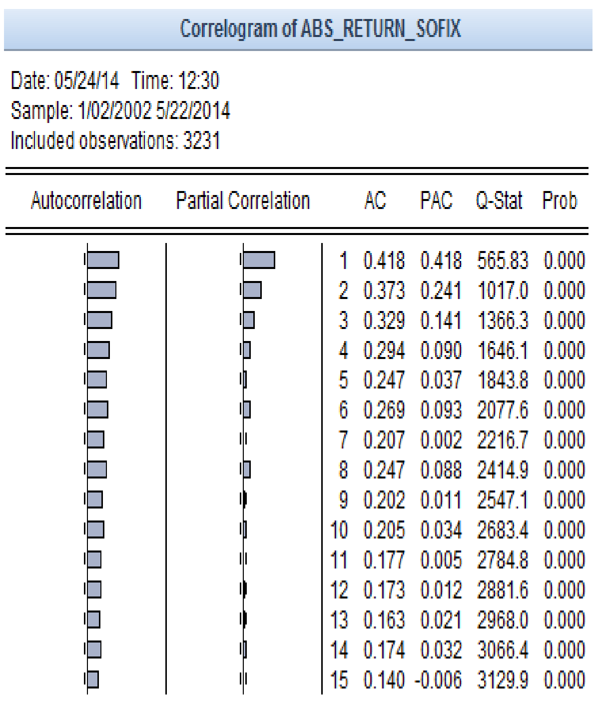
	AC	PAC	Q-Stat	Prob
1	0.214	0.214	148.77	0.000
2	0.282	0.247	405.51	0.000
3	0.283	0.206	664.06	0.000
4	0.273	0.160	904.58	0.000
5	0.294	0.161	1183.7	0.000
6	0.282	0.129	1441.2	0.000
7	0.249	0.071	1641.9	0.000
8	0.233	0.040	1818.5	0.000
9	0.266	0.080	2047.0	0.000
10	0.237	0.046	2229.2	0.000
11	0.250	0.057	2432.5	0.000
12	0.237	0.043	2614.7	0.000
13	0.248	0.058	2814.1	0.000
14	0.233	0.037	2990.9	0.000
15	0.247	0.051	3189.8	0.000



Correlogram of ABS_RETURN_NIKKEI_225

Date: 05/24/14 Time: 12:28
Sample: 1/02/2002 5/22/2014
Included observations: 3231

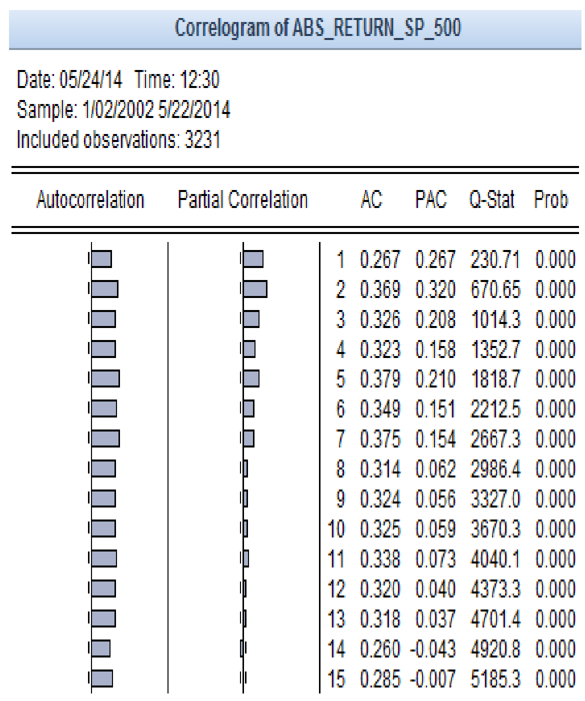
	AC	PAC	Q-Stat	Prob
1	0.187	0.187	113.21	0.000
2	0.272	0.245	352.39	0.000
3	0.248	0.181	551.76	0.000
4	0.272	0.175	790.50	0.000
5	0.228	0.103	958.51	0.000
6	0.263	0.126	1182.1	0.000
7	0.213	0.061	1329.3	0.000
8	0.237	0.077	1512.0	0.000
9	0.186	0.018	1624.3	0.000
10	0.214	0.046	1773.3	0.000
11	0.173	0.008	1870.6	0.000
12	0.209	0.047	2012.1	0.000
13	0.185	0.032	2122.7	0.000
14	0.176	0.014	2223.1	0.000
15	0.172	0.020	2319.7	0.000



Correlogram of ABS_RETURN_SOFIX

Date: 05/24/14 Time: 12:30
Sample: 1/02/2002 5/22/2014
Included observations: 3231

	AC	PAC	Q-Stat	Prob
1	0.418	0.418	565.83	0.000
2	0.373	0.241	1017.0	0.000
3	0.329	0.141	1366.3	0.000
4	0.294	0.090	1646.1	0.000
5	0.247	0.037	1843.8	0.000
6	0.269	0.093	2077.6	0.000
7	0.207	0.002	2216.7	0.000
8	0.247	0.088	2414.9	0.000
9	0.202	0.011	2547.1	0.000
10	0.205	0.034	2683.4	0.000
11	0.177	0.005	2784.8	0.000
12	0.173	0.012	2881.6	0.000
13	0.163	0.021	2968.0	0.000
14	0.174	0.032	3066.4	0.000
15	0.140	-0.006	3129.9	0.000



Correlogram of ABS_RETURN_SP_500

Date: 05/24/14 Time: 12:30
Sample: 1/02/2002 5/22/2014
Included observations: 3231

	AC	PAC	Q-Stat	Prob
1	0.267	0.267	230.71	0.000
2	0.369	0.320	670.65	0.000
3	0.326	0.208	1014.3	0.000
4	0.323	0.158	1352.7	0.000
5	0.379	0.210	1818.7	0.000
6	0.349	0.151	2212.5	0.000
7	0.375	0.154	2667.3	0.000
8	0.314	0.062	2986.4	0.000
9	0.324	0.056	3327.0	0.000
10	0.325	0.059	3670.3	0.000
11	0.338	0.073	4040.1	0.000
12	0.320	0.040	4373.3	0.000
13	0.318	0.037	4701.4	0.000
14	0.260	-0.043	4920.8	0.000
15	0.285	-0.007	5185.3	0.000

Figure 10. Absolute returns corellogram

APPENDIX 11

Variance Ratio Test on RETURN_BET_C

Null Hypothesis: RETURN_BET_C is a martingale
Date: 05/24/14 Time: 16:49
Sample: 1/02/2002 5/22/2014
Included observations: 3230 (after adjustments)
Heteroskedasticity robust standard error estimates
User-specified lags: 2 4 8 16

Joint Tests	Value	df	Probability
Max |z| (at period 2)*	10.37315	3230	0.0000

Individual Tests

Period	Var. Ratio	Std. Error	z-Statistic	Probability
2	0.537740	0.044563	-10.37315	0.0000
4	0.281595	0.078244	-9.181643	0.0000
8	0.132905	0.110311	-7.860431	0.0000
16	0.067472	0.147196	-6.335278	0.0000

*Probability approximation using studentized maximum modulus with parameter value 4 and infinite degrees of freedom

Test Details (Mean = -3.47681534859e-06)

Period	Variance	Var. Ratio	Obs.
1	0.00041	--	3230
2	0.00022	0.53774	3229
4	0.00012	0.28159	3227
8	5.5E-05	0.13290	3223
16	2.8E-05	0.06747	3215

Variance Ratio Test on RETURN_BUX

Null Hypothesis: RETURN_BUX is a martingale
Date: 05/24/14 Time: 16:50
Sample: 1/02/2002 5/22/2014
Included observations: 3230 (after adjustments)
Heteroskedasticity robust standard error estimates
User-specified lags: 2 4 8 16

Joint Tests	Value	df	Probability
Max |z| (at period 2)*	13.78791	3230	0.0000

Individual Tests

Period	Var. Ratio	Std. Error	z-Statistic	Probability
2	0.547839	0.032794	-13.78791	0.0000
4	0.237670	0.058410	-13.05129	0.0000
8	0.127081	0.084352	-10.34853	0.0000
16	0.063326	0.119971	-7.807518	0.0000

*Probability approximation using studentized maximum modulus with parameter value 4 and infinite degrees of freedom

Test Details (Mean = 1.35370148545e-06)

Period	Variance	Var. Ratio	Obs.
1	0.00049	--	3230
2	0.00027	0.54784	3229
4	0.00012	0.23767	3227
8	6.2E-05	0.12708	3223
16	3.1E-05	0.06333	3215

Variance Ratio Test on RETURN_DAX

Null Hypothesis: RETURN_DAX is a martingale
Date: 05/24/14 Time: 16:52
Sample: 1/02/2002 5/22/2014
Included observations: 3230 (after adjustments)
Heteroskedasticity robust standard error estimates
User-specified lags: 2 4 8 16

Joint Tests	Value	df	Probability
Max |z| (at period 2)*	14.87617	3230	0.0000

Individual Tests

Period	Var. Ratio	Std. Error	z-Statistic	Probability
2	0.494616	0.033973	-14.87617	0.0000
4	0.239734	0.060048	-12.66090	0.0000
8	0.120224	0.089642	-9.814330	0.0000
16	0.060632	0.129250	-7.267833	0.0000

*Probability approximation using studentized maximum modulus with parameter value 4 and infinite degrees of freedom

Test Details (Mean = -5.34052055234e-06)

Period	Variance	Var. Ratio	Obs.
1	0.00048	--	3230
2	0.00024	0.49462	3229
4	0.00012	0.23973	3227
8	5.8E-05	0.12022	3223
16	2.9E-05	0.06063	3215

Variance Ratio Test on RETURN_FTSE_100

Null Hypothesis: RETURN_FTSE_100 is a martingale
Date: 05/24/14 Time: 16:52
Sample: 1/02/2002 5/22/2014
Included observations: 3230 (after adjustments)
Heteroskedasticity robust standard error estimates
User-specified lags: 2 4 8 16

Joint Tests	Value	df	Probability
Max |z| (at period 2)*	13.90972	3230	0.0000

Individual Tests

Period	Var. Ratio	Std. Error	z-Statistic	Probability
2	0.488516	0.036772	-13.90972	0.0000
4	0.219040	0.066737	-11.70206	0.0000
8	0.115065	0.103257	-8.570192	0.0000
16	0.058527	0.148685	-6.332012	0.0000

*Probability approximation using studentized maximum modulus with parameter value 4 and infinite degrees of freedom

Test Details (Mean = -5.92404947165e-06)

Period	Variance	Var. Ratio	Obs.
1	0.00032	--	3230
2	0.00016	0.48852	3229
4	7.0E-05	0.21904	3227
8	3.7E-05	0.11506	3223
16	1.9E-05	0.05853	3215

Variance Ratio Test on RETURN_FTSE_MIB

Null Hypothesis: RETURN_FTSE_MIB is a martingale
Date: 05/24/14 Time: 16:54
Sample: 1/02/2002 5/22/2014
Included observations: 3230 (after adjustments)
Heteroskedasticity robust standard error estimates
User-specified lags: 2 4 8 16

Joint Tests	Value	df	Probability
Max |z| (at period 2)*	15.58898	3230	0.0000

Individual Tests

Period	Var. Ratio	Std. Error	z-Statistic	Probability
2	0.500461	0.032044	-15.58898	0.0000
4	0.236411	0.056086	-13.61448	0.0000
8	0.120067	0.085682	-10.26973	0.0000
16	0.058225	0.123369	-7.633796	0.0000

*Probability approximation using studentized maximum modulus with parameter value 4 and infinite degrees of freedom

Test Details (Mean = -1.02773375822e-05)

Period	Variance	Var. Ratio	Obs.
1	0.00047	--	3230
2	0.00023	0.50046	3229
4	0.00011	0.23641	3227
8	5.6E-05	0.12007	3223
16	2.7E-05	0.05822	3215

Variance Ratio Test on RETURN_NIKKEI_225

Null Hypothesis: RETURN_NIKKEI_225 is a martingale
Date: 05/24/14 Time: 16:54
Sample: 1/02/2002 5/22/2014
Included observations: 3230 (after adjustments)
Heteroskedasticity robust standard error estimates
User-specified lags: 2 4 8 16

Joint Tests	Value	df	Probability
Max |z| (at period 2)*	16.01136	3230	0.0000

Individual Tests

Period	Var. Ratio	Std. Error	z-Statistic	Probability
2	0.473389	0.032890	-16.01136	0.0000
4	0.236127	0.058615	-13.03203	0.0000
8	0.121883	0.091650	-9.581151	0.0000
16	0.060024	0.132779	-7.079245	0.0000

*Probability approximation using studentized maximum modulus with parameter value 4 and infinite degrees of freedom

Test Details (Mean = 3.94353493562e-06)

Period	Variance	Var. Ratio	Obs.
1	0.00048	--	3230
2	0.00023	0.47339	3229
4	0.00011	0.23613	3227
8	5.9E-05	0.12188	3223
16	2.9E-05	0.06002	3215

Variance Ratio Test on RETURN_SOFIX

Null Hypothesis: RETURN_SOFIX is a martingale
Date: 05/24/14 Time: 16:56
Sample: 1/02/2002 5/22/2014
Included observations: 3230 (after adjustments)
Heteroskedasticity robust standard error estimates
User-specified lags: 2 4 8 16

Joint Tests	Value	df	Probability
Max |z| (at period 2)*	10.54322	3230	0.0000

Individual Tests

Period	Var. Ratio	Std. Error	z-Statistic	Probability
2	0.492723	0.048114	-10.54322	0.0000
4	0.256709	0.082929	-8.963015	0.0000
8	0.130626	0.116445	-7.465947	0.0000
16	0.063665	0.158012	-5.925730	0.0000

*Probability approximation using studentized maximum modulus with parameter value 4 and infinite degrees of freedom

Test Details (Mean = -3.80508394084e-07)

Period	Variance	Var. Ratio	Obs.
1	0.00033	--	3230
2	0.00016	0.49272	3229
4	8.4E-05	0.25671	3227
8	4.3E-05	0.13063	3223
16	2.1E-05	0.06367	3215

Variance Ratio Test on RETURN_SP_500

Null Hypothesis: RETURN_SP_500 is a martingale
Date: 05/24/14 Time: 16:56
Sample: 1/02/2002 5/22/2014
Included observations: 3230 (after adjustments)
Heteroskedasticity robust standard error estimates
User-specified lags: 2 4 8 16

Joint Tests	Value	df	Probability
Max |z| (at period 2)*	12.46981	3230	0.0000

Individual Tests

Period	Var. Ratio	Std. Error	z-Statistic	Probability
2	0.451375	0.043996	-12.46981	0.0000
4	0.224858	0.077526	-9.998537	0.0000
8	0.106031	0.114849	-7.783887	0.0000
16	0.054117	0.166225	-5.690365	0.0000

*Probability approximation using studentized maximum modulus with parameter value 4 and infinite degrees of freedom

Test Details (Mean = -2.09868919077e-06)

Period	Variance	Var. Ratio	Obs.
1	0.00037	--	3230
2	0.00017	0.45137	3229
4	8.3E-05	0.22486	3227
8	3.9E-05	0.10603	3223
16	2.0E-05	0.05412	3215

Figure 11. VR test results

APPENDIX 12

	0	1	2	4	5	n / m
Akaike	-5.555547	-5.563302	-5.555593	-5.555286	-5.555559	0
Schwarz	-5.553665	-5.559538	-5.551829	-5.551522	-5.551795	
Log likelihood	8975.986	8989.514	8977.061	8976.564	8977.006	
R^2	0.000000	0.008339	0.000665	0.000358	0.000631	
Akaike	**-5.563557**	-5.562656	-5.563252	-5.563410	-5.563003	1
Schwarz	**-5.559792**	-5.558891	-5.557605	-5.557762	-5.559238	
Log likelihood	**8987.145**	8985.689	8987.652	8987.907	8986.249	
R^2	**0.008692**	0.007798	0.009003	0.009159	0.008142	
Akaike	-5.555336	-5.562506	-5.554577	-5.555142	-5.555366	2
Schwarz	-5.551570	-5.558740	-5.550811	-5.549493	-5.549718	
Log likelihood	8971.089	8982.666	8969.865	8971.776	8972.139	
R^2	0.000637	0.007778	-0.000121	0.001063	0.001287	
Akaike	-5.554477	-5.562305	-5.554552	-5.554661	-5.554634	4
Schwarz	-5.550710	-5.556653	-5.548901	-5.549010	-5.548983	
R^2	8964.149	8977.779	8965.270	8965.446	8965.403	
R-squared	0.000386	0.008794	0.001080	0.001189	0.001162	
Akaike	-5.555130	-5.562331	-5.555207	-5.554977	-5.561991	5
Schwarz	-5.551361	-5.558563	-5.549554	-5.549324	-5.558222	
Log likelihood	8962.424	8974.041	8963.549	8963.178	8973.491	
R-squared	0.000668	0.007839	0.001365	0.001135	0.007501	

Figure 12.1. ARMA - BET-C

	0	1	2	4	6	n / m
Akaike	-5.442397	-5.443294	-5.445023	-5.448646	-5.444391	0
Schwarz	-5.440515	-5.441412	-5.443141	-5.446764	-5.442509	
Log likelihood	8793.192	8794.641	8797.435	8803.288	8796.413	
R^2	0.000000	0.000897	0.002623	0.006230	0.001992	
Akaike	-5.442884	-5.443374	-5.445325	-5.448875	-5.444751	1
Schwarz	-5.441001	-5.439609	-5.441560	-5.445110	-5.440986	
Log likelihood	8791.257	8793.049	8796.200	8801.933	8795.273	
R^2	0.000753	0.001861	0.003807	0.007337	0.003235	
Akaike	-5.445593	-5.446146	**-5.453844**	-5.450685	-5.446827	2
Schwarz	-5.443711	-5.442380	**-5.450078**	-5.446919	-5.443061	
Log likelihood	8792.911	8794.803	**8807.231**	8802.131	8795.902	
R^2	0.003111	0.004279	**0.011914**	0.008788	0.004956	
Akaike	-5.448318	-5.448945	-5.450289	-5.449646	-5.449386	4
Schwarz	-5.446435	-5.445177	-5.446521	-5.445878	-5.445618	
R^2	8791.862	8793.872	8796.041	8795.004	8794.584	
R-squared	0.006436	0.007673	0.009006	0.008369	0.008111	
Akaike	-5.444134	-5.444801	-5.446051	-5.449373	-5.445155	6
Schwarz	-5.442249	-5.441031	-5.442281	-5.445603	-5.441385	
Log likelihood	8779.666	8781.741	8783.757	8789.114	8782.312	
R-squared	0.002171	0.003454	0.004699	0.008000	0.003806	

Figure 12.2. ARMA - BUX

	0	1	3	4	5	n / m
Akaike	-5.512637	-5.513193	-5.514202	-5.512768	-5.515060	0
Schwarz	-5.510755	-5.511311	-5.512320	-5.510886	-5.513178	
Log likelihood	8906.666	8907.564	8909.193	8906.877	8910.579	
R^2	0.000000	0.000556	0.001563	0.000131	0.002419	
Akaike	-5.513365	-5.514051	-5.514475	-5.512958	-5.515292	1
Schwarz	-5.511482	-5.510286	-5.510710	-5.509193	-5.511527	
Log likelihood	8905.084	8907.192	8907.877	8905.427	8909.197	
R^2	0.000542	0.001845	0.002269	0.000754	0.003084	
Akaike	-5.514194	-5.514294	-5.513902	-5.513845	**-5.516204**	3
Schwarz	-5.512310	-5.510527	-5.510135	-5.510078	**-5.512437**	
Log likelihood	8900.908	8902.070	8901.438	8901.345	**8905.153**	
R^2	0.001523	0.002242	0.001851	0.001793	**0.004146**	
Akaike	-5.512506	-5.512527	-5.513626	-5.513998	-5.514456	4
Schwarz	-5.510622	-5.508759	-5.509858	-5.510230	-5.510688	
R^2	8895.428	8896.463	8898.235	8898.835	8899.574	
R-squared	0.000146	0.000787	0.001884	0.002255	0.002712	
Akaike	-5.514540	-5.514626	-5.515660	-5.514160	-5.515034	5
Schwarz	-5.512656	-5.510857	-5.511891	-5.510391	-5.511266	
Log likelihood	8895.953	8897.092	8898.759	8896.339	8897.750	
R-squared	0.002366	0.003070	0.004099	0.002604	0.003476	

Figure 12.3. ARMA - DAX

	0	1	3	4	5	n / m
Akaike	-5.535881	-5.536004	-5.538186	-5.537506	-5.540864	0
Schwarz	-5.533999	-5.534122	-5.536304	-5.535624	-5.538982	
Log likelihood	8944.215	8944.414	8947.940	8946.840	8952.266	
R^2	0.000000	0.000123	0.002303	0.001624	0.004971	
Akaike	-5.536350	-5.536432	-5.538105	-5.537319	-5.540629	1
Schwarz	-5.534467	-5.532667	-5.534340	-5.533554	-5.536864	
Log likelihood	8942.205	8943.337	8946.039	8944.770	8950.116	
R^2	0.000108	0.000808	0.002479	0.001695	0.004993	
Akaike	-5.538074	-5.537623	-5.537500	-5.539203	**-5.542423**	3
Schwarz	-5.536191	-5.533857	-5.533733	-5.535436	**-5.538656**	
Log likelihood	8939.451	8939.724	8939.525	8942.273	**8947.470**	
R^2	0.002319	0.002488	0.002364	0.004062	**0.007264**	
Akaike	-5.537186	-5.536653	-5.539038	-5.540564	-5.541494	4
Schwarz	-5.535302	-5.532885	-5.535270	-5.536796	-5.537726	
R^2	8935.249	8935.390	8939.237	8941.699	8943.200	
R-squared	0.001706	0.001793	0.004171	0.005689	0.006613	
Akaike	-5.539871	-5.539370	-5.541624	-5.540846	-5.540888	5
Schwarz	-5.537987	-5.535602	-5.537855	-5.537077	-5.537119	
Log likelihood	8936.812	8937.004	8940.639	8939.384	8939.452	
R-squared	0.004655	0.004773	0.007013	0.006240	0.006282	

Figure 12.4. ARMA - FTSE MIB

	0	1	3	4	5	n / m
Akaike	-5.958670	-5.961901	-5.967217	-5.964259	-5.962085	0
Schwarz	-5.956788	-5.960019	-5.965335	-5.962378	-5.960203	
Log likelihood	9627.231	9632.451	9641.040	9636.261	9632.748	
R^2	0.000000	0.003226	0.008511	0.005574	0.003409	
Akaike	-5.962092	-5.965213	-5.969651	-5.966147	-5.964646	1
Schwarz	-5.960210	-5.961448	-5.965886	-5.962383	-5.960881	
Log likelihood	9629.779	9635.819	9642.986	9637.328	9634.902	
R^2	0.002989	0.006711	0.011109	0.007638	0.006147	
Akaike	-5.966488	-5.968807	-5.969132	-5.971420	-5.969386	3
Schwarz	-5.964604	-5.965040	-5.965365	-5.967653	-5.965619	
Log likelihood	9630.911	9635.654	9636.179	9639.872	9636.588	
R^2	0.007907	0.010818	0.011140	0.013400	0.011391	
Akaike	-5.964314	-5.965859	**-5.971979**	-5.964249	-5.966633	4
Schwarz	-5.962431	-5.962091	**-5.968212**	-5.960481	-5.962865	
R^2	9624.421	9627.914	**9637.789**	9625.315	9629.163	
R-squared	0.005918	0.008067	**0.014119**	0.006468	0.008835	
Akaike	-5.961377	-5.963816	-5.969496	-5.966012	-5.961385	5
Schwarz	-5.959493	-5.960047	-5.965727	-5.962244	-5.957617	
Log likelihood	9616.702	9621.635	9630.797	9625.178	9617.715	
R-squared	0.003266	0.006309	0.011938	0.008490	0.003891	

	3	3 si 5	1 si 3	1, 3 si 5	n / m
Akaike	-5.975925				1, 4 si 5
Schwarz	-5.968387				
Log likelihood	9643.167				
R^2	0.019486				
Akaike	-5.974082	-5.976048			1 si 4
Schwarz	-5.968430	-5.968512			
Log likelihood	9642.181	9646.353			
R^2	0.016799	0.019338			
Akaike	-5.974204		-5.976292		4 si 5
Schwarz	-5.968551		-5.968754		
Log likelihood	9639.391		9643.758		
R^2	0.017188		0.019846		
Akaike	**-5.971979**	-5.974168	-5.973894	**-5.976407**	4
Schwarz	**-5.968212**	-5.968517	-5.968242	**-5.968872**	
R^2	**9637.789**	9642.321	9641.878	**9646.933**	
R-squared	**0.014119**	0.016884	0.016615	**0.019691**	

Figure 12.5. ARMA - FTSE 100

	0	1	2	3	4	n / m
Akaike	-5.535568	-5.537541	-5.535632	-5.538276	-5.535718	0
Schwarz	-5.533686	-5.535659	-5.533750	-5.536394	-5.533836	
Log likelihood	8943.709	8946.897	8943.814	8948.085	8943.953	
R^2	0.000000	0.001971	0.000065	0.002705	0.000151	
Akaike	-5.537348	-5.537645	-5.536764	-5.539360	-5.536875	1
Schwarz	-5.535465	-5.533880	-5.532999	-5.535595	-5.533110	
Log likelihood	8943.816	8945.296	8943.874	8948.067	8944.053	
R^2	0.002002	0.002916	0.002038	0.004625	0.002148	
Akaike	-5.536370	-5.537778	-5.535993	-5.538431	-5.535958	2
Schwarz	-5.534487	-5.534013	-5.532227	-5.534665	-5.532192	
Log likelihood	8939.469	8942.743	8939.861	8943.797	8939.804	
R^2	0.000077	0.002103	0.000319	0.002754	0.000285	
Akaike	-5.538669	**-5.540014**	-5.538139	-5.538446	-5.538209	3
Schwarz	-5.536785	**-5.536248**	-5.534372	-5.534679	-5.534442	
R^2	8940.411	**8943.583**	8940.556	8941.051	8940.669	
R-squared	0.002627	**0.004585**	0.002717	0.003023	0.002786	
Akaike	-5.536582	-5.537982	-5.536083	-5.538652	-5.539219	4
Schwarz	-5.534698	-5.534215	-5.532315	-5.534884	-5.535451	
Log likelihood	8934.275	8937.534	8934.470	8938.614	8939.530	
R-squared	0.000150	0.002167	0.000271	0.002835	0.003401	

Figure 12.6. ARMA - NIKKEI 225

	0	1	2	5	8	n / m
Akaike	-5.882762	-5.899205	-5.882859	-5.883257	-5.884887	0
Schwarz	-5.880880	-5.897323	-5.880977	-5.881375	-5.883005	
Log likelihood	9504.602	9531.166	9504.758	9505.401	9508.035	
R^2	0.000000	0.016309	0.000097	0.000495	0.002123	
Akaike	-5.898290	-5.898680	-5.898653	-5.898463	**-5.899436**	1
Schwarz	-5.896407	-5.894916	-5.894888	-5.894699	**-5.895671**	
Log likelihood	9526.738	9528.369	9528.325	9528.018	**9529.589**	
R^2	0.015562	0.016556	0.016529	0.016342	**0.017298**	
Akaike	-5.882465	-5.898420	-5.881846	-5.882499	-5.884149	2
Schwarz	-5.880582	-5.894654	-5.878081	-5.878733	-5.880383	
Log likelihood	9498.240	9524.999	9498.241	9499.294	9501.958	
R^2	0.000100	0.016536	0.000101	0.000753	0.002401	
Akaike	-5.882109	-5.898298	-5.881751	-5.882263	-5.883868	5
Schwarz	-5.880224	-5.894530	-5.877982	-5.878494	-5.880099	
R^2	9488.841	9515.955	9489.265	9490.090	9492.679	
R-squared	0.000516	0.017176	0.000778	0.001289	0.002891	
Akaike	-5.883188	-5.898672	-5.882830	-5.883308	-5.882979	8
Schwarz	-5.881302	-5.894900	-5.879058	-5.879536	-5.879208	
Log likelihood	9481.757	9507.709	9482.181	9482.951	9482.421	
R-squared	0.002265	0.018204	0.002527	0.003004	0.002676	

Figure 12.7. ARMA - S&P 500

	0	1	2	4	6	n / m
Akaike	-5.771775	-5.780023	-5.783069	-5.776710	-5.774912	0
Schwarz	-5.769893	-5.778141	-5.781187	-5.774829	-5.773030	
Log likelihood	9325.303	9338.627	9343.548	9333.276	9330.370	
R^2	0.000000	0.008214	0.011231	0.004923	0.003132	
Akaike	-5.781823	-5.798788	-5.790819	-5.786786	-5.786786	1
Schwarz	-5.779940	-5.795023	-5.787054	-5.783021	-5.783022	
Log likelihood	9338.644	9367.042	9354.173	9347.660	9347.660	
R^2	0.010304	0.027555	0.019775	0.015813	0.015814	
Akaike	-5.788402	-5.796229	**-5.803634**	-5.791544	-5.790308	2
Schwarz	-5.786519	-5.792463	**-5.799868**	-5.787779	-5.786542	
Log likelihood	9346.375	9360.012	**9371.967**	9352.449	9350.452	
R^2	0.012873	0.021176	**0.028397**	0.016579	0.015362	
Akaike	-5.780473	-5.789172	-5.791362	-5.791750	-5.782732	4
Schwarz	-5.778589	-5.785404	-5.787594	-5.787982	-5.778964	
R^2	9327.792	9342.829	9346.363	9346.988	9332.438	
R-squared	0.005553	0.014777	0.016932	0.017313	0.008412	
Akaike	-5.778089	-5.788266	-5.788331	-5.782323	-5.788015	6
Schwarz	-5.776204	-5.784497	-5.784561	-5.778554	-5.784245	
Log likelihood	9318.168	9335.580	9335.684	9325.996	9335.174	
R-squared	0.003685	0.014385	0.014449	0.008510	0.014137	

	2	2 si 5	1 si 2	1, 2 si 5	n / m
Akaike	-5.814182				1, 2 si 5
Schwarz	-5.806644				
Log likelihood	9382.275				
R^2	0.040529				
Akaike	-5.806947	**-5.819696**			1 si 2
Schwarz	-5.801298	**-5.812165**			
Log likelihood	9378.316	**9399.9**			
R^2	0.03221	**0.045062**			
Akaike	-5.804322		-5.811911		2 si 5
Schwarz	-5.798669		-5.804373		
Log likelihood	9365.371		9378.612		
R^2	0.030421		0.038347		
Akaike	**-5.803634**	-5.804049	-5.806926	-5.814646	2
Schwarz	**-5.799868**	-5.7984	-5.801277	-5.807114	
R^2	**9371.967**	9373.636	9378.282	9391.746	
R-squared	**0.028397**	0.029401	0.03219	0.040227	

Figure 12.8. ARMA - SOFIX

APPENDIX 13

Dependent Variable: RETURN_BET_C
Method: Least Squares
Date: 05/25/14 Time: 00:46
Sample (adjusted): 1/04/2002 5/22/2014
Included observations: 3230 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000526	0.000264	1.994930	0.0461
RETURN_BET_C(-1)	0.093221	0.017522	5.320100	0.0000

R-squared	0.008692	Mean dependent var	0.000581
Adjusted R-squared	0.008385	S.D. dependent var	0.015043
S.E. of regression	0.014980	Akaike info criterion	-5.563557
Sum squared resid	0.724383	Schwarz criterion	-5.559792
Log likelihood	8987.145	Hannan-Quinn criter.	-5.562208
F-statistic	28.30347	Durbin-Watson stat	2.003313
Prob(F-statistic)	0.000000		

Figure 13.1. AR(1) – BET-C

Dependent Variable: RETURN_BUX
Method: Least Squares
Date: 05/24/14 Time: 22:25
Sample (adjusted): 1/07/2002 5/22/2014
Included observations: 3229 after adjustments
Convergence achieved after 7 iterations
MA Backcast: 1/03/2002 1/04/2002

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RETURN_BUX(-2)	-0.821603	0.058591	-14.02265	0.0000
MA(2)	0.759593	0.066906	11.35309	0.0000

R-squared	0.011914	Mean dependent var	0.000296
Adjusted R-squared	0.011608	S.D. dependent var	0.015918
S.E. of regression	0.015825	Akaike info criterion	-5.453844
Sum squared resid	0.808130	Schwarz criterion	-5.450078
Log likelihood	8807.231	Hannan-Quinn criter.	-5.452494
Durbin-Watson stat	1.928910		

Figure 13.2. ARMA(2,2) – BUX

Dependent Variable: RETURN_DAX
Method: Least Squares
Date: 05/24/14 Time: 23:27
Sample (adjusted): 1/08/2002 5/22/2014
Included observations: 3228 after adjustments
Convergence achieved after 5 iterations
MA Backcast: 1/01/2002 1/07/2002

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RETURN_DAX(-3)	-0.041036	0.017585	-2.333569	0.0197
MA(5)	-0.052012	0.017585	-2.957750	0.0031

R-squared	0.004146	Mean dependent var	0.000192
Adjusted R-squared	0.003837	S.D. dependent var	0.015369
S.E. of regression	0.015339	Akaike info criterion	-5.516204
Sum squared resid	0.759039	Schwarz criterion	-5.512437
Log likelihood	8905.153	Hannan-Quinn criter.	-5.514854
Durbin-Watson stat	2.052431		

Figure 13.3. ARMA(3,5) – DAX

Dependent Variable: RETURN_FTSE_100
Method: Least Squares
Date: 05/25/14 Time: 02:24
Sample (adjusted): 1/09/2002 5/22/2014
Included observations: 3227 after adjustments
Convergence achieved after 6 iterations
MA Backcast: 1/02/2002 1/08/2002

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RETURN_FTSE_100(-4)	0.077791	0.017665	4.403690	0.0000
MA(1)	-0.055223	0.017566	-3.143675	0.0017
MA(3)	-0.094939	0.017521	-5.418468	0.0000
MA(5)	-0.056580	0.017488	-3.235391	0.0012

R-squared	0.019691	Mean dependent var	8.11E-05
Adjusted R-squared	0.018778	S.D. dependent var	0.012298
S.E. of regression	0.012182	Akaike info criterion	-5.976407
Sum squared resid	0.478330	Schwarz criterion	-5.968872
Log likelihood	9646.933	Hannan-Quinn criter.	-5.973707
Durbin-Watson stat	1.993533		

Figure 13.4. AR(4)MA(1)MA(3)MA(5) – FTSE 10

Dependent Variable: RETURN_FTSE_MIB
Method: Least Squares
Date: 05/25/14 Time: 00:22
Sample (adjusted): 1/08/2002 5/22/2014
Included observations: 3228 after adjustments
Convergence achieved after 6 iterations
MA Backcast: 1/01/2002 1/07/2002

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RETURN_FTSE_MIB(-3)	-0.048347	0.017584	-2.749433	0.0060
MA(5)	-0.072727	0.017570	-4.139407	0.0000

R-squared	0.007264	Mean dependent var	-0.000143
Adjusted R-squared	0.006956	S.D. dependent var	0.015192
S.E. of regression	0.015139	Akaike info criterion	-5.542423
Sum squared resid	0.739396	Schwarz criterion	-5.538656
Log likelihood	8947.470	Hannan-Quinn criter.	-5.541073
Durbin-Watson stat	2.018552		

Figure 13.5. ARMA(3,5) – FTSE MIB

Dependent Variable: RETURN_NIKKEI_225
Method: Least Squares
Date: 05/25/14 Time: 00:43
Sample (adjusted): 1/08/2002 5/22/2014
Included observations: 3228 after adjustments
Convergence achieved after 4 iterations
MA Backcast: 1/07/2002

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RETURN_NIKKEI_225(-3)	-0.050663	0.017577	-2.882340	0.0040
MA(1)	-0.043985	0.017595	-2.499861	0.0125

R-squared	0.004585	Mean dependent var	8.37E-05
Adjusted R-squared	0.004277	S.D. dependent var	0.015190
S.E. of regression	0.015158	Akaike info criterion	-5.540014
Sum squared resid	0.741179	Schwarz criterion	-5.536248
Log likelihood	8943.583	Hannan-Quinn criter.	-5.538665
Durbin-Watson stat	1.999490		

Figure 13.6. ARMA(3,1) – NIKKEI 225

Dependent Variable: RETURN_SOFIX
Method: Least Squares
Date: 05/25/14 Time: 11:55
Sample (adjusted): 1/07/2002 5/22/2014
Included observations: 3229 after adjustments
Convergence achieved after 9 iterations
MA Backcast: 12/31/2001 1/04/2002

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RETURN_SOFIX(-1)	0.088230	0.012913	6.832638	0.0000
RETURN_SOFIX(-2)	0.878080	0.019701	44.56972	0.0000
MA(2)	-0.805026	0.025074	-32.10617	0.0000
MA(5)	-0.106856	0.015035	-7.106910	0.0000

R-squared	0.045062	Mean dependent var	0.000514
Adjusted R-squared	0.044174	S.D. dependent var	0.013476
S.E. of regression	0.013175	Akaike info criterion	-5.819696
Sum squared resid	0.559829	Schwarz criterion	-5.812165
Log likelihood	9399.900	Hannan-Quinn criter.	-5.816997
Durbin-Watson stat	1.994088		

Figure 13.7. AR(1)AR(2)MA(2)MA(5) – SOFIX

Dependent Variable: RETURN_SP_500
Method: Least Squares
Date: 05/25/14 Time: 01:15
Sample (adjusted): 1/04/2002 5/22/2014
Included observations: 3230 after adjustments
Convergence achieved after 6 iterations
MA Backcast: 12/25/2001 1/03/2002

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RETURN_SP_500(-1)	-0.123455	0.017470	-7.066685	0.0000
MA(8)	0.040574	0.017596	2.305947	0.0212

R-squared	0.017298	Mean dependent var	0.000150
Adjusted R-squared	0.016994	S.D. dependent var	0.012773
S.E. of regression	0.012664	Akaike info criterion	-5.899436
Sum squared resid	0.517724	Schwarz criterion	-5.895671
Log likelihood	9529.589	Hannan-Quinn criter.	-5.898087
Durbin-Watson stat	2.007995		

Figure 13.8. ARMA(1,8) – S&P 500

APPENDIX 14

Correlogram of Residuals

Date: 05/25/14 Time: 22:28
Sample: 1/04/2002 5/22/2014
Included observations: 3230

	AC	PAC	Q-Stat	Prob
1	-0.002	-0.002	0.0097	0.921
2	0.017	0.017	0.9703	0.616
3	-0.004	-0.004	1.0205	0.796
4	-0.022	-0.023	2.6562	0.617
5	0.026	0.026	4.7657	0.445
6	0.027	0.028	7.0603	0.315
7	0.001	0.000	7.0639	0.422
8	0.037	0.036	11.460	0.177
9	0.020	0.021	12.703	0.176
10	0.028	0.028	15.255	0.123
11	0.009	0.007	15.505	0.161
12	0.029	0.029	18.249	0.108
13	0.018	0.018	19.321	0.113
14	0.052	0.050	28.035	0.014
15	0.050	0.049	36.249	0.002

Figure 14.1. AR(1) – BET-C correlogram

Correlogram of Residuals

Date: 05/25/14 Time: 23:14
Sample: 1/07/2002 5/22/2014
Included observations: 3229
Q-statistic probabilities adjusted for 1 ARMA term(s)

	AC	PAC	Q-Stat	Prob
1	0.035	0.035	3.9831	
2	0.014	0.012	4.5867	0.032
3	-0.019	-0.020	5.7141	0.057
4	0.024	0.026	7.6315	0.054
5	0.021	0.020	9.0541	0.060
6	-0.002	-0.004	9.0660	0.106
7	0.007	0.007	9.2098	0.162
8	-0.022	-0.022	10.716	0.152
9	-0.022	-0.022	12.258	0.140
10	-0.010	-0.008	12.563	0.183
11	0.019	0.019	13.731	0.186
12	0.003	0.002	13.765	0.246
13	-0.007	-0.006	13.903	0.307
14	-0.010	-0.008	14.244	0.357
15	-0.007	-0.007	14.421	0.419

Figure 14.2. ARMA(2,2) – BUX correlogram

Correlogram of Residuals

Date: 05/25/14 Time: 22:41
Sample: 1/08/2002 5/22/2014
Included observations: 3228
Q-statistic probabilities adjusted for 1 ARMA term(s)

	AC	PAC	Q-Stat	Prob
1	-0.026	-0.026	2.2540	
2	-0.017	-0.018	3.1732	0.075
3	-0.001	-0.002	3.1771	0.204
4	0.015	0.015	3.8926	0.273
5	0.001	0.001	3.8936	0.421
6	-0.013	-0.012	4.4327	0.489
7	0.013	0.013	4.9915	0.545
8	0.016	0.016	5.8401	0.559
9	-0.006	-0.005	5.9478	0.653
10	-0.011	-0.011	6.3693	0.702
11	0.037	0.036	10.863	0.368
12	0.004	0.005	10.910	0.451
13	-0.001	0.001	10.911	0.537
14	0.008	0.009	11.119	0.601
15	-0.013	-0.014	11.644	0.635

Figure 14.3. ARMA(3,5) – DAX correlogram

Correlogram of Residuals

Date: 05/25/14 Time: 22:41
Sample: 1/09/2002 5/22/2014
Included observations: 3227
Q-statistic probabilities adjusted for 3 ARMA term(s)

	AC	PAC	Q-Stat	Prob
1	0.003	0.003	0.0318	
2	-0.034	-0.034	3.7018	
3	0.001	0.001	3.7042	
4	-0.002	-0.003	3.7198	0.054
5	-0.000	-0.000	3.7202	0.156
6	-0.026	-0.026	5.9229	0.115
7	0.033	0.033	9.3609	0.053
8	0.029	0.027	12.072	0.034
9	-0.008	-0.006	12.289	0.056
10	-0.013	-0.011	12.842	0.076
11	-0.010	-0.010	13.153	0.107
12	-0.018	-0.020	14.249	0.114
13	0.009	0.011	14.536	0.150
14	-0.007	-0.008	14.699	0.197
15	-0.015	-0.017	15.463	0.217

Figure 14.4. AR(4)MA(1)MA(3)MA(5) – FTSE correlogram

Correlogram of Residuals

Date: 05/25/14 Time: 22:50
Sample: 1/08/2002 5/22/2014
Included observations: 3228
Q-statistic probabilities adjusted for 1 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.010	-0.010	0.2948	
		2	-0.017	-0.017	1.2140	0.271
		3	-0.000	-0.001	1.2143	0.545
		4	0.041	0.041	6.7321	0.081
		5	0.002	0.003	6.7464	0.150
		6	0.004	0.006	6.8043	0.236
		7	0.022	0.023	8.4130	0.209
		8	0.027	0.025	10.689	0.153
		9	-0.009	-0.008	10.939	0.205
		10	-0.026	-0.026	13.197	0.154
		11	0.025	0.022	15.146	0.127
		12	0.025	0.022	17.180	0.103
		13	0.021	0.022	18.562	0.100
		14	0.005	0.008	18.645	0.135
		15	-0.008	-0.010	18.876	0.170

Figure 14.5. ARMA(3,5) – FTSE MIB correlogram

Correlogram of Residuals

Date: 05/25/14 Time: 22:50
Sample: 1/08/2002 5/22/2014
Included observations: 3228
Q-statistic probabilities adjusted for 1 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.000	-0.000	0.0005	
		2	0.010	0.010	0.2955	0.587
		3	-0.000	-0.000	0.2959	0.862
		4	0.013	0.013	0.8337	0.841
		5	0.011	0.011	1.2250	0.874
		6	-0.016	-0.016	2.0272	0.845
		7	0.010	0.010	2.3494	0.885
		8	-0.015	-0.015	3.0809	0.877
		9	-0.018	-0.018	4.1137	0.847
		10	0.001	0.002	4.1172	0.904
		11	0.015	0.015	4.8396	0.902
		12	0.011	0.011	5.2569	0.918
		13	0.023	0.024	6.9912	0.858
		14	-0.011	-0.012	7.4054	0.880
		15	0.003	0.001	7.4264	0.917

Figure 14.6. ARMA(3,1) – NIKKEI 225 correlogram

Correlogram of Residuals

Date: 05/25/14 Time: 22:58
Sample: 1/07/2002 5/22/2014
Included observations: 3229
Q-statistic probabilities adjusted for 2 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.003	0.003	0.0222	
		2	0.007	0.007	0.1666	
		3	-0.026	-0.026	2.4098	0.121
		4	-0.010	-0.010	2.7589	0.252
		5	0.002	0.002	2.7715	0.428
		6	-0.009	-0.010	3.0389	0.551
		7	-0.042	-0.043	8.7748	0.118
		8	-0.003	-0.003	8.8070	0.185
		9	-0.004	-0.004	8.8638	0.263
		10	-0.011	-0.014	9.2785	0.319
		11	0.050	0.049	17.353	0.043
		12	0.013	0.012	17.867	0.057
		13	0.013	0.011	18.439	0.072
		14	0.004	0.004	18.484	0.102
		15	0.008	0.009	18.685	0.133

Figure 14.7. AR(1)AR(2)MA(2)MA(5) – SOFIX correlogram

Correlogram of Residuals

Date: 05/25/14 Time: 22:58
Sample: 1/04/2002 5/22/2014
Included observations: 3230
Q-statistic probabilities adjusted for 1 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.004	-0.004	0.0576	
		2	-0.031	-0.031	3.1638	0.075
		3	0.005	0.005	3.2558	0.196
		4	-0.012	-0.013	3.7309	0.292
		5	-0.030	-0.030	6.6198	0.157
		6	-0.011	-0.012	7.0172	0.219
		7	-0.026	-0.028	9.2681	0.159
		8	0.001	0.000	9.2739	0.234
		9	-0.018	-0.021	10.349	0.241
		10	0.020	0.019	11.701	0.231
		11	0.018	0.016	12.803	0.235
		12	-0.012	-0.012	13.269	0.276
		13	0.023	0.023	14.987	0.242
		14	-0.026	-0.028	17.183	0.191
		15	-0.029	-0.027	19.912	0.133

Figure 14.8. ARMA(1,8) – S&P 500 correlogram

APPENDIX 15

Correlogram of Residuals Squared

Date: 05/25/14 Time: 22:30
Sample: 1/04/2002 5/22/2014
Included observations: 3230

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.336	0.336	364.07	0.000
		2	0.246	0.150	559.90	0.000
		3	0.196	0.087	684.25	0.000
		4	0.155	0.048	761.62	0.000
		5	0.149	0.058	833.69	0.000
		6	0.120	0.026	880.46	0.000
		7	0.140	0.063	943.62	0.000
		8	0.137	0.049	1004.4	0.000
		9	0.152	0.064	1079.0	0.000
		10	0.180	0.084	1183.6	0.000
		11	0.226	0.121	1349.7	0.000
		12	0.189	0.041	1465.1	0.000
		13	0.158	0.017	1545.7	0.000
		14	0.118	-0.013	1590.6	0.000
		15	0.154	0.060	1667.6	0.000

Figure 15.1. AR(1) – BET-C Square return correlogram

Correlogram of Residuals Squared

Date: 05/25/14 Time: 22:30
Sample: 1/07/2002 5/22/2014
Included observations: 3229
Q-statistic probabilities adjusted for 1 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.341	0.341	376.59	
		2	0.229	0.128	546.39	0.000
		3	0.173	0.070	643.16	0.000
		4	0.137	0.045	704.27	0.000
		5	0.154	0.079	780.85	0.000
		6	0.186	0.104	893.38	0.000
		7	0.191	0.082	1010.9	0.000
		8	0.242	0.134	1200.4	0.000
		9	0.257	0.120	1415.0	0.000
		10	0.246	0.092	1611.1	0.000
		11	0.146	-0.030	1679.8	0.000
		12	0.143	0.023	1746.1	0.000
		13	0.195	0.093	1868.8	0.000
		14	0.171	0.026	1963.7	0.000
		15	0.172	0.023	2059.2	0.000

Figure 15.2. ARMA(2,2) – BUX Square return correlogram

Correlogram of Residuals Squared

Date: 07/01/12 Time: 16:18
Sample: 1/09/2002 1/03/2012
Included observations: 2605
Q-statistic probabilities adjusted for 1 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.176	0.176	80.432	
		2	0.209	0.184	194.40	0.000
		3	0.288	0.241	411.16	0.000
		4	0.145	0.049	465.96	0.000
		5	0.094	-0.020	488.86	0.000
		6	0.091	-0.016	510.65	0.000
		7	0.068	0.003	522.63	0.000
		8	0.104	0.071	550.70	0.000
		9	0.057	0.014	559.16	0.000
		10	0.069	0.022	571.45	0.000
		11	0.087	0.030	591.26	0.000
		12	0.062	0.013	601.33	0.000
		13	0.066	0.018	612.79	0.000
		14	0.083	0.033	630.81	0.000
		15	0.121	0.082	669.27	0.000
		16	0.061	-0.002	679.05	0.000
		17	0.065	-0.005	690.30	0.000
		18	0.134	0.068	737.17	0.000
		19	0.087	0.031	757.19	0.000
		20	0.080	0.021	773.97	0.000

Figure 15.3. ARMA(3,5) – DAX Square return correlogram

Correlogram of Residuals Squared

Date: 07/01/12 Time: 16:18
Sample: 1/07/2002 1/03/2012
Included observations: 2607
Q-statistic probabilities adjusted for 1 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.220	0.220	126.51	
		2	0.208	0.168	239.63	0.000
		3	0.237	0.175	386.66	0.000
		4	0.163	0.067	456.37	0.000
		5	0.180	0.087	540.88	0.000
		6	0.067	-0.048	552.69	0.000
		7	0.135	0.063	600.24	0.000
		8	0.138	0.059	649.81	0.000
		9	0.138	0.072	699.65	0.000
		10	0.167	0.081	772.31	0.000
		11	0.159	0.068	838.25	0.000
		12	0.160	0.046	905.62	0.000
		13	0.152	0.039	966.03	0.000
		14	0.208	0.105	1079.0	0.000
		15	0.157	0.033	1144.1	0.000
		16	0.109	-0.015	1175.5	0.000
		17	0.149	0.034	1233.5	0.000
		18	0.152	0.047	1294.3	0.000
		19	0.183	0.080	1382.3	0.000
		20	0.166	0.059	1454.7	0.000

Figure 15.4. AR(4)MA(1)MA(3)MA(5) – FTSE 100 Square return correlogram

Correlogram of Residuals Squared

Date: 05/25/14 Time: 22:52
Sample: 1/08/2002 5/22/2014
Included observations: 3228
Q-statistic probabilities adjusted for 1 ARMA term(s)

	AC	PAC	Q-Stat	Prob
1	0.179	0.179	103.27	
2	0.226	0.200	268.07	0.000
3	0.249	0.195	468.87	0.000
4	0.258	0.177	683.82	0.000
5	0.271	0.168	920.81	0.000
6	0.216	0.084	1072.0	0.000
7	0.173	0.017	1169.4	0.000
8	0.153	-0.014	1245.1	0.000
9	0.253	0.114	1452.9	0.000
10	0.187	0.045	1565.7	0.000
11	0.181	0.041	1671.9	0.000
12	0.214	0.082	1819.8	0.000
13	0.186	0.042	1931.5	0.000
14	0.178	0.015	2034.2	0.000
15	0.189	0.030	2150.1	0.000

Figure 15.5. ARMA(3,5) – FTSE MIB la Square return correlogram

Correlogram of Residuals Squared

Date: 05/25/14 Time: 22:52
Sample: 1/08/2002 5/22/2014
Included observations: 3228
Q-statistic probabilities adjusted for 1 ARMA term(s)

	AC	PAC	Q-Stat	Prob
1	0.225	0.225	163.09	
2	0.384	0.351	640.09	0.000
3	0.222	0.104	798.91	0.000
4	0.378	0.245	1261.7	0.000
5	0.193	0.026	1381.6	0.000
6	0.367	0.184	1817.4	0.000
7	0.183	0.007	1925.8	0.000
8	0.277	0.038	2173.6	0.000
9	0.189	0.032	2289.0	0.000
10	0.314	0.108	2608.1	0.000
11	0.177	0.013	2709.4	0.000
12	0.240	0.003	2895.9	0.000
13	0.190	0.038	3013.0	0.000
14	0.187	-0.040	3126.7	0.000
15	0.181	0.032	3233.1	0.000

Figure 15.6. ARMA(3,1) – NIKKEI 225 Square return correlogram

Correlogram of Residuals Squared

Date: 05/25/14 Time: 22:59
Sample: 1/07/2002 5/22/2014
Included observations: 3229
Q-statistic probabilities adjusted for 2 ARMA term(s)

	AC	PAC	Q-Stat	Prob
1	0.351	0.351	397.62	
2	0.325	0.230	738.41	
3	0.319	0.181	1067.6	0.000
4	0.262	0.083	1288.9	0.000
5	0.172	-0.024	1384.2	0.000
6	0.220	0.082	1541.1	0.000
7	0.146	-0.010	1610.5	0.000
8	0.254	0.164	1820.1	0.000
9	0.145	-0.029	1888.2	0.000
10	0.144	0.000	1955.7	0.000
11	0.154	0.025	2032.2	0.000
12	0.115	-0.021	2075.4	0.000
13	0.082	-0.006	2097.1	0.000
14	0.124	0.028	2146.7	0.000
15	0.083	0.005	2169.2	0.000

Figure 15.7. AR(1)AR(2)MA(2)MA(5) – SOFIX Square return correlogram

Correlogram of Residuals Squared

Date: 05/25/14 Time: 22:59
Sample: 1/04/2002 5/22/2014
Included observations: 3230
Q-statistic probabilities adjusted for 1 ARMA term(s)

	AC	PAC	Q-Stat	Prob
1	0.195	0.195	122.78	
2	0.405	0.382	653.64	0.000
3	0.213	0.110	800.70	0.000
4	0.293	0.129	1077.6	0.000
5	0.352	0.250	1479.6	0.000
6	0.300	0.139	1771.9	0.000
7	0.343	0.134	2152.7	0.000
8	0.248	0.043	2351.9	0.000
9	0.300	0.069	2643.9	0.000
10	0.277	0.070	2892.5	0.000
11	0.352	0.138	3293.5	0.000
12	0.298	0.071	3580.9	0.000
13	0.238	-0.042	3765.2	0.000
14	0.163	-0.134	3851.4	0.000
15	0.210	-0.043	3994.9	0.000

Figure 15.8.ARMA(1,8) – S&P 500 Square return correlogram

APPENDIX 16

Heteroskedasticity Test: ARCH

F-statistic	409.5185	Prob. F(1,3227)	0.0000
Obs*R-squared	363.6267	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 05/25/14 Time: 22:32
Sample (adjusted): 1/07/2002 5/22/2014
Included observations: 3229 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000149	1.34E-05	11.13881	0.0000
RESID^2(-1)	0.335581	0.016583	20.23656	0.0000

R-squared	0.112613	Mean dependent var	0.000224
Adjusted R-squared	0.112338	S.D. dependent var	0.000775
S.E. of regression	0.000730	Akaike info criterion	-11.60577
Sum squared resid	0.001721	Schwarz criterion	-11.60201
Log likelihood	18739.52	Hannan-Quinn criter.	-11.60442
F-statistic	409.5185	Durbin-Watson stat	2.100940
Prob(F-statistic)	0.000000		

Figure 16.1. ARCH test results AR(1) – BET-C

Heteroskedasticity Test: ARCH

F-statistic	425.4928	Prob. F(1,3226)	0.0000
Obs*R-squared	376.1450	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 05/25/14 Time: 22:32
Sample (adjusted): 1/08/2002 5/22/2014
Included observations: 3228 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000165	1.24E-05	13.34787	0.0000
RESID^2(-1)	0.341354	0.016549	20.62748	0.0000

R-squared	0.116526	Mean dependent var	0.000250
Adjusted R-squared	0.116252	S.D. dependent var	0.000703
S.E. of regression	0.000661	Akaike info criterion	-11.80415
Sum squared resid	0.001411	Schwarz criterion	-11.80038
Log likelihood	19053.89	Hannan-Quinn criter.	-11.80280
F-statistic	425.4928	Durbin-Watson stat	2.087048
Prob(F-statistic)	0.000000		

Figure 16.2. ARCH test results ARMA(2,2) – BUX

Heteroskedasticity Test: ARCH

F-statistic	114.9481	Prob. F(1,3225)	0.0000
Obs*R-squared	111.0609	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 05/25/14 Time: 22:44
Sample (adjusted): 1/09/2002 5/22/2014
Included observations: 3227 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000192	1.13E-05	16.88719	0.0000
RESID^2(-1)	0.185516	0.017303	10.72139	0.0000

R-squared	0.034416	Mean dependent var	0.000235
Adjusted R-squared	0.034117	S.D. dependent var	0.000612
S.E. of regression	0.000602	Akaike info criterion	-11.99351
Sum squared resid	0.001167	Schwarz criterion	-11.98974
Log likelihood	19353.53	Hannan-Quinn criter.	-11.99216
F-statistic	114.9481	Durbin-Watson stat	2.096485
Prob(F-statistic)	0.000000		

Figure 16.3. ARCH test results ARMA(3,5) – DAX

Heteroskedasticity Test: ARCH

F-statistic	172.0325	Prob. F(1,3224)	0.0000
Obs*R-squared	163.4191	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 05/25/14 Time: 22:44
Sample (adjusted): 1/10/2002 5/22/2014
Included observations: 3226 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000115	7.86E-06	14.61900	0.0000
RESID^2(-1)	0.225072	0.017160	13.11611	0.0000

R-squared	0.050657	Mean dependent var	0.000148
Adjusted R-squared	0.050362	S.D. dependent var	0.000433
S.E. of regression	0.000422	Akaike info criterion	-12.70092
Sum squared resid	0.000575	Schwarz criterion	-12.69715
Log likelihood	20488.58	Hannan-Quinn criter.	-12.69957
F-statistic	172.0325	Durbin-Watson stat	2.124894
Prob(F-statistic)	0.000000		

Figure 16.4. ARCH test results AR(4)MA(1)MA(3)MA(5) – FTSE 100

Heteroskedasticity Test: ARCH

F-statistic	106.4900	Prob. F(1,3225)	0.0000
Obs*R-squared	103.1500	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 05/25/14 Time: 22:53
Sample (adjusted): 1/09/2002 5/22/2014
Included observations: 3227 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000188	1.10E-05	17.12582	0.0000
RESID^2(-1)	0.178784	0.017325	10.31940	0.0000

R-squared	0.031965	Mean dependent var	0.000229
Adjusted R-squared	0.031665	S.D. dependent var	0.000591
S.E. of regression	0.000582	Akaike info criterion	-12.05956
Sum squared resid	0.001092	Schwarz criterion	-12.05580
Log likelihood	19460.11	Hannan-Quinn criter.	-12.05821
F-statistic	106.4900	Durbin-Watson stat	2.071637
Prob(F-statistic)	0.000000		

Figure 16.5. ARCH test results ARMA(3,5) – FTSE MIB

Heteroskedasticity Test: ARCH

F-statistic	171.4602	Prob. F(1,3225)	0.0000
Obs*R-squared	162.9055	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 05/25/14 Time: 22:53
Sample (adjusted): 1/09/2002 5/22/2014
Included observations: 3227 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000178	1.19E-05	14.93270	0.0000
RESID^2(-1)	0.224679	0.017159	13.09428	0.0000

R-squared	0.050482	Mean dependent var	0.000230
Adjusted R-squared	0.050188	S.D. dependent var	0.000656
S.E. of regression	0.000639	Akaike info criterion	-11.87300
Sum squared resid	0.001316	Schwarz criterion	-11.86923
Log likelihood	19159.09	Hannan-Quinn criter.	-11.87165
F-statistic	171.4602	Durbin-Watson stat	2.157703
Prob(F-statistic)	0.000000		

Figure 16.6. ARCH test results ARMA(3,1) – NIKKEI 225

Heteroskedasticity Test: ARCH

F-statistic	452.5793	Prob. F(1,3226)	0.0000
Obs*R-squared	397.1441	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 05/25/14 Time: 23:00
Sample (adjusted): 1/08/2002 5/22/2014
Included observations: 3228 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000113	9.15E-06	12.30561	0.0000
RESID^2(-1)	0.350762	0.016488	21.27391	0.0000

R-squared	0.123031	Mean dependent var	0.000173
Adjusted R-squared	0.122759	S.D. dependent var	0.000527
S.E. of regression	0.000494	Akaike info criterion	-12.38870
Sum squared resid	0.000786	Schwarz criterion	-12.38493
Log likelihood	19997.35	Hannan-Quinn criter.	-12.38735
F-statistic	452.5793	Durbin-Watson stat	2.161298
Prob(F-statistic)	0.000000		

Figure 16.7. ARCH test results AR(1)AR(2)MA(2)MA(5) – SOFIX

Heteroskedasticity Test: ARCH

F-statistic	127.3903	Prob. F(1,3227)	0.0000
Obs*R-squared	122.6283	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 05/25/14 Time: 23:00
Sample (adjusted): 1/07/2002 5/22/2014
Included observations: 3229 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000129	9.58E-06	13.47935	0.0000
RESID^2(-1)	0.194879	0.017266	11.28673	0.0000

R-squared	0.037977	Mean dependent var	0.000160
Adjusted R-squared	0.037679	S.D. dependent var	0.000531
S.E. of regression	0.000521	Akaike info criterion	-12.28143
Sum squared resid	0.000876	Schwarz criterion	-12.27766
Log likelihood	19830.37	Hannan-Quinn criter.	-12.28008
F-statistic	127.3903	Durbin-Watson stat	2.148756
Prob(F-statistic)	0.000000		

Figure 16.8. ARCH test results ARMA(1,8) – S&P 500

APPENDIX 17

Heteroskedasticity Test: White

F-statistic	221.5869	Prob. F(2,3227)	0.0000
Obs*R-squared	390.0228	Prob. Chi-Square(2)	0.0000
Scaled explained SS	2324.938	Prob. Chi-Square(2)	0.0000

Figure 17.1. WHITE test results AR(1) – BET-C

Heteroskedasticity Test: White

F-statistic	93.35216	Prob. F(3,3225)	0.0000
Obs*R-squared	257.9994	Prob. Chi-Square(3)	0.0000
Scaled explained SS	1017.242	Prob. Chi-Square(3)	0.0000

Figure 17.2. WHITE test results ARMA(2,2) – BUX

Heteroskedasticity Test: White

F-statistic	161.9008	Prob. F(3,3224)	0.0000
Obs*R-squared	422.6342	Prob. Chi-Square(3)	0.0000
Scaled explained SS	1429.228	Prob. Chi-Square(3)	0.0000

Figure 17.3. WHITE test results ARMA(3,5) – DAX

Heteroskedasticity Test: White

F-statistic	97.07056	Prob. F(10,3216)	0.0000
Obs*R-squared	748.1937	Prob. Chi-Square(10)	0.0000
Scaled explained SS	3188.237	Prob. Chi-Square(10)	0.0000

Figure 17.4. WHITE test results AR(4)MA(1)MA(3)MA(5) – FTSE 100

Heteroskedasticity Test: White

F-statistic	162.8288	Prob. F(3,3224)	0.0000
Obs*R-squared	424.7381	Prob. Chi-Square(3)	0.0000
Scaled explained SS	1413.315	Prob. Chi-Square(3)	0.0000

Figure 17.5. WHITE test results ARMA(3,5) – FTSE MIB

Heteroskedasticity Test: White

F-statistic	86.84533	Prob. F(3,3224)	0.0000
Obs*R-squared	241.3550	Prob. Chi-Square(3)	0.0000
Scaled explained SS	981.9791	Prob. Chi-Square(3)	0.0000

Figure 17.6. WHITE test results ARMA(3,1) – NIKKEI 225

Heteroskedasticity Test: White

F-statistic	107.6564	Prob. F(10,3218)	0.0000
Obs*R-squared	809.4476	Prob. Chi-Square(10)	0.0000
Scaled explained SS	3729.383	Prob. Chi-Square(10)	0.0000

Figure 17.7. WHITE test results AR(1)AR(2)MA(2)MA(5) – SOFIX

Heteroskedasticity Test: White

F-statistic	88.35302	Prob. F(3,3226)	0.0000
Obs*R-squared	245.2381	Prob. Chi-Square(3)	0.0000
Scaled explained SS	1343.214	Prob. Chi-Square(3)	0.0000

Figure 17.8. WHITE test results ARMA(1,8) – S&P 500

APPENDIX 18

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	1.089652	Prob. F(1,3227)	0.2966
Obs*R-squared	1.090297	Prob. Chi-Square(1)	0.2964

Figure 18.1. BG test results AR(1) – BET-C

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	4.069430	Prob. F(1,3226)	0.0437
Obs*R-squared	2.833148	Prob. Chi-Square(1)	0.0923

Figure 18.2. BG test results ARMA(2,2) – BUX

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	2.219754	Prob. F(1,3225)	0.1364
Obs*R-squared	1.611008	Prob. Chi-Square(1)	0.2044

Figure 18.3. BG test results ARMA(3,5) – DAX

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	4.986172	Prob. F(1,3222)	0.0256
Obs*R-squared	4.792880	Prob. Chi-Square(1)	0.0286

Figure 18.4. BG test results AR(4)MA(1)MA(3)MA(5) – FTSE 100

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.287987	Prob. F(1,3225)	0.5916
Obs*R-squared	0.000000	Prob. Chi-Square(1)	1.0000

Figure 18.5. BG test results ARMA(3,5) – FTSE MIB

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.218754	Prob. F(1,3225)	0.6400
Obs*R-squared	0.099708	Prob. Chi-Square(1)	0.7522

Figure 18.6. BG test results ARMA(3,1) – NIKKEI 225

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.056391	Prob. F(1,3224)	0.8123
Obs*R-squared	0.000000	Prob. Chi-Square(1)	1.0000

Figure 18.7. BG test results AR(1)AR(2)MA(2)MA(5) – SOFIX

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	3.454432	Prob. F(1,3227)	0.0632
Obs*R-squared	2.923412	Prob. Chi-Square(1)	0.0873

Figure 18.9. BG test results ARMA(1,8) – S&P 500

APPENDIX 19

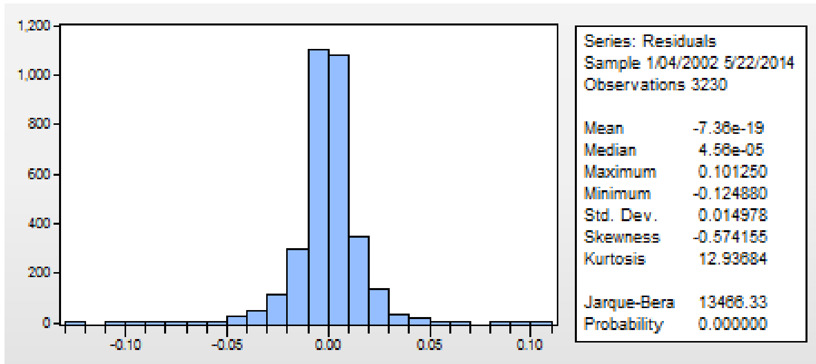


Figure 19.3 AR(1) – BET-C Histogram

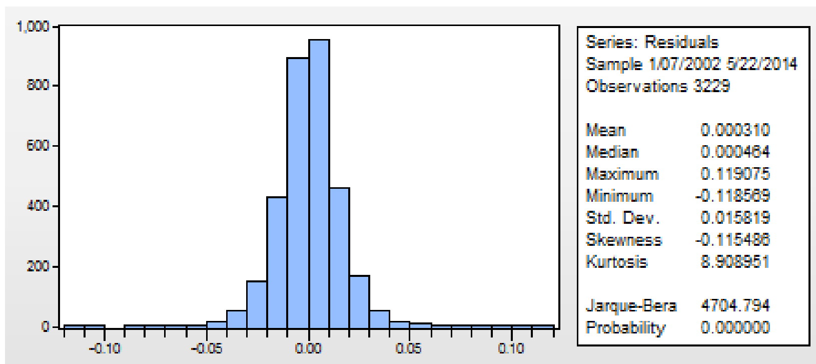


Figure 19.3 ARMA(2,2) – BUX Histogram

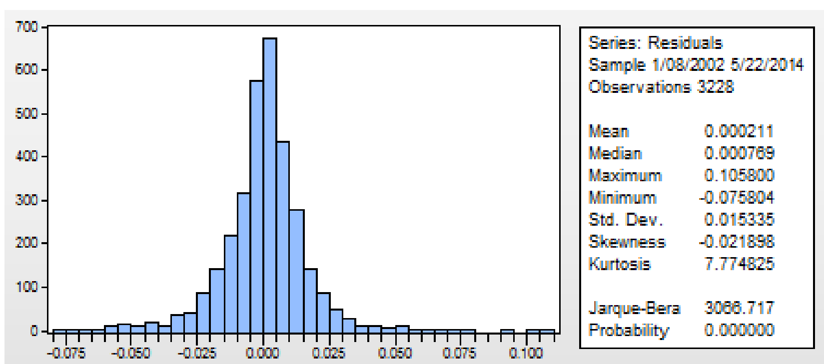


Figure 19.3. ARMA(3,5) – DAX Histogram

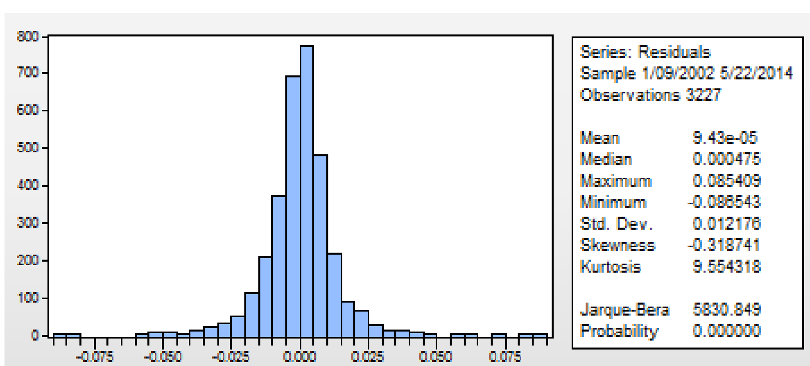


Figure 19.4. AR(4)MA(1)MA(3)MA(5) – FTSE 100 Histogram

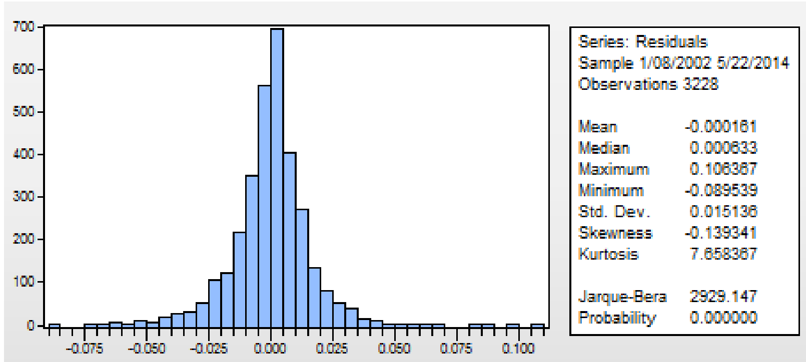


Figure 19.5. ARMA(3,5) – FTSE MIB Histogram

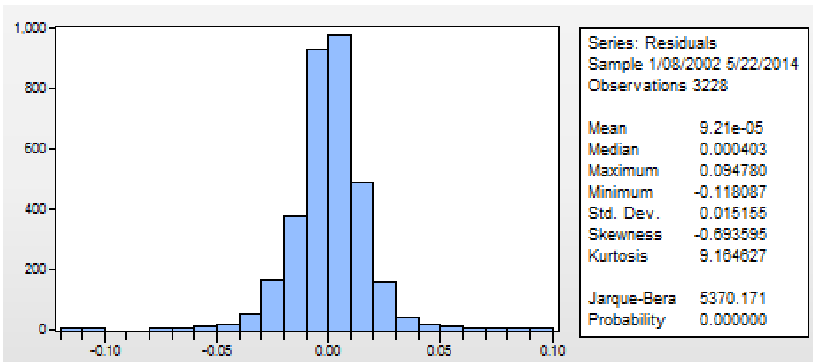


Figure 19.6. ARMA(3,1) – NIKKEI 225 Histogram

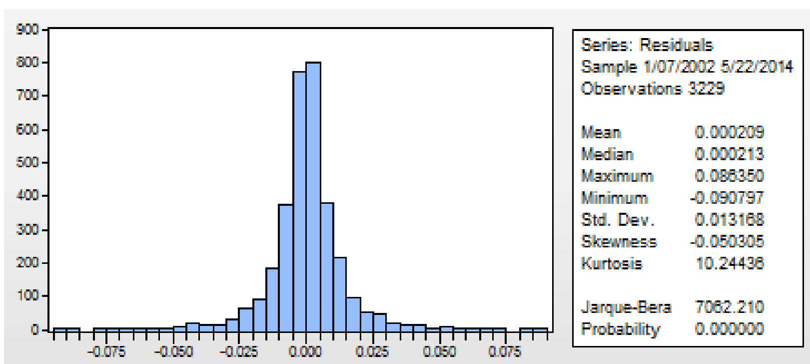


Figure 19.7. AR(1)AR(2)MA(2)MA(5) – SOFIX Histogram

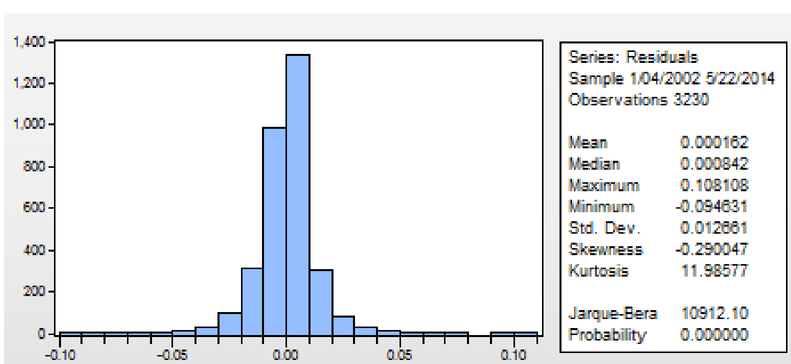


Figure 19.8. ARMA(1,8) – S&P 500 Histogram

APPENDIX 20

Dependent Variable: RETURN_BET_C
Method: ML - ARCH (Marquardt) - Student's t distribution
Date: 05/26/14 Time: 14:58
Sample (adjusted): 1/04/2002 5/22/2014
Included observations: 3230 after adjustments
Convergence achieved after 18 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000672	0.000159	4.230653	0.0000
RETURN_BET_C(-1)	0.084770	0.017815	4.758422	0.0000
Variance Equation				
C	5.11E-06	1.01E-06	5.083107	0.0000
RESID(-1)^2	0.197431	0.021614	9.134232	0.0000
GARCH(-1)	0.801247	0.016230	49.36748	0.0000
T-DIST. DOF	4.568967	0.388571	11.75839	0.0000

R-squared	0.008533	Mean dependent var	0.000581
Adjusted R-squared	0.008225	S.D. dependent var	0.015043
S.E. of regression	0.014981	Akaike info criterion	-6.114347
Sum squared resid	0.724499	Schwarz criterion	-6.103052
Log likelihood	9880.670	Hannan-Quinn criter.	-6.110299
Durbin-Watson stat	1.985939		

Figure 20.1. GARCH(1,1) – BET-C

Dependent Variable: RETURN_BUX
Method: ML - ARCH (Marquardt) - Generalized error distribution (GED)
Date: 05/25/14 Time: 07:49
Sample (adjusted): 1/07/2002 5/22/2014
Included observations: 3229 after adjustments
Convergence achieved after 18 iterations
MA Backcast: 1/03/2002 1/04/2002
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
RETURN_BUX(-2)	-0.718548	0.152990	-4.696697	0.0000
MA(2)	0.699326	0.157730	4.433693	0.0000
Variance Equation				
C(3)	-0.306997	0.046946	-6.539299	0.0000
C(4)	0.163172	0.017929	9.100933	0.0000
C(5)	-0.046085	0.009730	-4.736632	0.0000
C(6)	0.978582	0.004672	209.4540	0.0000
GED PARAMETER	1.474325	0.049864	29.56721	0.0000

R-squared	0.005701	Mean dependent var	0.000296
Adjusted R-squared	0.005393	S.D. dependent var	0.015918
S.E. of regression	0.015875	Akaike info criterion	-5.737157
Sum squared resid	0.813212	Schwarz criterion	-5.723976
Log likelihood	9269.640	Hannan-Quinn criter.	-5.732433
Durbin-Watson stat	1.931968		

Figure 20.2. EGARCH(1,1,1) – BUX

Dependent Variable: RETURN_DAX
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 05/26/14 Time: 15:57
Sample (adjusted): 1/08/2002 5/22/2014
Included observations: 3228 after adjustments
Convergence achieved after 10 iterations
MA Backcast: 1/01/2002 1/07/2002
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
RETURN_DAX(-3)	-0.030194	0.017900	-1.686807	0.0916
MA(5)	-0.034794	0.018126	-1.919559	0.0549
Variance Equation				
C	2.02E-06	3.63E-07	5.550711	0.0000
RESID(-1)^2	0.080019	0.006506	12.29916	0.0000
GARCH(-1)	0.910248	0.007261	125.3595	0.0000

R-squared	0.003741	Mean dependent var	0.000192
Adjusted R-squared	0.003433	S.D. dependent var	0.015369
S.E. of regression	0.015342	Akaike info criterion	-5.926265
Sum squared resid	0.759347	Schwarz criterion	-5.916848
Log likelihood	9569.992	Hannan-Quinn criter.	-5.922890
Durbin-Watson stat	2.052525		

Figure 20.3. GARCH(1,1) – DAX

Dependent Variable: RETURN_FTSE_100
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 05/26/14 Time: 16:52
Sample (adjusted): 1/09/2002 5/22/2014
Included observations: 3227 after adjustments
Convergence achieved after 15 iterations
MA Backcast: 1/02/2002 1/08/2002
Presample variance: backcast (parameter = 0.7)
GARCH = C(5) + C(6)*RESID(-1)^2 + C(7)*RESID(-2)^2 + C(8)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
RETURN_FTSE_100(-4)	0.019962	0.018442	1.082450	0.2791
MA(1)	-0.062726	0.017798	-3.524289	0.0004
MA(3)	-0.036008	0.018305	-1.967142	0.0492
MA(5)	-0.015422	0.017872	-0.862891	0.3882
Variance Equation				
C	1.50E-06	2.91E-07	5.145118	0.0000
RESID(-1)^2	0.059448	0.014494	4.101586	0.0000
RESID(-2)^2	0.049735	0.017287	2.877082	0.0040
GARCH(-1)	0.880604	0.010487	83.97148	0.0000

R-squared	0.011920	Mean dependent var	8.11E-05
Adjusted R-squared	0.011001	S.D. dependent var	0.012298
S.E. of regression	0.012231	Akaike info criterion	-6.439699
Sum squared resid	0.482121	Schwarz criterion	-6.424628
Log likelihood	10398.46	Hannan-Quinn criter.	-6.434299
Durbin-Watson stat	1.982960		

Figure 20.4. GARCH(2,1) – FTSE 100

Dependent Variable: RETURN_FTSE_MIB
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 05/26/14 Time: 18:09
Sample (adjusted): 1/08/2002 5/22/2014
Included observations: 3228 after adjustments
Convergence achieved after 10 iterations
MA Backcast: 1/01/2002 1/07/2002
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
RETURN_FTSE_MIB(-3)	-0.026186	0.017877	-1.464733	0.1430
MA(5)	-0.029998	0.017617	-1.702764	0.0886
Variance Equation				
C	9.82E-07	2.05E-07	4.783036	0.0000
RESID(-1)^2	0.074271	0.005128	14.48447	0.0000
GARCH(-1)	0.923379	0.005134	179.8604	0.0000

R-squared	0.005046	Mean dependent var	-0.000143
Adjusted R-squared	0.004737	S.D. dependent var	0.015192
S.E. of regression	0.015156	Akaike info criterion	-5.944030
Sum squared resid	0.741048	Schwarz criterion	-5.934613
Log likelihood	9598.664	Hannan-Quinn criter.	-5.940655
Durbin-Watson stat	2.023722		

Figure 20.5. GARCH(1,1) – FTSE MIB

Dependent Variable: RETURN_NIKKEI_225
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 05/26/14 Time: 16:40
Sample (adjusted): 1/08/2002 5/22/2014
Included observations: 3228 after adjustments
Convergence achieved after 12 iterations
MA Backcast: 1/07/2002
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
RETURN_NIKKEI_225(-3)	0.009170	0.017046	0.537972	0.5906
MA(1)	-0.035908	0.020379	-1.762060	0.0781
Variance Equation				
C	3.41E-06	7.02E-07	4.853801	0.0000
RESID(-1)^2	0.091090	0.006477	14.06333	0.0000
GARCH(-1)	0.895046	0.007820	114.4521	0.0000

R-squared	0.000923	Mean dependent var	8.37E-05
Adjusted R-squared	0.000613	S.D. dependent var	0.015190
S.E. of regression	0.015185	Akaike info criterion	-5.797217
Sum squared resid	0.743907	Schwarz criterion	-5.787800
Log likelihood	9361.708	Hannan-Quinn criter.	-5.793842
Durbin-Watson stat	2.018050		

Figure 20.6. GARCH(1,1) – NIKKEI 225

Dependent Variable: RETURN_SOFIX
Method: ML - ARCH (Marquardt) - Generalized error distribution (GED)
Date: 05/26/14 Time: 16:44
Sample (adjusted): 1/07/2002 5/22/2014
Included observations: 3229 after adjustments
Convergence achieved after 13 iterations
MA Backcast: 12/31/2001 1/04/2002
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(5) + C(6)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(7)
*RESID(-1)/@SQRT(GARCH(-1)) + C(8)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
RETURN_SOFIX(-1)	0.060901	0.012354	4.929717	0.0000
RETURN_SOFIX(-2)	0.840472	0.030344	27.69822	0.0000
MA(2)	-0.810569	0.034791	-23.29811	0.0000
MA(5)	-0.024328	0.011946	-2.036396	0.0417
Variance Equation				
C(5)	-0.943369	0.101942	-9.254008	0.0000
C(6)	0.464986	0.033229	13.99346	0.0000
C(7)	-0.044994	0.020830	-2.160092	0.0308
C(8)	0.933315	0.009858	94.67300	0.0000
GED PARAMETER	1.068054	0.032699	32.66368	0.0000

R-squared	0.029677	Mean dependent var	0.000514
Adjusted R-squared	0.028774	S.D. dependent var	0.013476
S.E. of regression	0.013281	Akaike info criterion	-6.422657
Sum squared resid	0.568849	Schwarz criterion	-6.405711
Log likelihood	10378.38	Hannan-Quinn criter.	-6.416584
Durbin-Watson stat	1.950120		

Figure 20.7. EGARCH(1,1,1) – SOFIX

Dependent Variable: RETURN_SP_500
Method: ML - ARCH (Marquardt) - Generalized error distribution (GED)
Date: 05/26/14 Time: 16:50
Sample (adjusted): 1/04/2002 5/22/2014
Included observations: 3230 after adjustments
Convergence achieved after 17 iterations
MA Backcast: 12/25/2001 1/03/2002
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)
*ABS(RESID(-2)/@SQRT(GARCH(-2))) + C(6)*RESID(-1)
/@SQRT(GARCH(-1)) + C(7)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
RETURN_SP_500(-1)	-0.076468	0.015192	-5.033414	0.0000
MA(8)	0.009361	0.016613	0.563484	0.5731
Variance Equation				
C(3)	-0.294224	0.032138	-9.154999	0.0000
C(4)	-0.125145	0.044809	-2.792831	0.0052
C(5)	0.252789	0.046067	5.487443	0.0000
C(6)	-0.139159	0.011503	-12.09819	0.0000
C(7)	0.978603	0.002751	355.7206	0.0000
GED PARAMETER	1.460838	0.047950	30.46610	0.0000

R-squared	0.013929	Mean dependent var	0.000150
Adjusted R-squared	0.013623	S.D. dependent var	0.012773
S.E. of regression	0.012686	Akaike info criterion	-6.495199
Sum squared resid	0.519499	Schwarz criterion	-6.480140
Log likelihood	10497.75	Hannan-Quinn criter.	-6.489803
Durbin-Watson stat	2.102132		

Figure 20.8. EGARCH(2,1,1) – S&P

APPENDIX 21

Wald Test:
Equation: GARCH_BUX

Test Statistic	Value	df	Probability
t-statistic	-4.736632	3222	0.0000
F-statistic	22.43568	(1, 3222)	0.0000
Chi-square	22.43568	1	0.0000

Null Hypothesis: C(5)=0
Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(5)	-0.046085	0.009730

Wald Test:
Equation: GARCH_SOFIX

Test Statistic	Value	df	Probability
t-statistic	-2.160092	3220	0.0308
F-statistic	4.665999	(1, 3220)	0.0308
Chi-square	4.665999	1	0.0308

Null Hypothesis: C(7)=0
Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(7)	-0.044994	0.020830

Wald Test:
Equation: GARCH_SP_500

Test Statistic	Value	df	Probability
t-statistic	-12.09819	3222	0.0000
F-statistic	146.3661	(1, 3222)	0.0000
Chi-square	146.3661	1	0.0000

Null Hypothesis: C(6)=0
Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(6)	-0.139159	0.011503

Figure 21. Wald test results

APPENDIX 22

Correlogram of Standardized Residuals Squared

Date: 05/26/14 Time: 20:16
Sample: 1/04/2002 5/22/2014
Included observations: 3230

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.032	0.032	3.2436	0.072
		2	0.011	0.010	3.6253	0.163
		3	-0.014	-0.014	4.2352	0.237
		4	-0.024	-0.023	6.1308	0.190
		5	-0.019	-0.017	7.2958	0.200
		6	-0.035	-0.033	11.184	0.083
		7	-0.024	-0.022	12.974	0.073
		8	-0.024	-0.024	14.912	0.061
		9	-0.031	-0.031	17.940	0.036
		10	-0.018	-0.018	18.946	0.041
		11	0.040	0.039	24.258	0.012
		12	-0.004	-0.010	24.299	0.019
		13	-0.004	-0.009	24.347	0.028
		14	-0.002	-0.004	24.360	0.041
		15	0.009	0.007	24.606	0.055

Figure 22.1. GARCH(1,1) – BET-C Square return correlogram

Correlogram of Standardized Residuals Squared

Date: 05/26/14 Time: 20:16
Sample: 1/07/2002 5/22/2014
Included observations: 3229
Q-statistic probabilities adjusted for 1 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.014	0.014	0.5901	
		2	-0.000	-0.000	0.5901	0.442
		3	0.030	0.030	3.5271	0.171
		4	0.003	0.003	3.5646	0.312
		5	-0.007	-0.007	3.7293	0.444
		6	-0.006	-0.006	3.8287	0.574
		7	0.013	0.013	4.3859	0.625
		8	-0.018	-0.018	5.3887	0.613
		9	0.010	0.011	5.7402	0.676
		10	0.024	0.023	7.5799	0.577
		11	-0.007	-0.007	7.7541	0.653
		12	-0.007	-0.008	7.9343	0.719
		13	0.046	0.045	14.795	0.253
		14	0.002	0.001	14.806	0.320
		15	0.003	0.004	14.833	0.390

Figure 22.2. EGARCH(1,1,1) – BUX Square return correlogram

Correlogram of Standardized Residuals Squared

Date: 05/26/14 Time: 20:56
Sample: 1/08/2002 5/22/2014
Included observations: 3228
Q-statistic probabilities adjusted for 1 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.030	-0.030	2.9750	
		2	0.027	0.026	5.2699	0.022
		3	0.037	0.038	9.6270	0.008
		4	0.021	0.022	11.031	0.012
		5	-0.003	-0.003	11.055	0.026
		6	-0.006	-0.009	11.175	0.048
		7	0.004	0.002	11.237	0.081
		8	-0.016	-0.016	12.058	0.099
		9	-0.001	-0.002	12.063	0.148
		10	0.001	0.002	12.069	0.209
		11	0.036	0.038	16.371	0.089
		12	-0.017	-0.014	17.346	0.098
		13	0.001	-0.002	17.349	0.137
		14	-0.011	-0.013	17.736	0.168
		15	-0.018	-0.019	18.748	0.175

Figure 22.3. GARCH(1,1) – DAX Square return correlogram

Correlogram of Standardized Residuals Squared

Date: 05/26/14 Time: 20:56
Sample: 1/09/2002 5/22/2014
Included observations: 3227
Q-statistic probabilities adjusted for 3 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.012	0.012	0.4984	
		2	0.006	0.005	0.6001	
		3	0.016	0.016	1.4417	
		4	0.011	0.011	1.8657	0.172
		5	-0.000	-0.001	1.8665	0.393
		6	0.004	0.004	1.9154	0.590
		7	-0.012	-0.013	2.4038	0.662
		8	0.010	0.010	2.7430	0.740
		9	-0.015	-0.016	3.5109	0.743
		10	-0.000	0.000	3.5113	0.834
		11	0.050	0.050	11.502	0.175
		12	-0.003	-0.004	11.526	0.241
		13	-0.005	-0.005	11.606	0.312
		14	-0.040	-0.042	16.770	0.115
		15	-0.005	-0.004	16.841	0.156

Figure 22.4. GARCH(2,1) – FTSE 100 Square return correlogram

Correlogram of Standardized Residuals Squared

Date: 05/26/14 Time: 21:06
Sample: 1/08/2002 5/22/2014
Included observations: 3228
Q-statistic probabilities adjusted for 1 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.026	-0.026	2.1209	
		2	0.008	0.007	2.3206	0.128
		3	0.034	0.034	5.9530	0.051
		4	0.041	0.043	11.475	0.009
		5	0.002	0.004	11.486	0.022
		6	0.015	0.014	12.228	0.032
		7	-0.009	-0.011	12.490	0.052
		8	-0.022	-0.024	13.998	0.051
		9	0.006	0.003	14.105	0.079
		10	-0.013	-0.013	14.618	0.102
		11	0.037	0.039	19.175	0.038
		12	-0.024	-0.020	21.063	0.033
		13	0.002	0.002	21.083	0.049
		14	-0.008	-0.008	21.269	0.068
		15	-0.005	-0.008	21.359	0.093

Figure 22.5. GARCH(1,1) – FTSE MIB Square return correlogram

Correlogram of Standardized Residuals Squared

Date: 05/26/14 Time: 21:06
Sample: 1/08/2002 5/22/2014
Included observations: 3228
Q-statistic probabilities adjusted for 1 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.018	0.018	1.0277	
		2	0.016	0.016	1.8661	0.172
		3	-0.001	-0.002	1.8690	0.393
		4	0.000	-0.000	1.8691	0.600
		5	0.006	0.006	1.9877	0.738
		6	0.024	0.024	3.9260	0.560
		7	-0.001	-0.002	3.9297	0.686
		8	0.006	0.006	4.0553	0.773
		9	-0.006	-0.006	4.1624	0.842
		10	-0.000	-0.000	4.1626	0.900
		11	-0.028	-0.028	6.7702	0.747
		12	0.038	0.038	11.398	0.411
		13	-0.009	-0.010	11.668	0.473
		14	0.003	0.001	11.690	0.553
		15	0.034	0.035	15.484	0.346

Figure 22.6. GARCH(1,1) – NIKKEI 225 Square return correlogram

Correlogram of Standardized Residuals Squared

Date: 05/26/14 Time: 21:16
Sample: 1/07/2002 5/22/2014
Included observations: 3229
Q-statistic probabilities adjusted for 2 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.004	-0.004	0.0625	
		2	-0.007	-0.007	0.2120	
		3	0.000	0.000	0.2125	0.645
		4	-0.023	-0.023	1.9832	0.371
		5	-0.022	-0.023	3.6211	0.305
		6	0.006	0.005	3.7371	0.443
		7	-0.004	-0.004	3.7905	0.580
		8	-0.019	-0.020	4.9820	0.546
		9	0.014	0.013	5.6176	0.585
		10	-0.012	-0.013	6.1209	0.634
		11	-0.005	-0.005	6.2072	0.719
		12	-0.007	-0.008	6.3667	0.784
		13	0.007	0.006	6.5154	0.837
		14	0.014	0.014	7.1192	0.850
		15	-0.009	-0.010	7.3818	0.881

Figure 22.7. EGARCH(1,1,1) – SOFIX Square return correlogram

Correlogram of Standardized Residuals Squared

Date: 05/26/14 Time: 21:16
Sample: 1/04/2002 5/22/2014
Included observations: 3230
Q-statistic probabilities adjusted for 1 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.005	-0.005	0.0809	
		2	-0.002	-0.002	0.0995	0.752
		3	-0.016	-0.016	0.9395	0.625
		4	-0.006	-0.006	1.0405	0.791
		5	0.024	0.024	2.9309	0.569
		6	-0.010	-0.010	3.2465	0.662
		7	0.028	0.028	5.7434	0.453
		8	-0.011	-0.010	6.1445	0.523
		9	0.025	0.025	8.1345	0.420
		10	0.028	0.028	10.652	0.300
		11	-0.004	-0.004	10.715	0.380
		12	0.029	0.028	13.359	0.271
		13	-0.007	-0.005	13.531	0.332
		14	0.002	0.000	13.542	0.407
		15	-0.007	-0.006	13.702	0.472

Figure 22.8. EGARCH(2,1,1) – S&P 500 Square return correlogram

APPENDIX 23

Heteroskedasticity Test: ARCH

F-statistic	3.241016	Prob. F(1,3227)	0.0719
Obs*R-squared	3.239771	Prob. Chi-Square(1)	0.0719

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 05/26/14 Time: 20:19
Sample (adjusted): 1/07/2002 5/22/2014
Included observations: 3229 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.954169	0.046342	20.58956	0.0000
WGT_RESID^2(-1)	0.031676	0.017595	1.800282	0.0719

R-squared	0.001003	Mean dependent var	0.985390
Adjusted R-squared	0.000694	S.D. dependent var	2.442880
S.E. of regression	2.442033	Akaike info criterion	4.624158
Sum squared resid	19244.29	Schwarz criterion	4.627924
Log likelihood	-7463.703	Hannan-Quinn criter.	4.625507
F-statistic	3.241016	Durbin-Watson stat	2.000548
Prob(F-statistic)	0.071909		

Figure 23.1. GARCH(1,1) – BET-C ARCH LM test results

Heteroskedasticity Test: ARCH

F-statistic	0.589139	Prob. F(1,3226)	0.4428
Obs*R-squared	0.589397	Prob. Chi-Square(1)	0.4427

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 05/26/14 Time: 20:19
Sample (adjusted): 1/08/2002 5/22/2014
Included observations: 3228 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.988545	0.035278	28.02154	0.0000
WGT_RESID^2(-1)	0.013512	0.017604	0.767554	0.4428

R-squared	0.000183	Mean dependent var	1.002081
Adjusted R-squared	-0.000127	S.D. dependent var	1.735850
S.E. of regression	1.735961	Akaike info criterion	3.941619
Sum squared resid	9721.746	Schwarz criterion	3.945385
Log likelihood	-6359.773	Hannan-Quinn criter.	3.942969
F-statistic	0.589139	Durbin-Watson stat	1.999837
Prob(F-statistic)	0.442808		

Figure EGARCH(1,1,1) – BUX ARCH LM test results

Heteroskedasticity Test: ARCH

F-statistic	2.972783	Prob. F(1,3225)	0.0848
Obs*R-squared	2.971887	Prob. Chi-Square(1)	0.0847

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 05/26/14 Time: 20:58
Sample (adjusted): 1/09/2002 5/22/2014
Included observations: 3227 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.031723	0.036175	28.52015	0.0000
WGT_RESID^2(-1)	-0.030347	0.017601	-1.724176	0.0848

R-squared	0.000921	Mean dependent var	1.001336
Adjusted R-squared	0.000611	S.D. dependent var	1.795173
S.E. of regression	1.794624	Akaike info criterion	4.008088
Sum squared resid	10386.68	Schwarz criterion	4.011856
Log likelihood	-6465.050	Hannan-Quinn criter.	4.009438
F-statistic	2.972783	Durbin-Watson stat	1.998385
Prob(F-statistic)	0.084772		

Figure 23.3. GARCH(1,1) – DAX ARCH LM test results

Heteroskedasticity Test: ARCH

F-statistic	0.497674	Prob. F(1,3224)	0.4806
Obs*R-squared	0.497906	Prob. Chi-Square(1)	0.4804

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 05/26/14 Time: 20:58
Sample (adjusted): 1/10/2002 5/22/2014
Included observations: 3226 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.988086	0.033862	29.17963	0.0000
WGT_RESID^2(-1)	0.012424	0.017611	0.705460	0.4806

R-squared	0.000154	Mean dependent var	1.000518
Adjusted R-squared	-0.000156	S.D. dependent var	1.642216
S.E. of regression	1.642344	Akaike info criterion	3.830746
Sum squared resid	8696.076	Schwarz criterion	3.834515
Log likelihood	-6176.993	Hannan-Quinn criter.	3.832097
F-statistic	0.497674	Durbin-Watson stat	2.000020
Prob(F-statistic)	0.480575		

Figure 23.4. GARCH(2,1) – FTSE 100 ARCH LM test

Heteroskedasticity Test: ARCH

F-statistic	2.118539	Prob. F(1,3225)	0.1456
Obs*R-squared	2.118461	Prob. Chi-Square(1)	0.1455

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 05/26/14 Time: 21:08
Sample (adjusted): 1/09/2002 5/22/2014
Included observations: 3227 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.026002	0.036257	28.29822	0.0000
WGT_RESID^2(-1)	-0.025621	0.017603	-1.455520	0.1456

R-squared	0.000656	Mean dependent var	1.000376
Adjusted R-squared	0.000347	S.D. dependent var	1.800796
S.E. of regression	1.800484	Akaike info criterion	4.014608
Sum squared resid	10454.62	Schwarz criterion	4.018376
Log likelihood	-6475.570	Hannan-Quinn criter.	4.015958
F-statistic	2.118539	Durbin-Watson stat	1.999686
Prob(F-statistic)	0.145623		

Figure 23.5. GARCH(1,1) – FTSE MIB ARCH LM test results

Heteroskedasticity Test: ARCH

F-statistic	1.026956	Prob. F(1,3225)	0.3110
Obs*R-squared	1.027265	Prob. Chi-Square(1)	0.3108

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 05/26/14 Time: 21:08
Sample (adjusted): 1/09/2002 5/22/2014
Included observations: 3227 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.981837	0.035272	27.83614	0.0000
WGT_RESID^2(-1)	0.017846	0.017611	1.013388	0.3110

R-squared	0.000318	Mean dependent var	0.999670
Adjusted R-squared	0.000008	S.D. dependent var	1.736509
S.E. of regression	1.736502	Akaike info criterion	3.942242
Sum squared resid	9724.787	Schwarz criterion	3.946009
Log likelihood	-6358.807	Hannan-Quinn criter.	3.943592
F-statistic	1.026956	Durbin-Watson stat	1.999584
Prob(F-statistic)	0.310951		

Figure 23.6. GARCH(1,1) – NIKKEI 225 ARCH LM test

Heteroskedasticity Test: ARCH

F-statistic	0.062340	Prob. F(1,3226)	0.8029
Obs*R-squared	0.062378	Prob. Chi-Square(1)	0.8028

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 05/26/14 Time: 21:19
Sample (adjusted): 1/08/2002 5/22/2014
Included observations: 3228 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.019818	0.046139	22.10341	0.0000
WGT_RESID^2(-1)	-0.004396	0.017606	-0.249681	0.8029

R-squared	0.000019	Mean dependent var	1.015354
Adjusted R-squared	-0.000291	S.D. dependent var	2.416204
S.E. of regression	2.416555	Akaike info criterion	4.603183
Sum squared resid	18839.00	Schwarz criterion	4.606949
Log likelihood	-7427.537	Hannan-Quinn criter.	4.604533
F-statistic	0.062340	Durbin-Watson stat	2.000021
Prob(F-statistic)	0.802850		

Figure 23.7. EGARCH(1,1,1) – SOFIX ARCH LM test results

Heteroskedasticity Test: ARCH

F-statistic	0.080758	Prob. F(1,3227)	0.7763
Obs*R-squared	0.080806	Prob. Chi-Square(1)	0.7762

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 05/26/14 Time: 21:19
Sample (adjusted): 1/07/2002 5/22/2014
Included observations: 3229 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.006402	0.036029	27.93311	0.0000
WGT_RESID^2(-1)	-0.005003	0.017604	-0.284180	0.7763

R-squared	0.000025	Mean dependent var	1.001391
Adjusted R-squared	-0.000285	S.D. dependent var	1.785103
S.E. of regression	1.785357	Akaike info criterion	3.997733
Sum squared resid	10286.06	Schwarz criterion	4.001499
Log likelihood	-6452.340	Hannan-Quinn criter.	3.999082
F-statistic	0.080758	Durbin-Watson stat	1.999957
Prob(F-statistic)	0.776291		

Figure 23.8. EGARCH(2,1,1) – S&P 500 ARCH LM test

APPENDIX 24

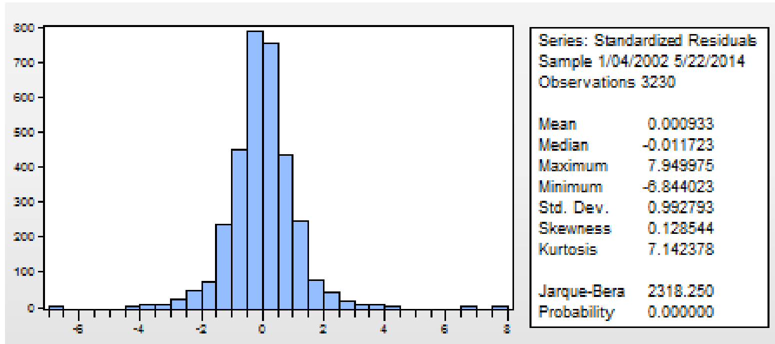


Figure 24.1. GARCH(1,1) – BET-C Histogram

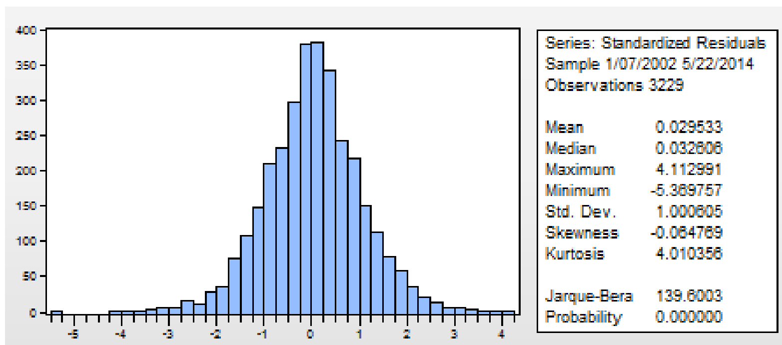


Figure 24.2. EGARCH(1,1,1) – BUX Histogram

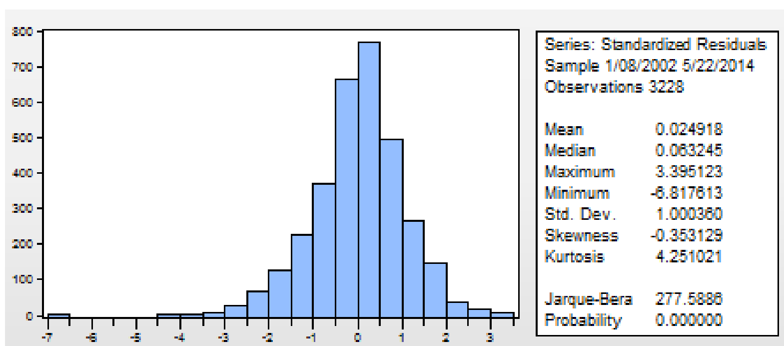


Figure 24.3. GARCH(1,1) – DAX Histogram

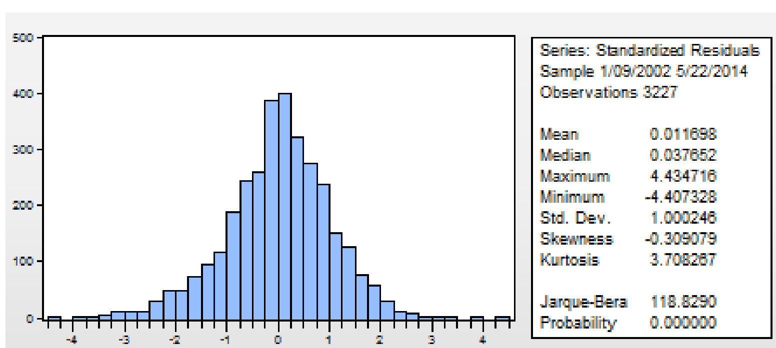


Figure 24.4. GARCH(2,1) – FTSE 100 Histogram

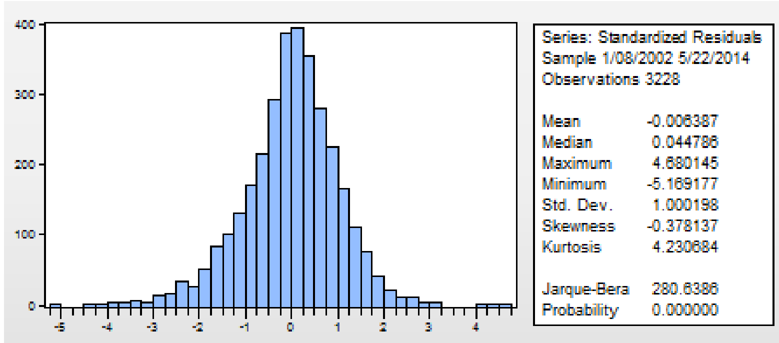


Figure 24.5. GARCH(1,1) – FTSE MIB Histogram

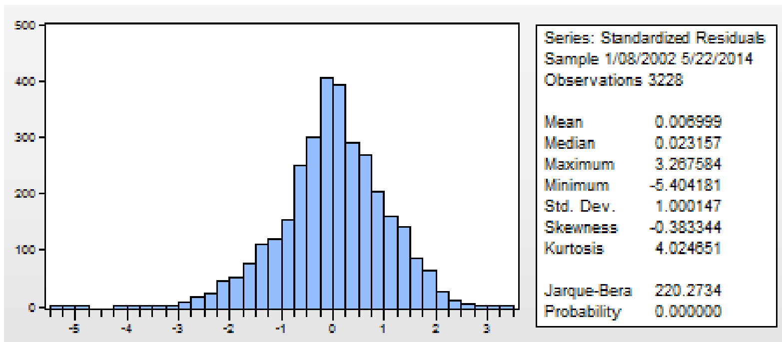


Figure 24.6. GARCH(1,1) – NIKKEI 225 Histogram

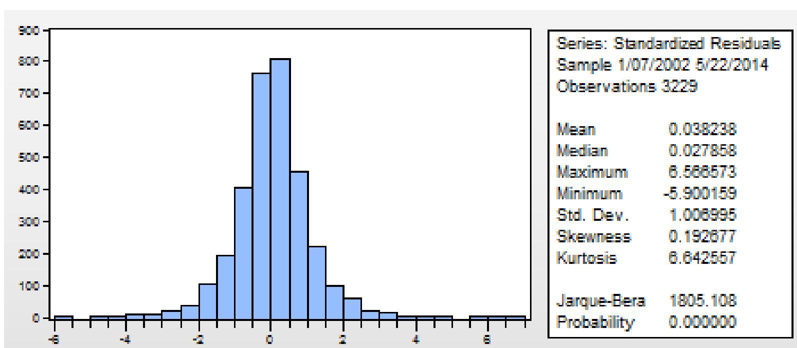


Figure 24.7. EGARCH(1,1,1) – SOFIX Histogram

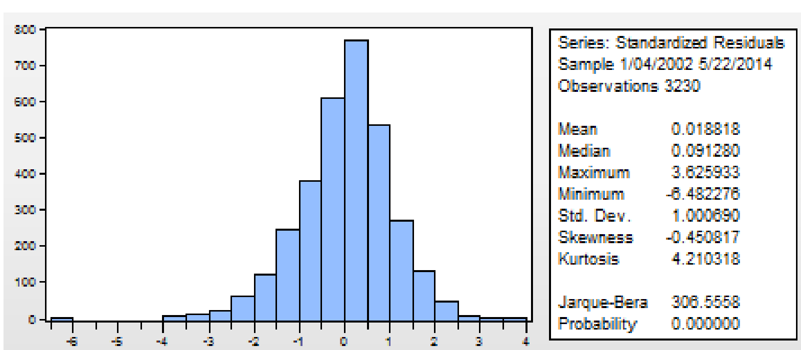


Figure 24.8. EGARCH(2,1,1) – S&P 500 Histogram

APPENDIX 25

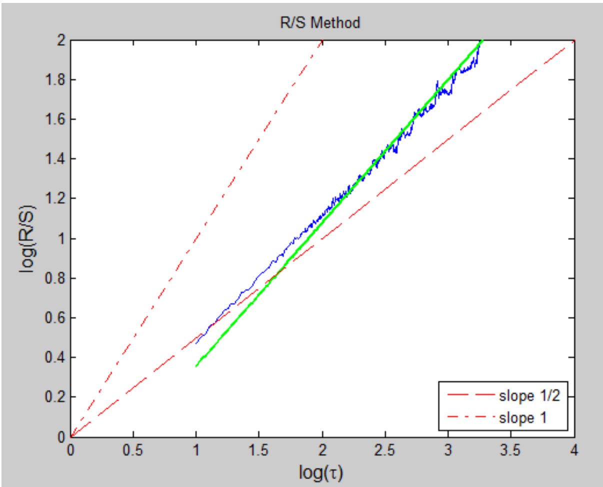


Figure 25.1. R/S analysis BET-C

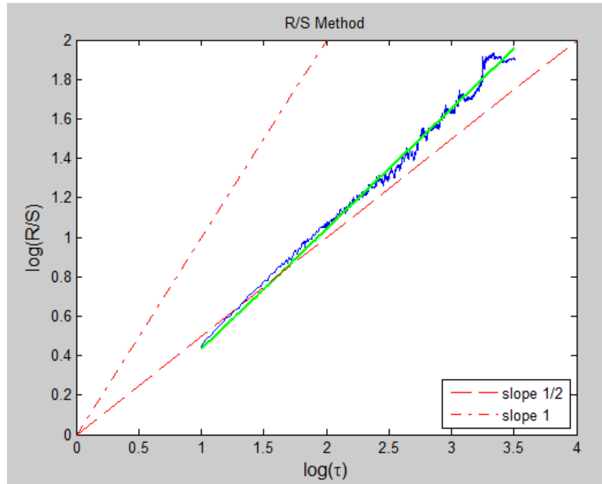


Figure 25.2. R/S analysis BUX

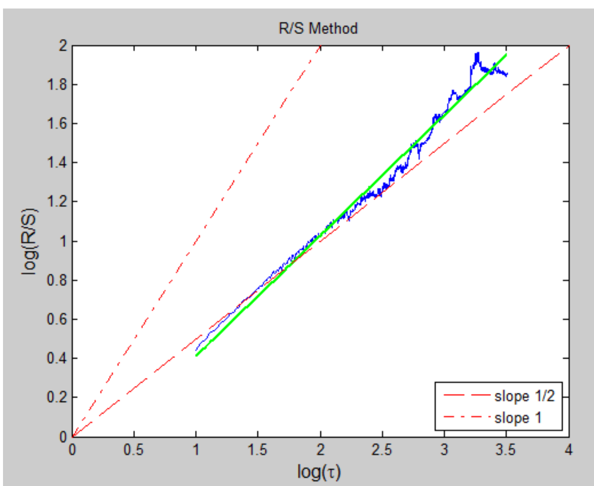


Figure 25.3. R/S analysis DAX

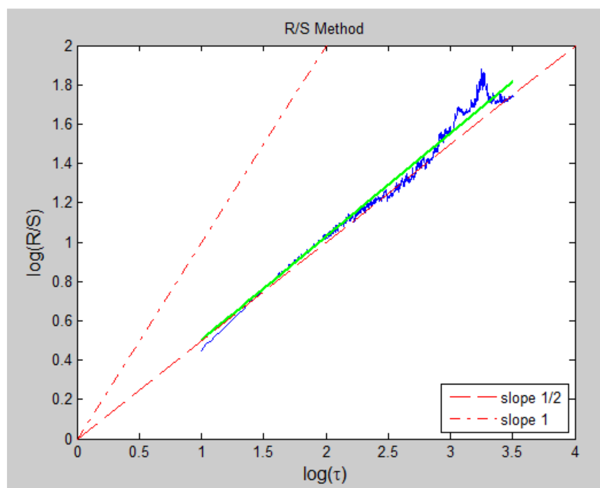


Figure 25.4. R/S analysis FTSE 100

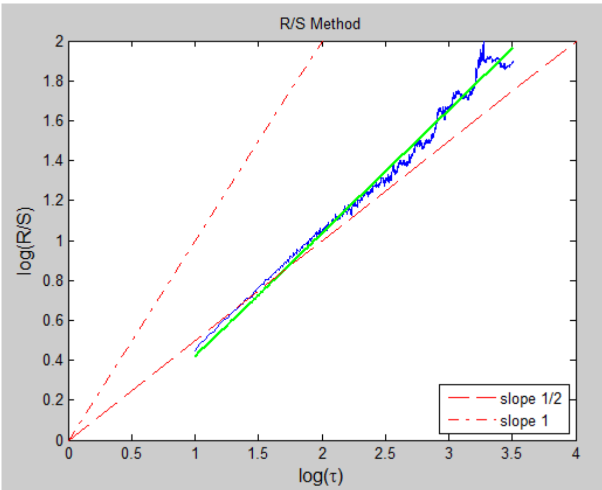


Figure 25.5. R/S analysis FTSE MIB

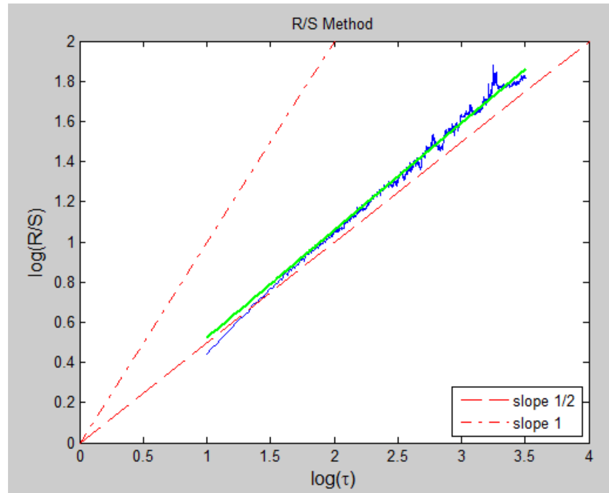


Figure 25.6. R/S analysis NIKKEI 225

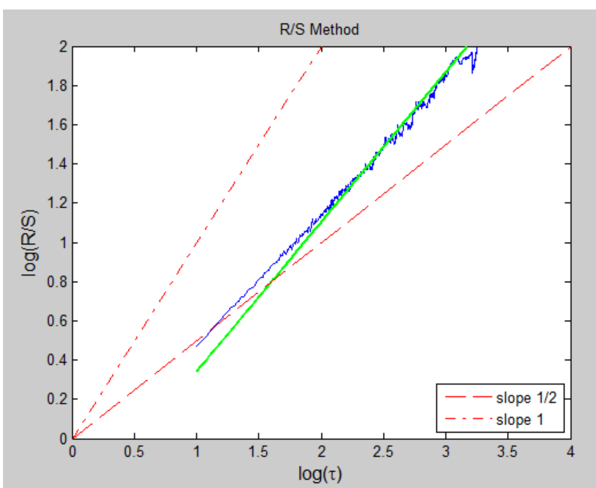


Figure 25.7. R/S analysis SOFIX

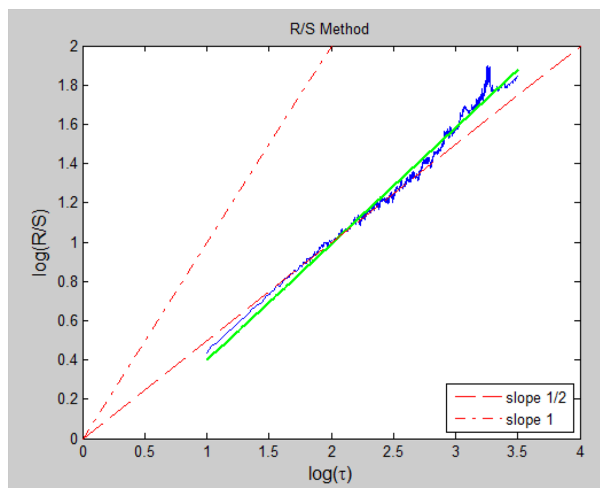


Figure 25.8. R/S analysis S&P 500

APPENDIX 26

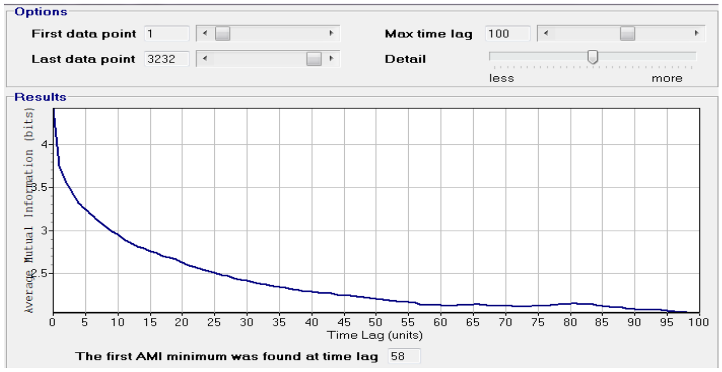


Figure 26.1. Mutual Information – BET-C

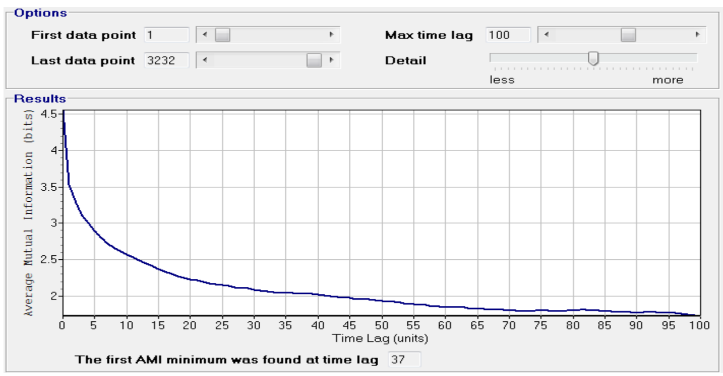


Figure 26.2. Mutual Information – BUX

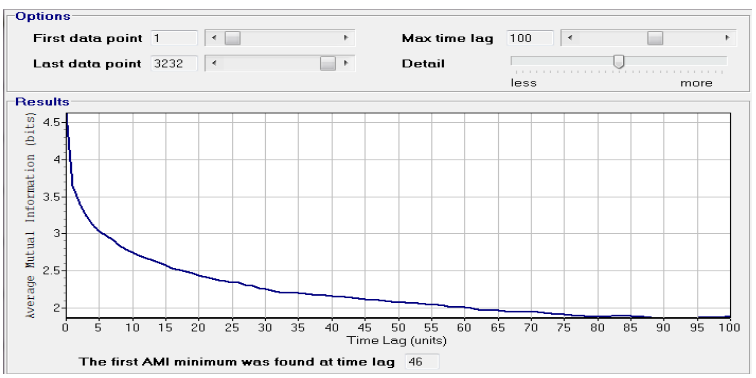


Figure 26.3. Mutual Information – DAX

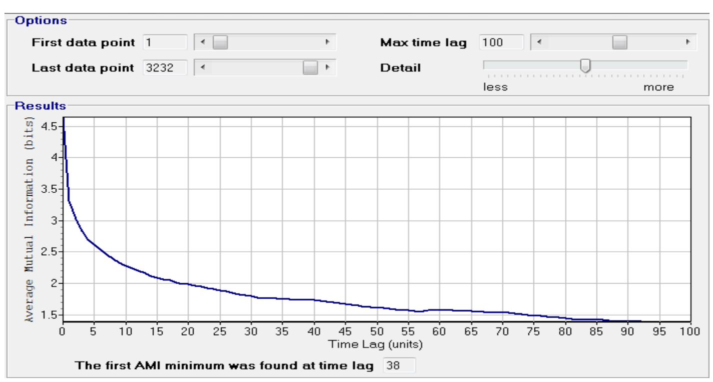


Figure 26.4. Mutual Information – FTSE 100

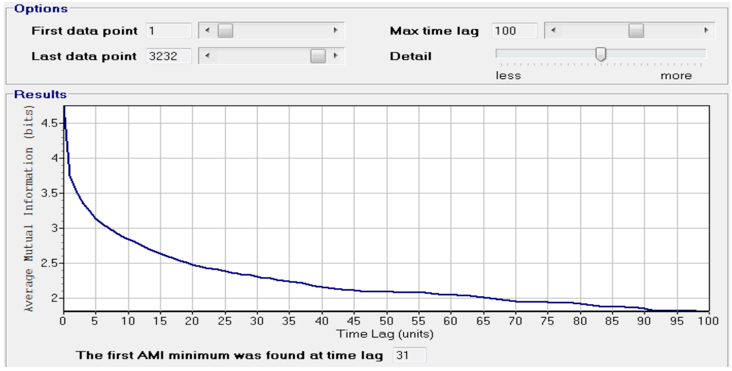


Figure 26.5. Mutual Information – FTSE MIB

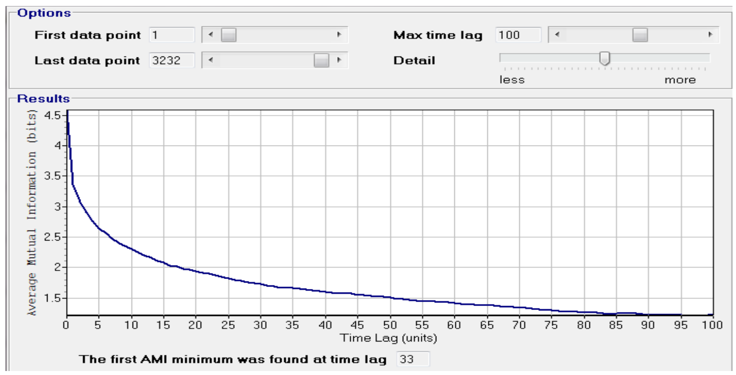


Figure 26.6. Mutual Information – NIKKEI 225

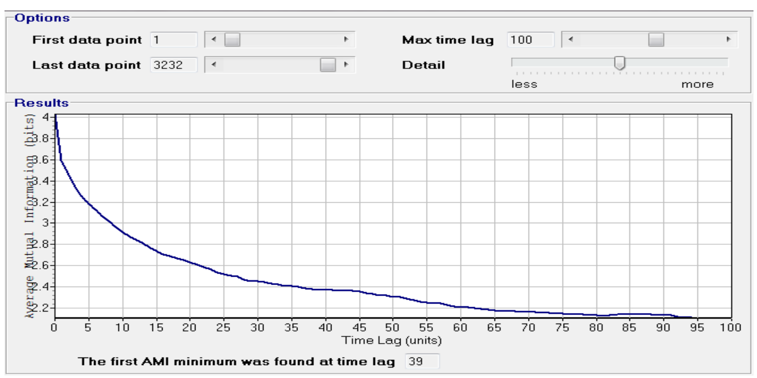


Figure 26.7. Mutual Information – SOFIX

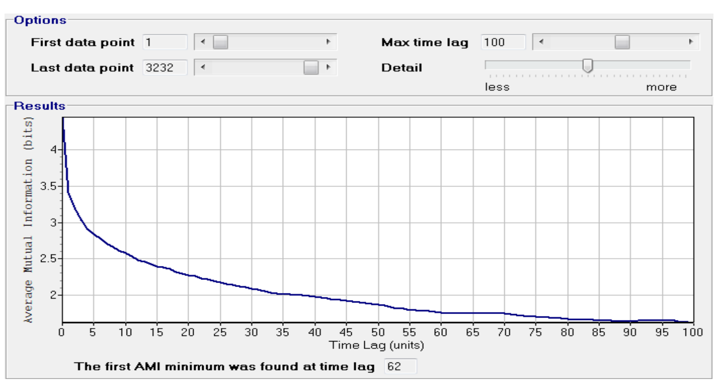


Figure 26.8. Mutual Information – S&P 500

APPENDIX 27

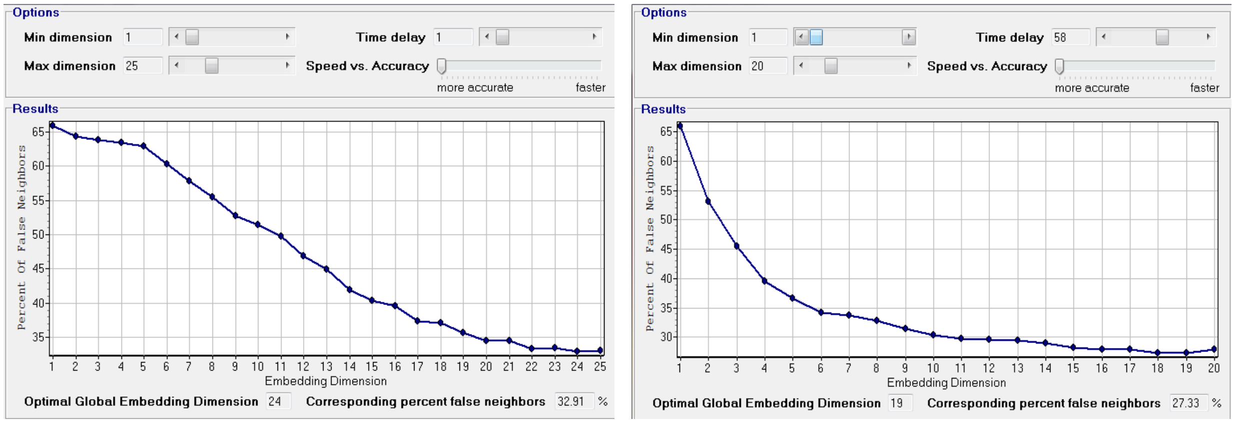


Figure 27.1. False Nearest Neighbors – BET-C (D=1 and D=58)

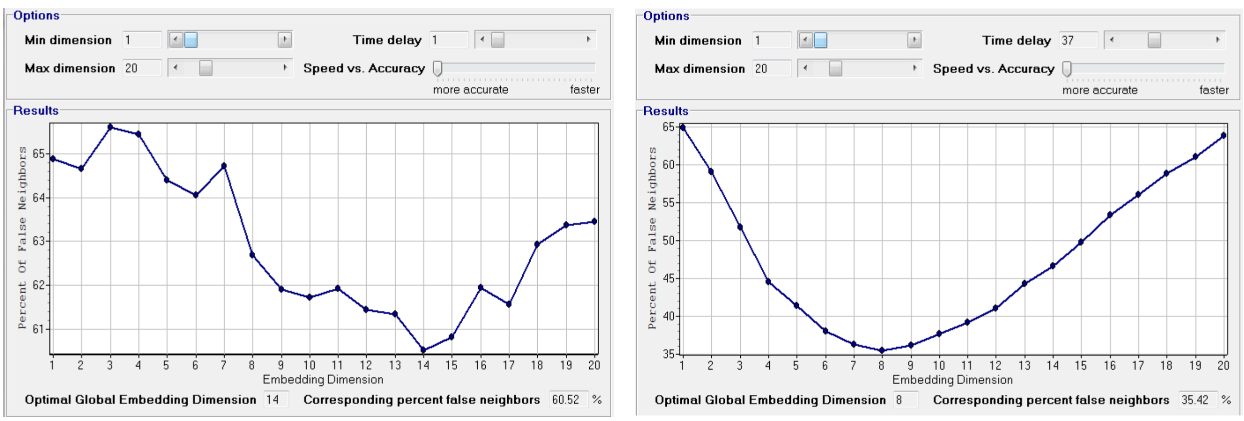


Figure 27.2. False Nearest Neighbors – BUX (D=1 and D=37)

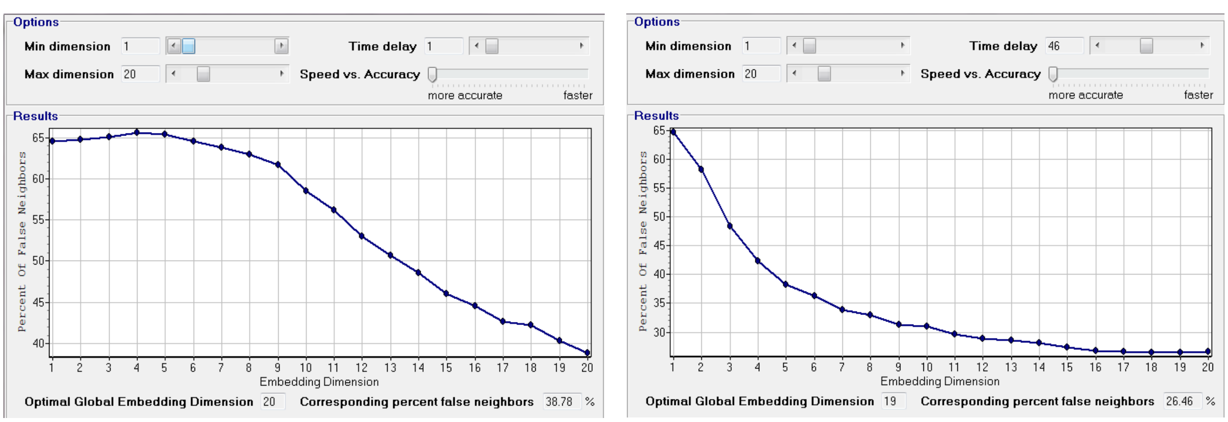


Figure 27.3. False Nearest Neighbors – DAX (D=1 and D=46)

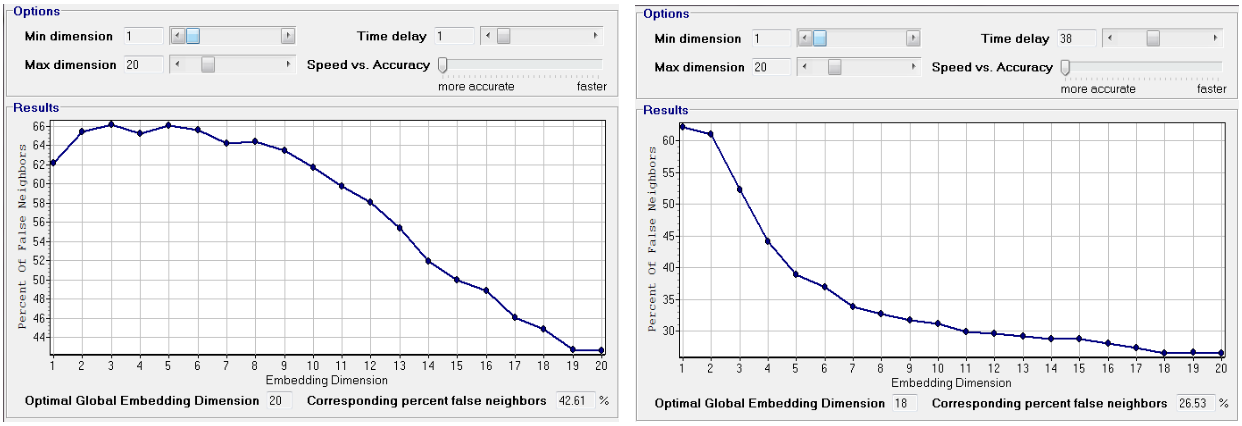


Figure 27.4. False Nearest Neighbors – FTSE 100 (D=1 and D=38)

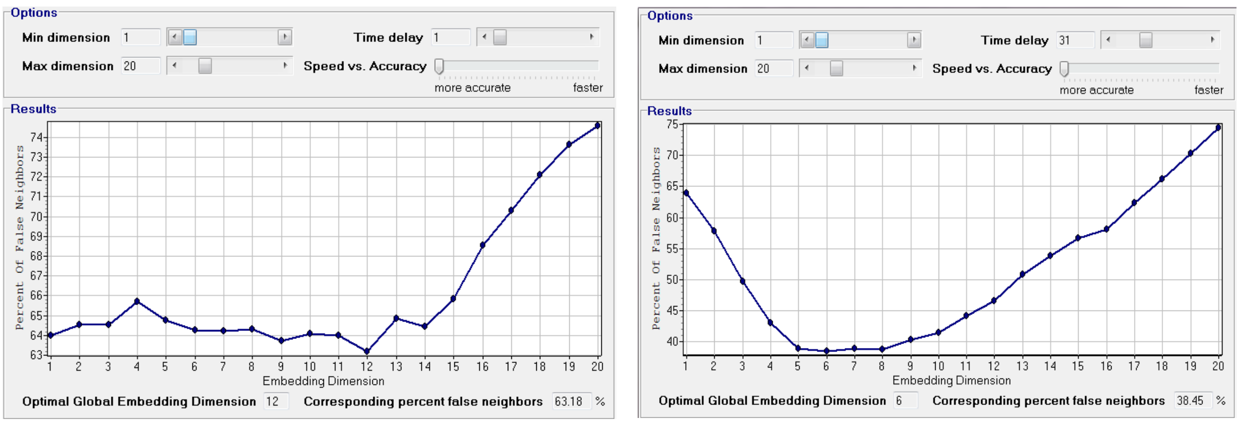


Figure 27.5. False Nearest Neighbors – FTSE MIB (D=1 and D=31)

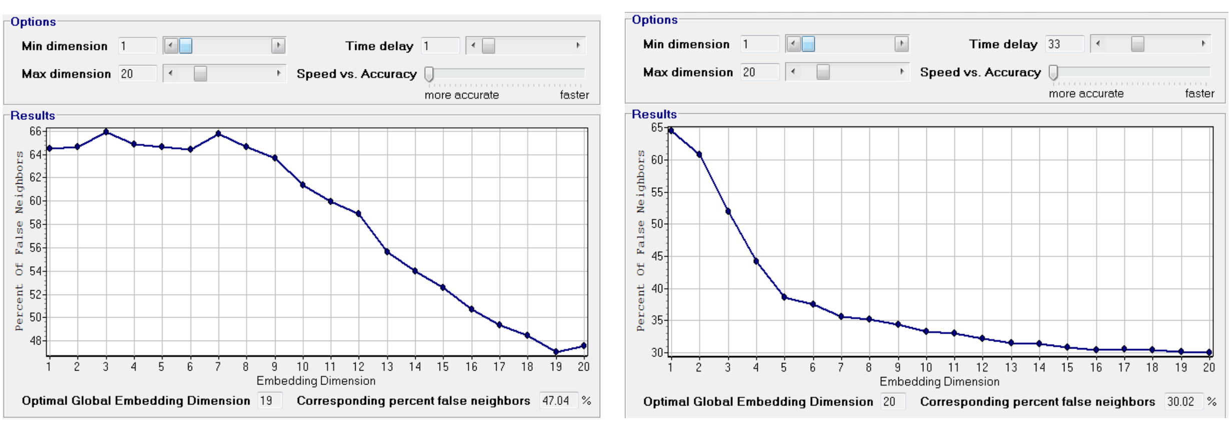


Figure 27.6. False Nearest Neighbors – NIKKEI (D=1 and D=33)

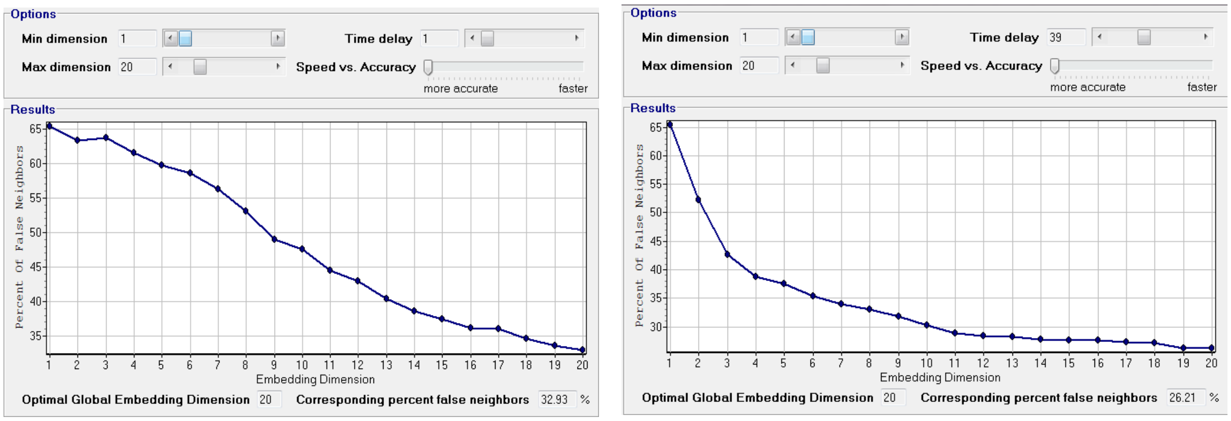


Figure 27.7. False Nearest Neighbors – SOFIX (D=1 and D=39)

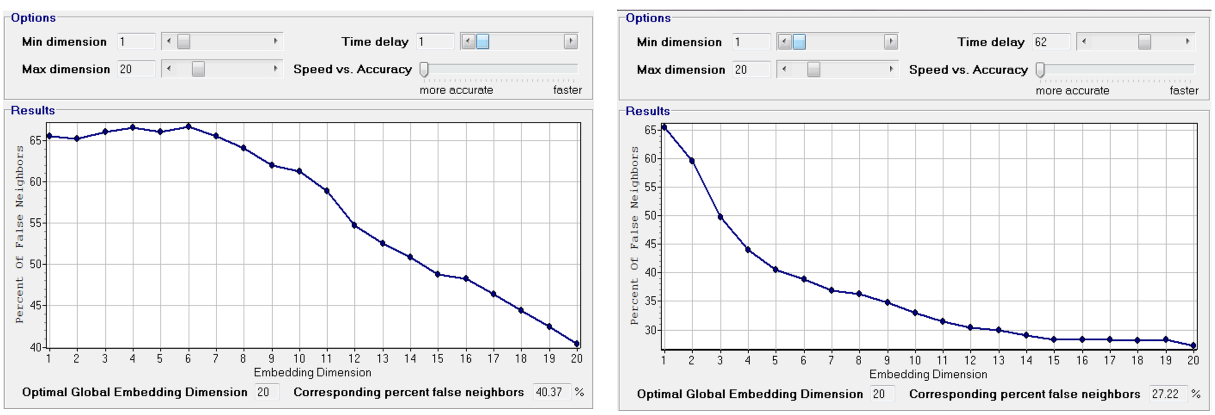


Figure 27.8. False Nearest Neighbors – S&P 500 (D=1 and D=62)

APPENDIX 28

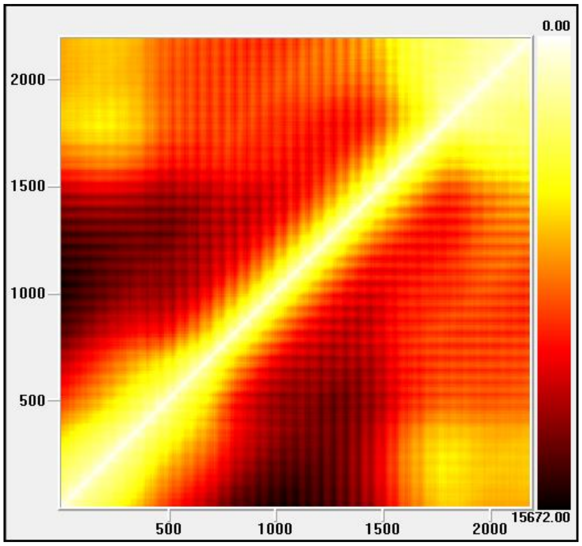


Figure 28.1. Recurrence Plot – BET-C
M=19 si D=58

Figure 28.2. BET-C evolution

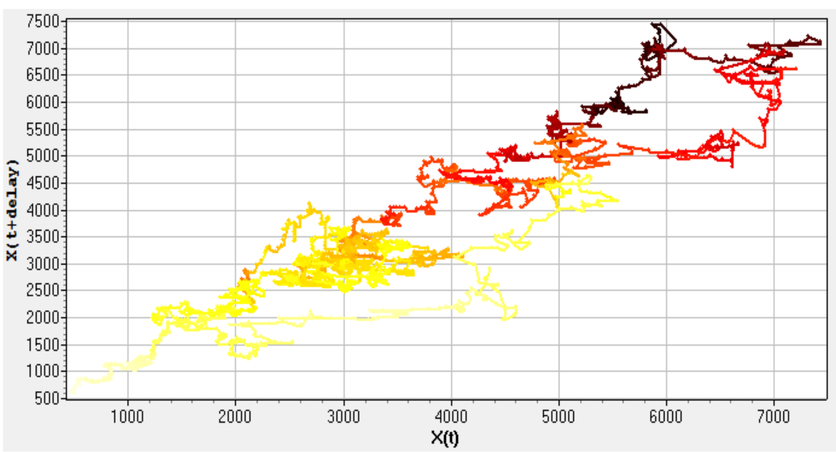


Figure 28.3. Phase Space Plot – BET-C

APPENDIX 29

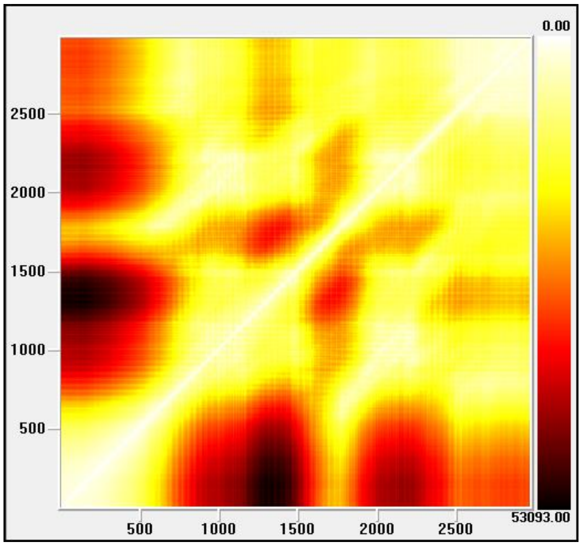


Figura nr.29.1. Recurrence Plot – BUX
M=8 si D=37

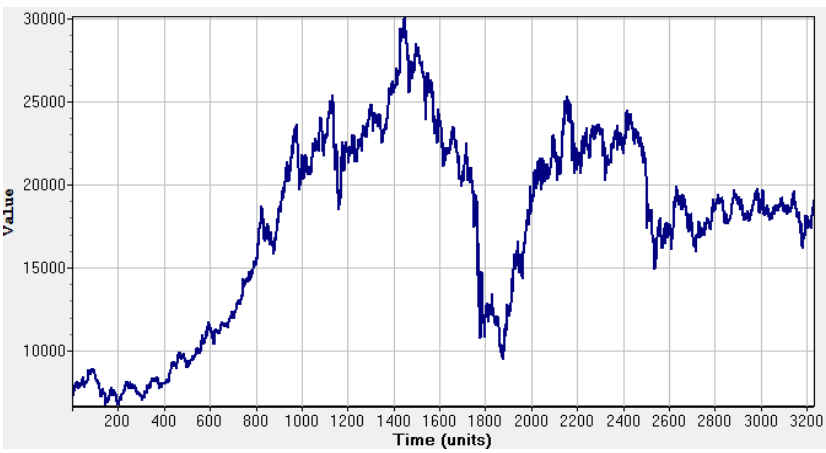


Figure 29.2. BUX evolution

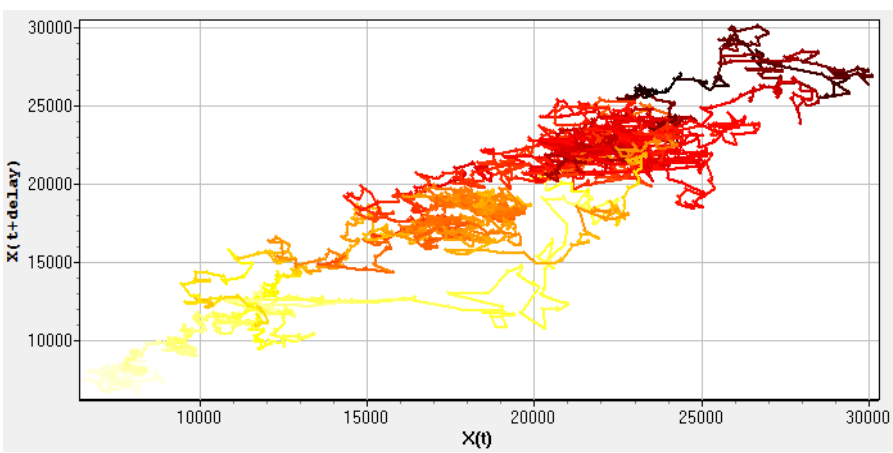


Figure 29.3. Phase Space Plot – BUX

APPENDIX 30

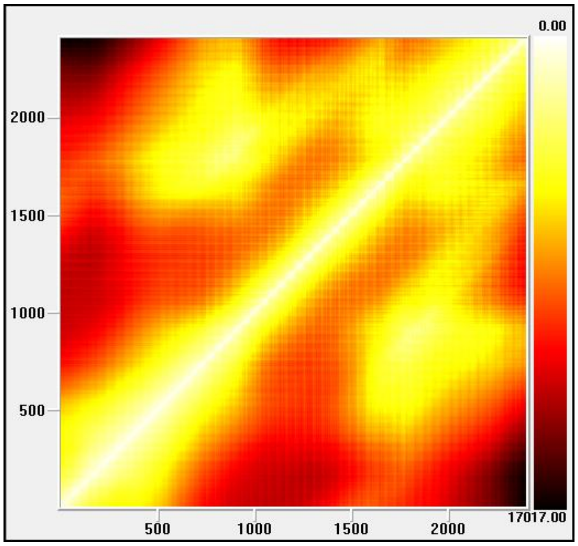


Figura nr.30.1. Recurrence Plot – DAX
M=19 si D=46

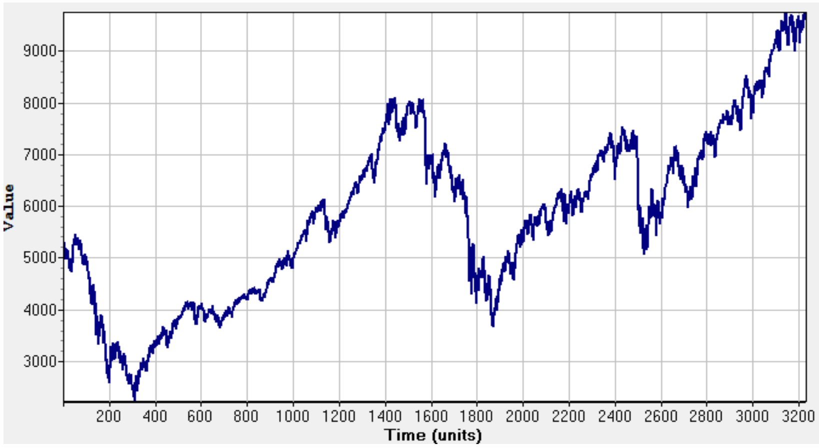


Figure 30.2. DAX evolution

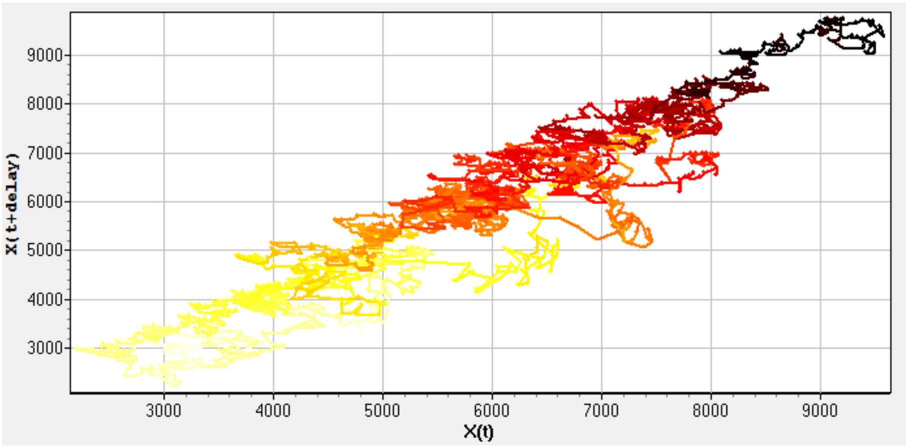


Figure 30.3. Phase Space Plot – DAX

APPENDIX 31

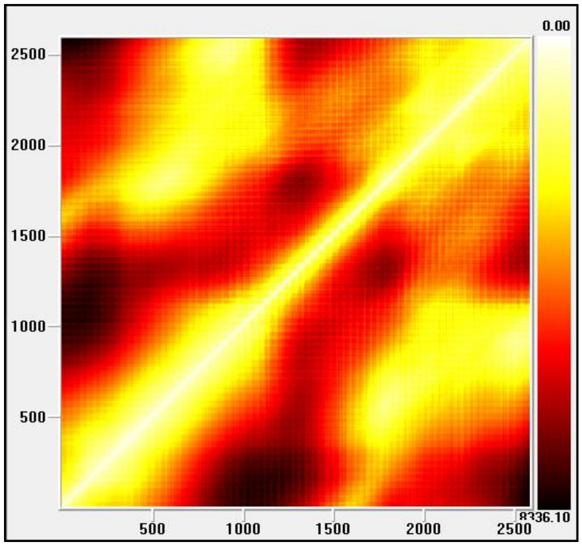


Figure 31.1. Recurrence Plot – FTSE 100
M=18 si D=38

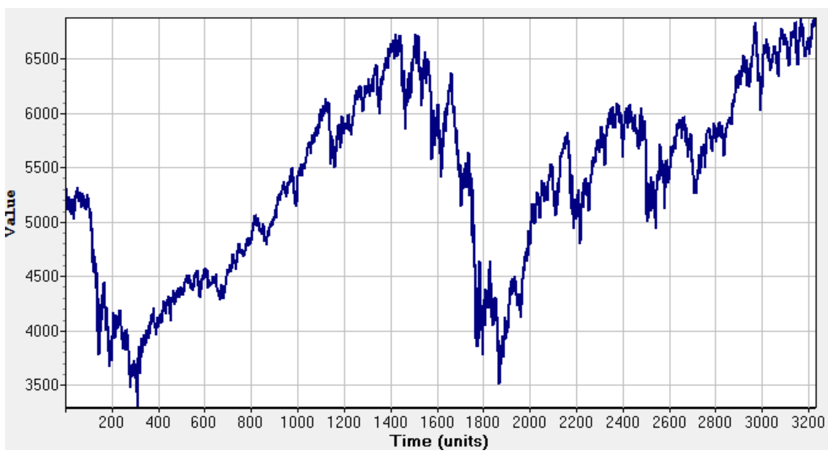


Figure 31.2. FTSE 100 evolution

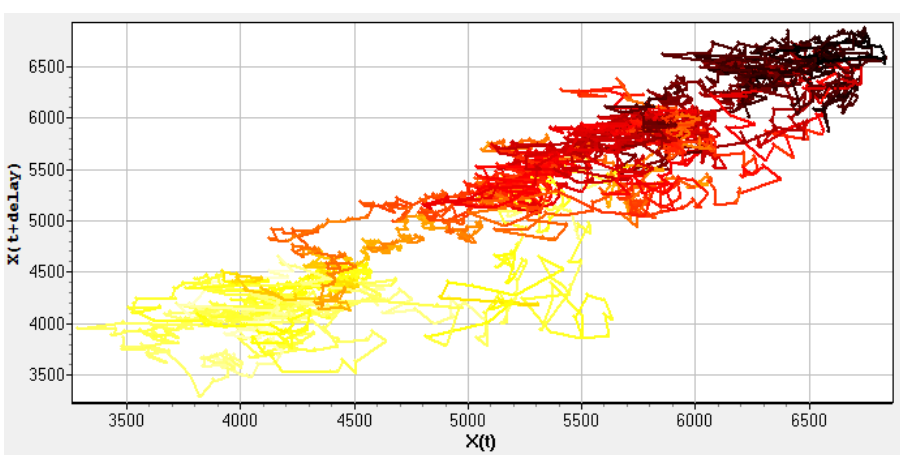


Figure 31.3. Phase Space Plot – FTSE 100

APPENDIX 32

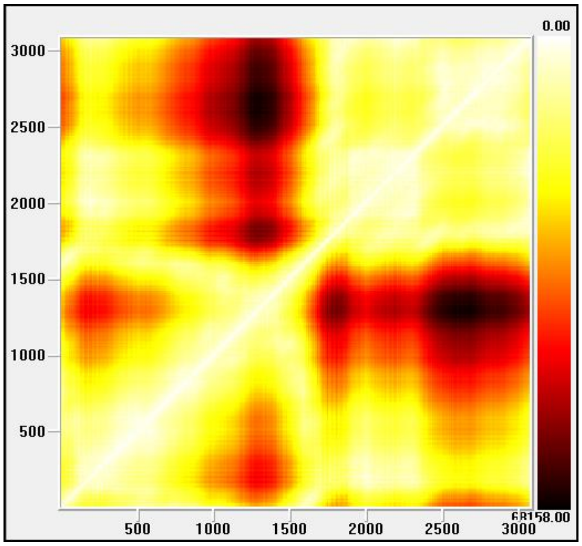


Figura nr.32.1. Recurrence Plot – FTSE MIB M=6 si D=31

Figure 32.2. FTSE MIB evolution

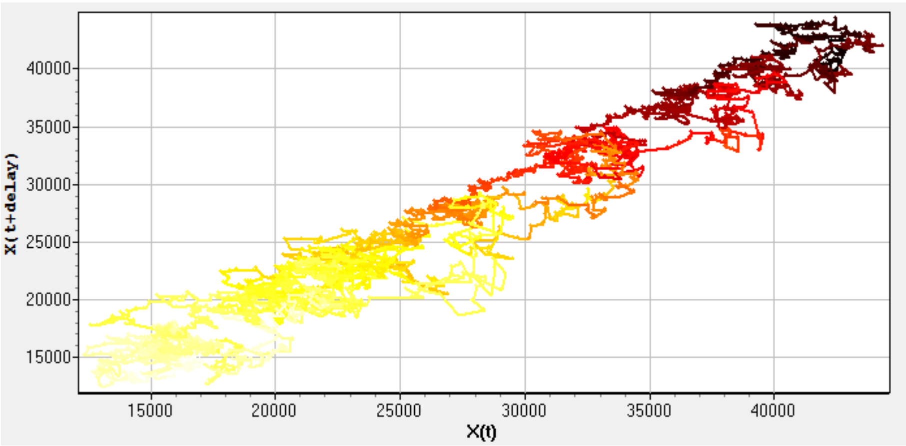


Figure 32.3. Phase Space Plot – FTSE MIB

APPENDIX 33

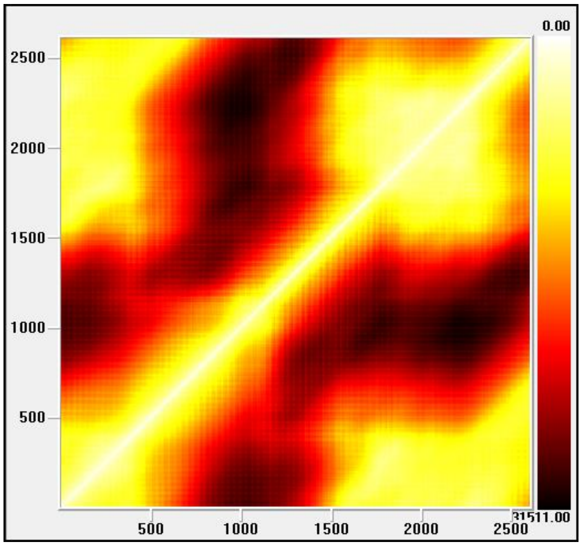


Figura nr.33.1. Recurrence Plot – NIKKEI M=20 si D=33

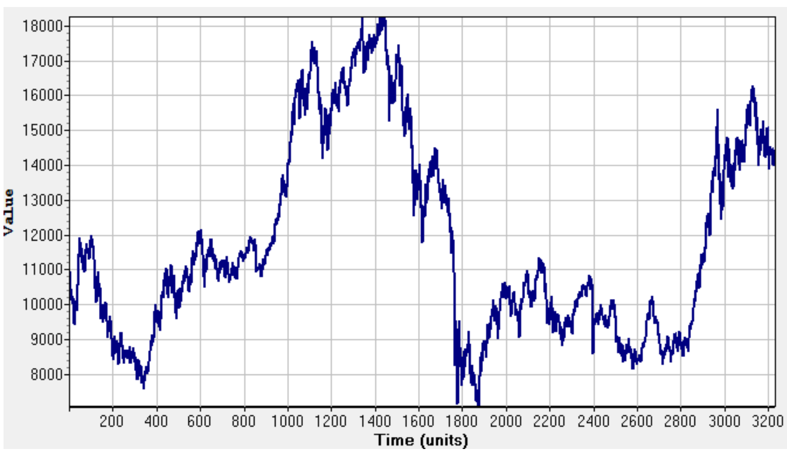


Figure 33.2. NIKKEI 225 evolution

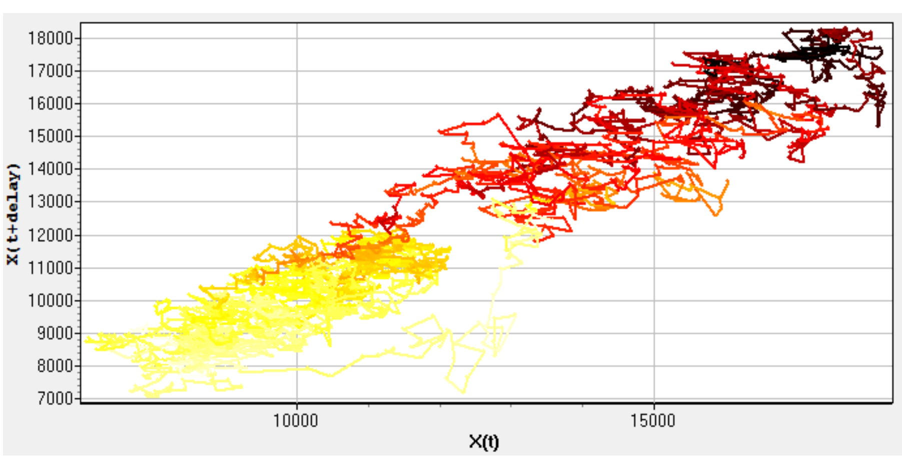


Figure 33.3. Phase Space Plot – NIKKEI 225

APPENDIX 34

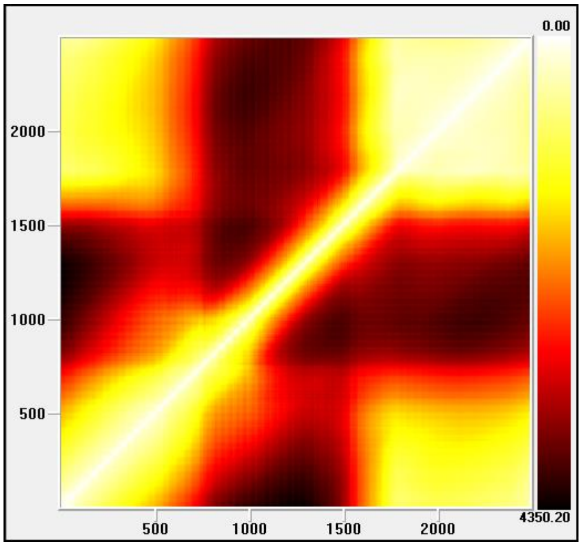


Figura nr.34.2. Recurrence Plot – SOFIX M=20 si D=39

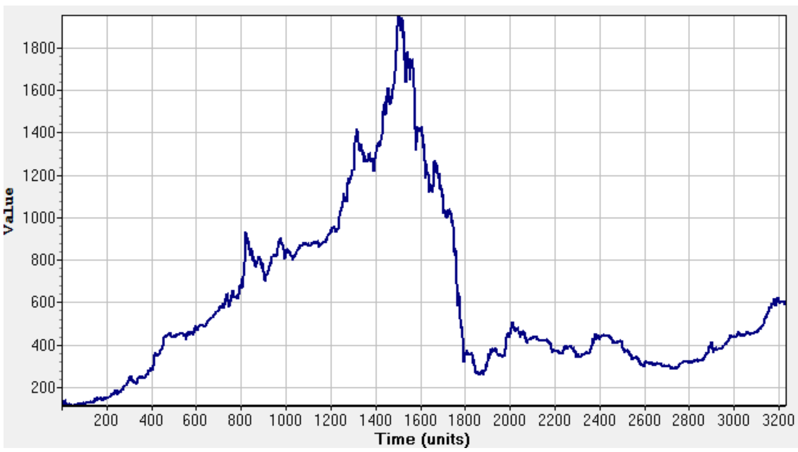


Figure 34.3. SOFIX evolution

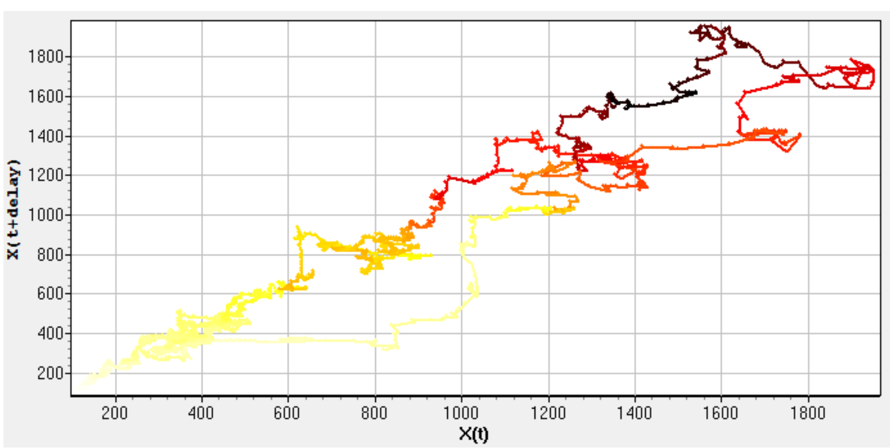


Figure 34.4. Phase Space Plot – SOFIX

APPENDIX 35

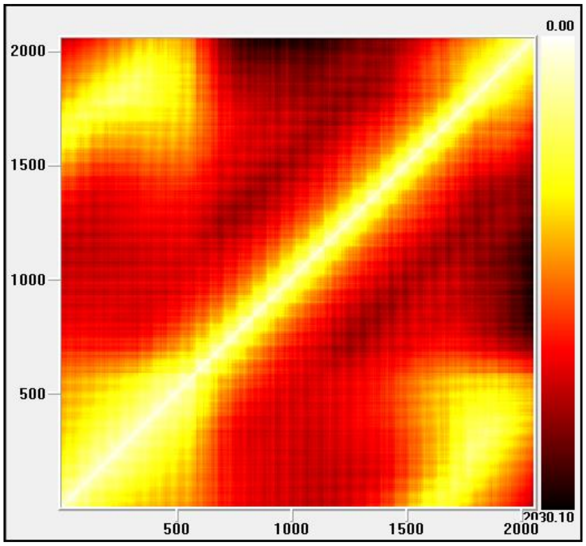


Figura nr.35.2. Recurrence Plot – S&P 500
M=20 si D=62

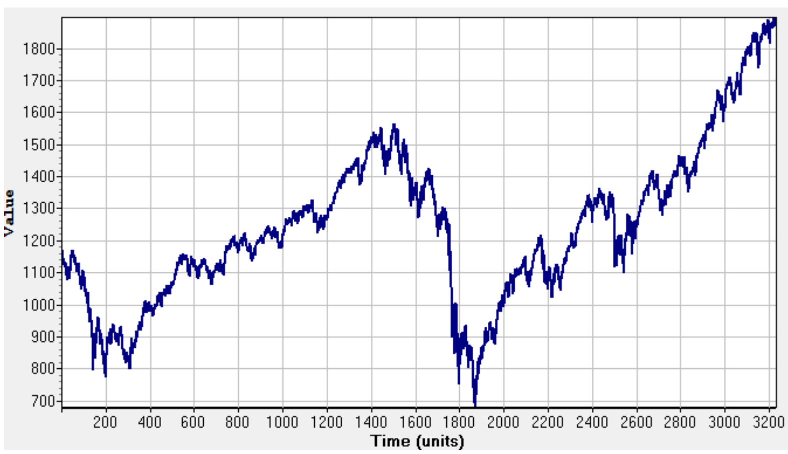


Figure 35.3. S&P 500 evolution

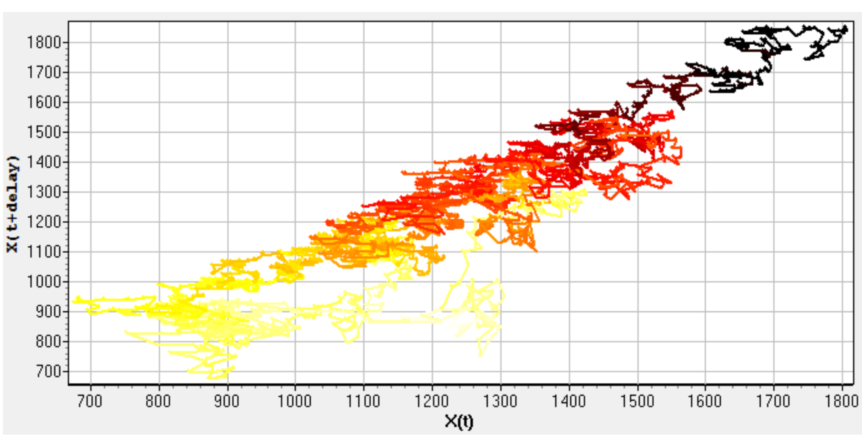


Figure 35.4. Phase Space Plot – S&P 500

APPENDIX 36

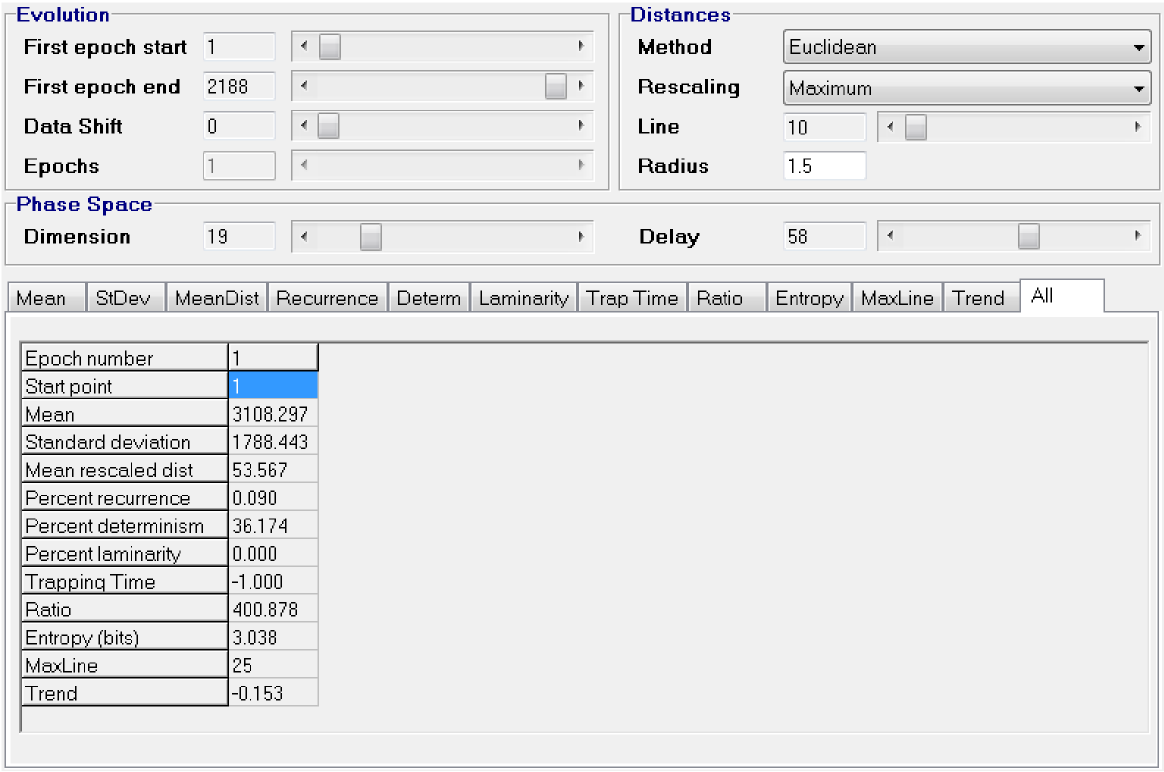


Figure 36.1. RQA - BET-C

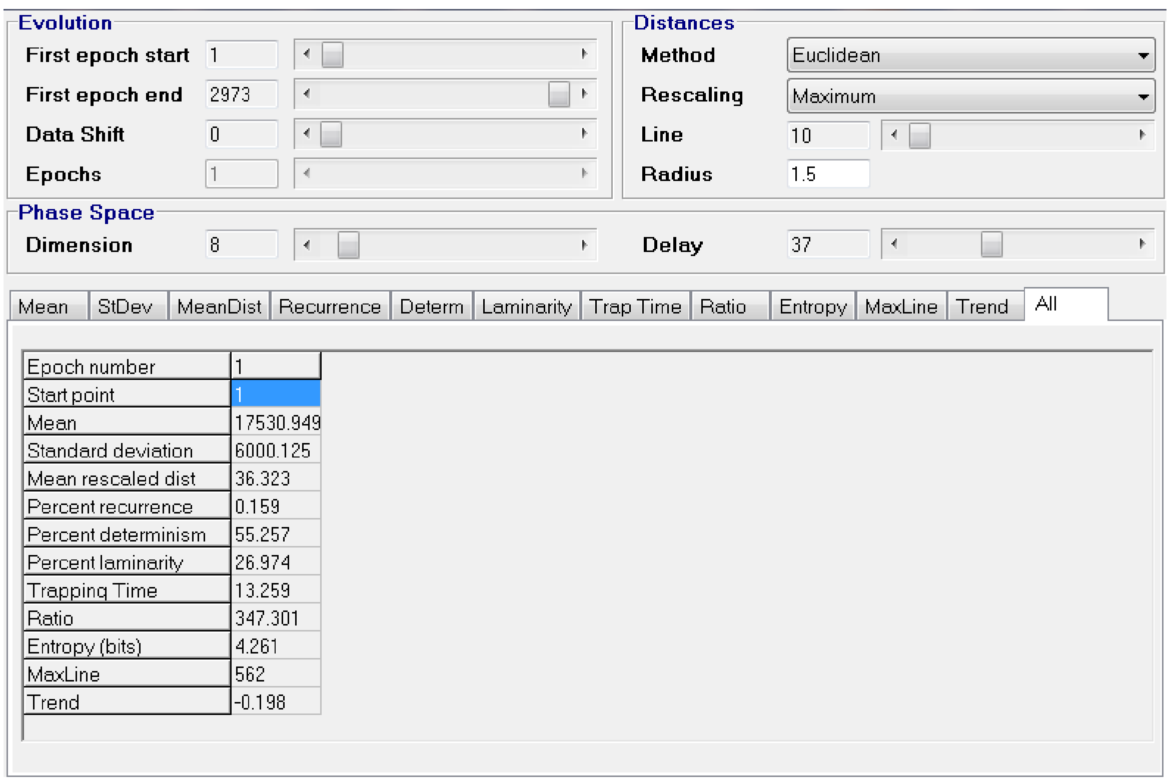


Figure 36.2. RQA – BUX

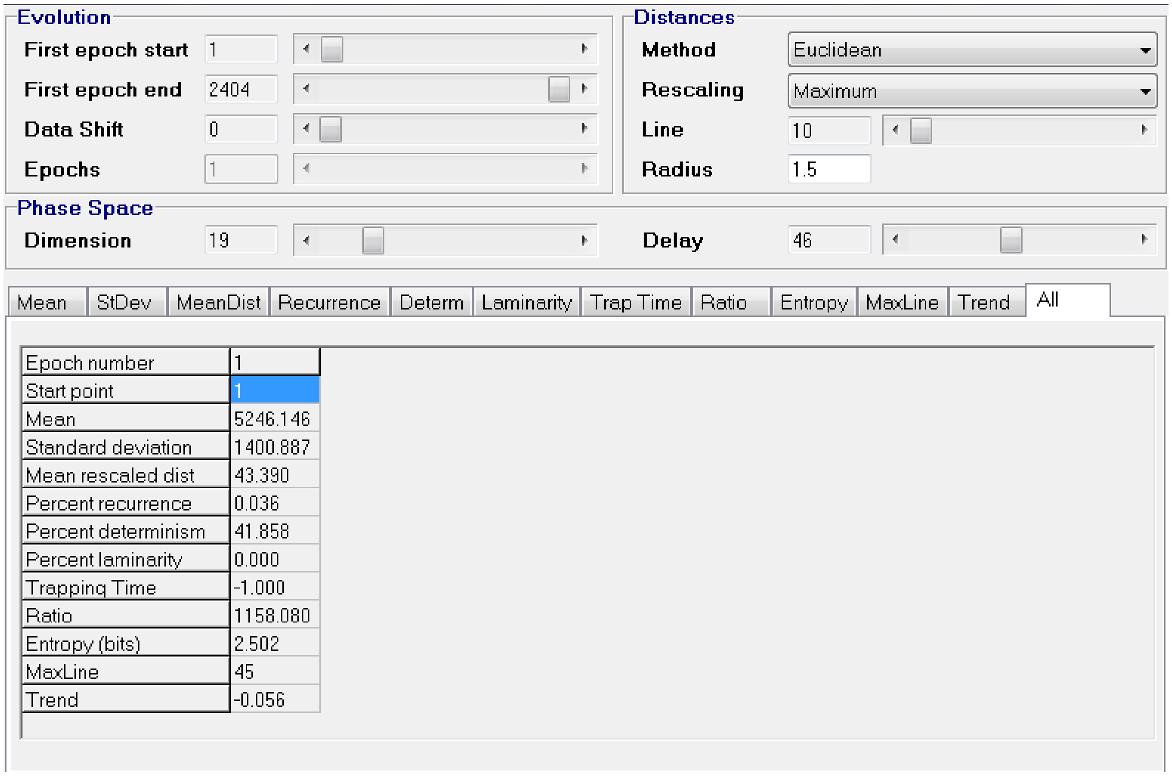


Figure 36.3. RQA - DAX

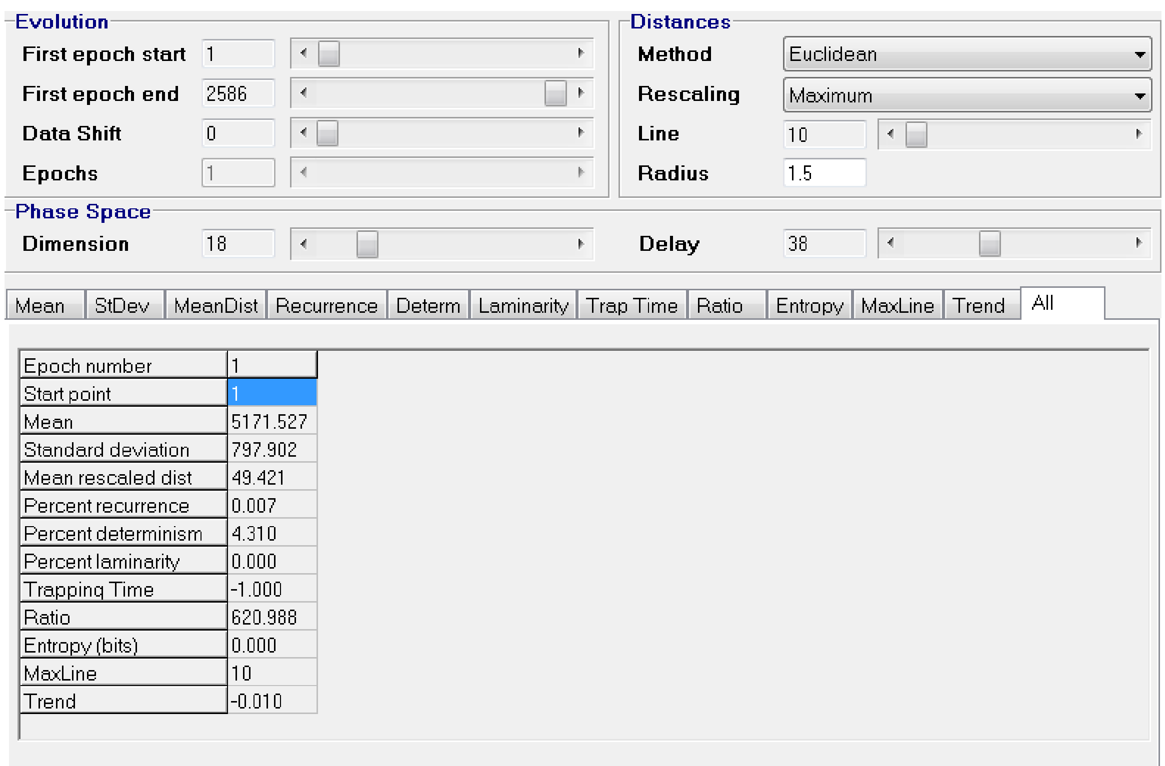


Figure 36.4. RQA – FTSE 100

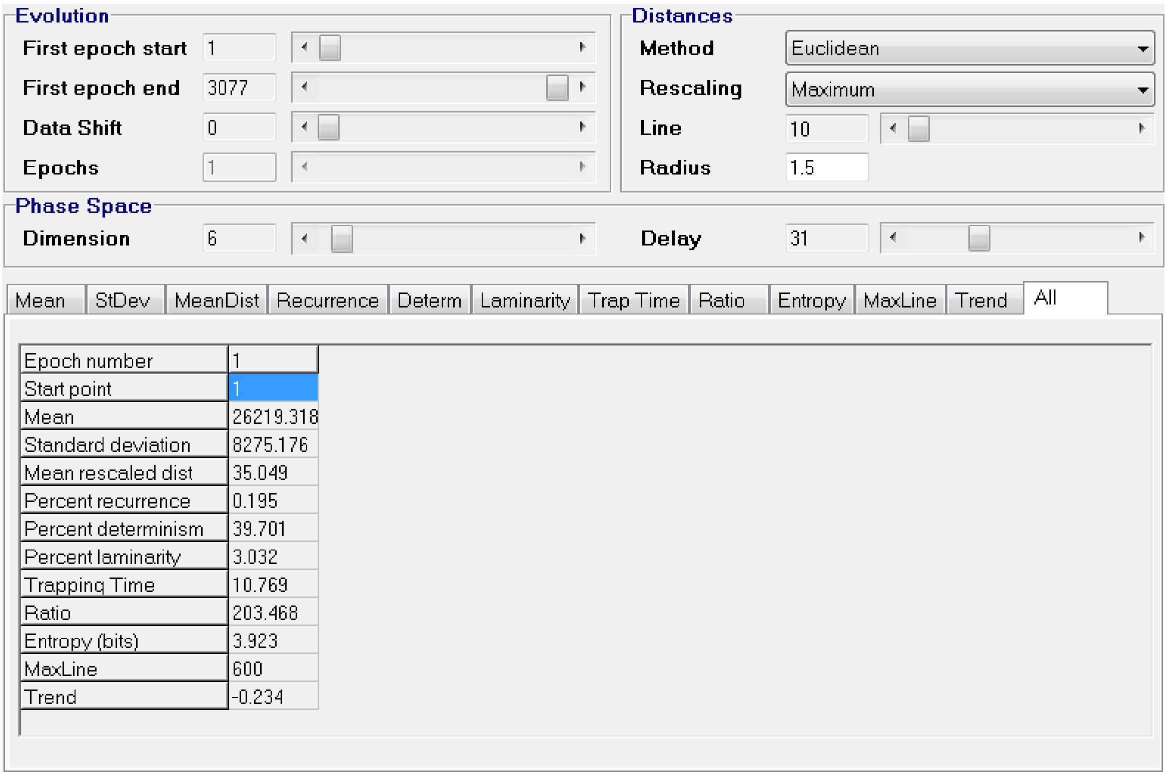


Figure 36.5. RQA – FTSE MIB

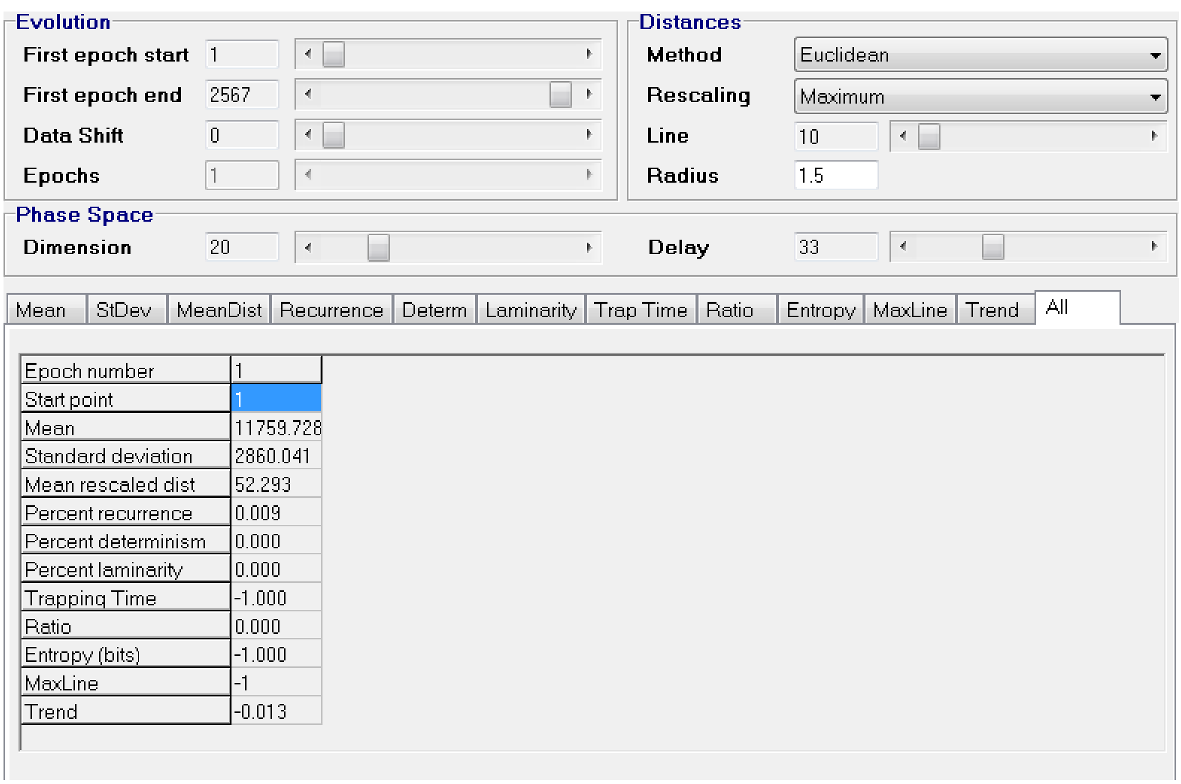


Figure 36.6. RQA – NIKKEI 225

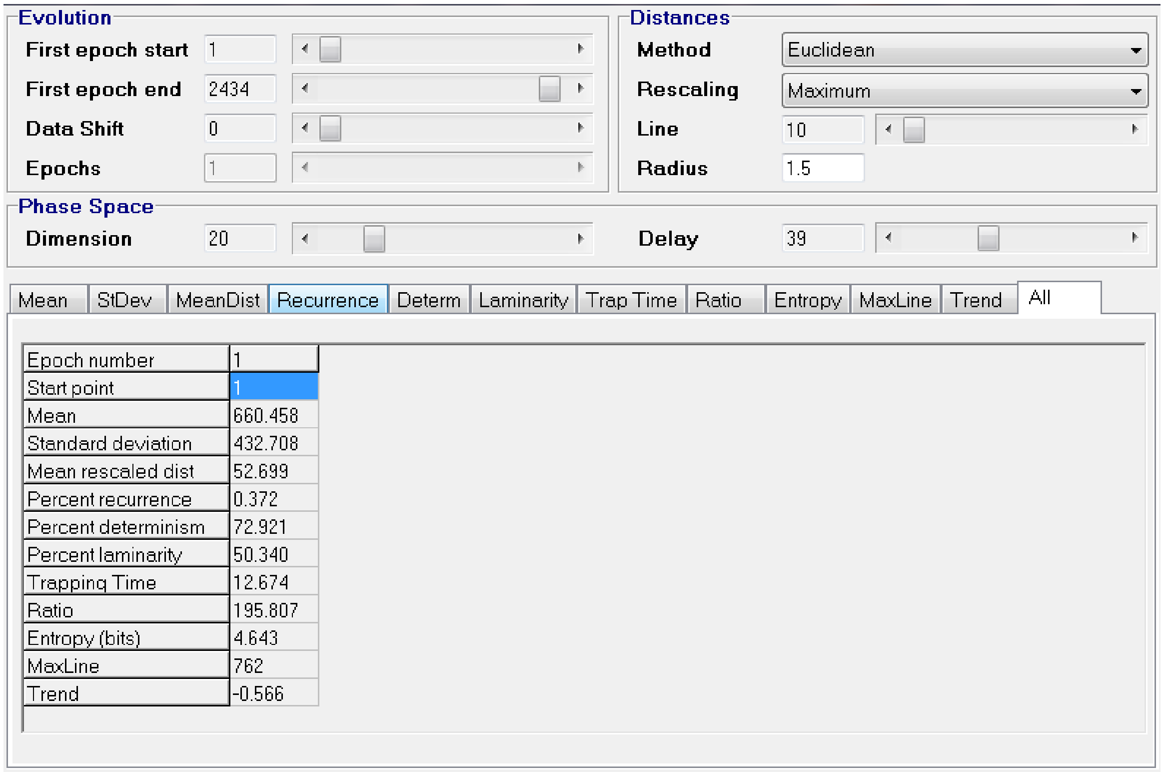


Figure.36.7. RQA – SOFIX

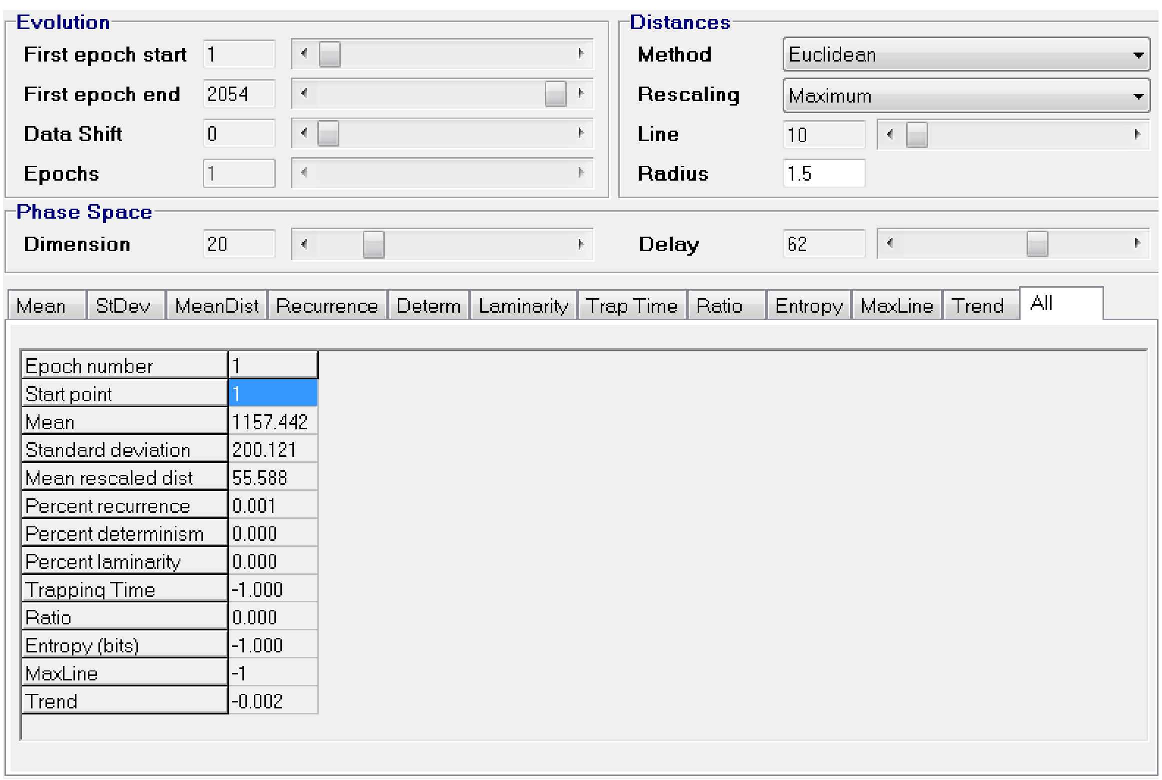


Figure 36.8. RQA – S&P 500

APPENDIX 37

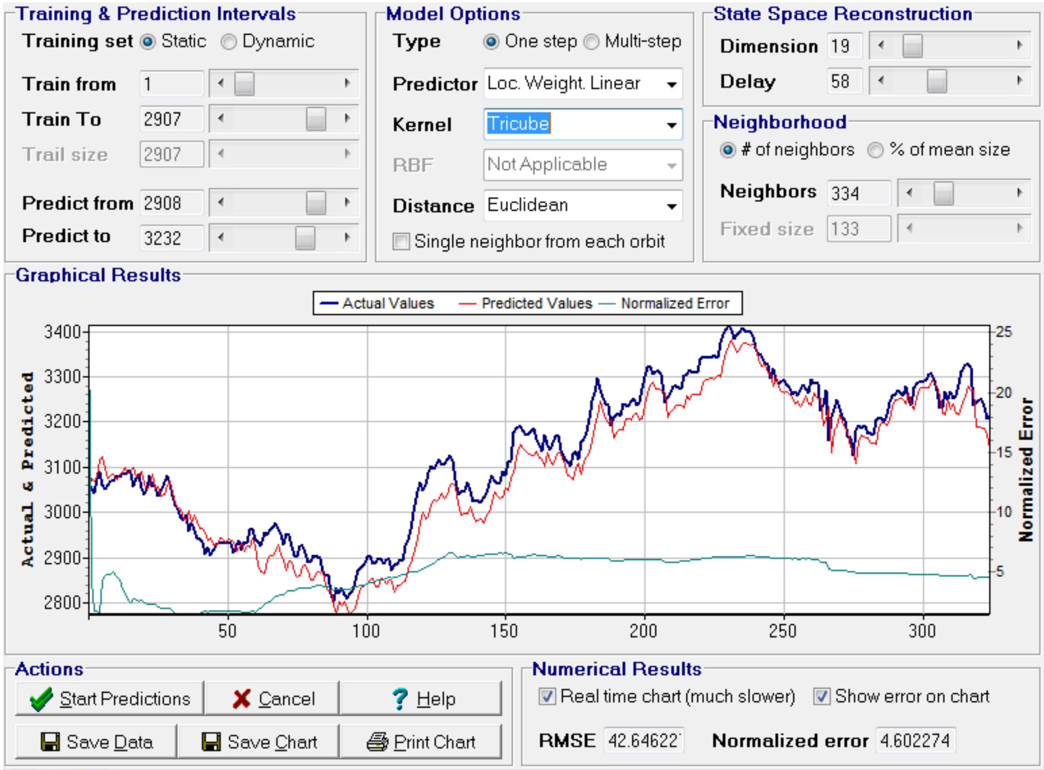


Figure 37.1. Prediction - BET-C

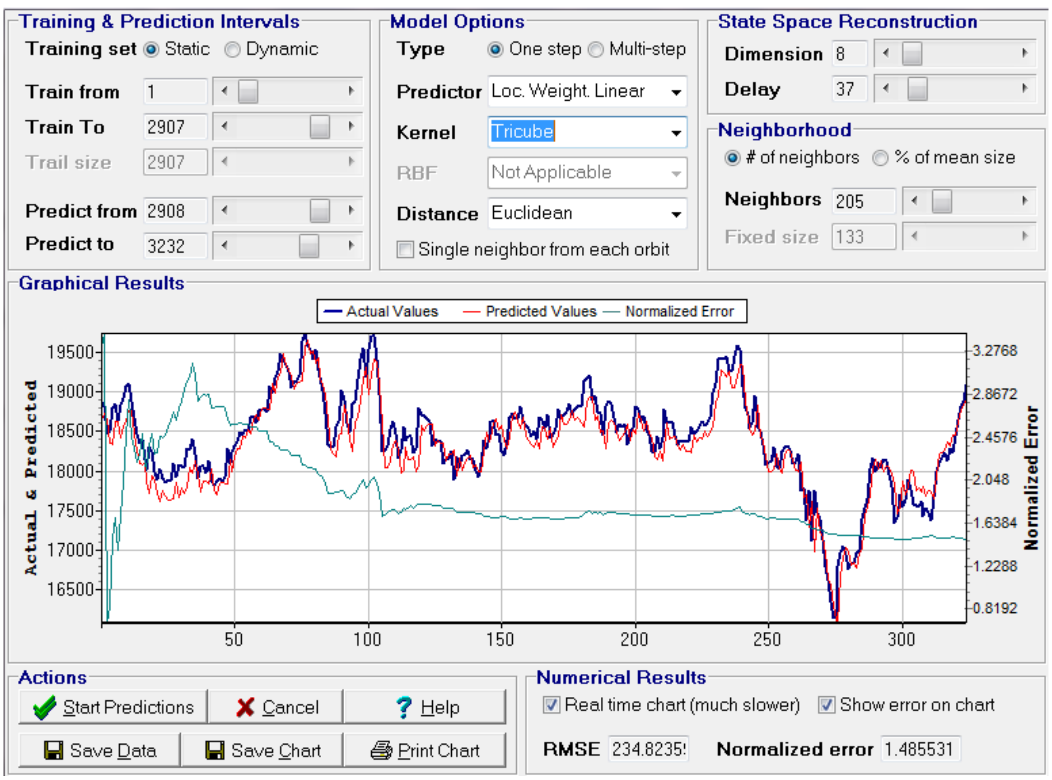


Figure 37.2. Prediction – BUX

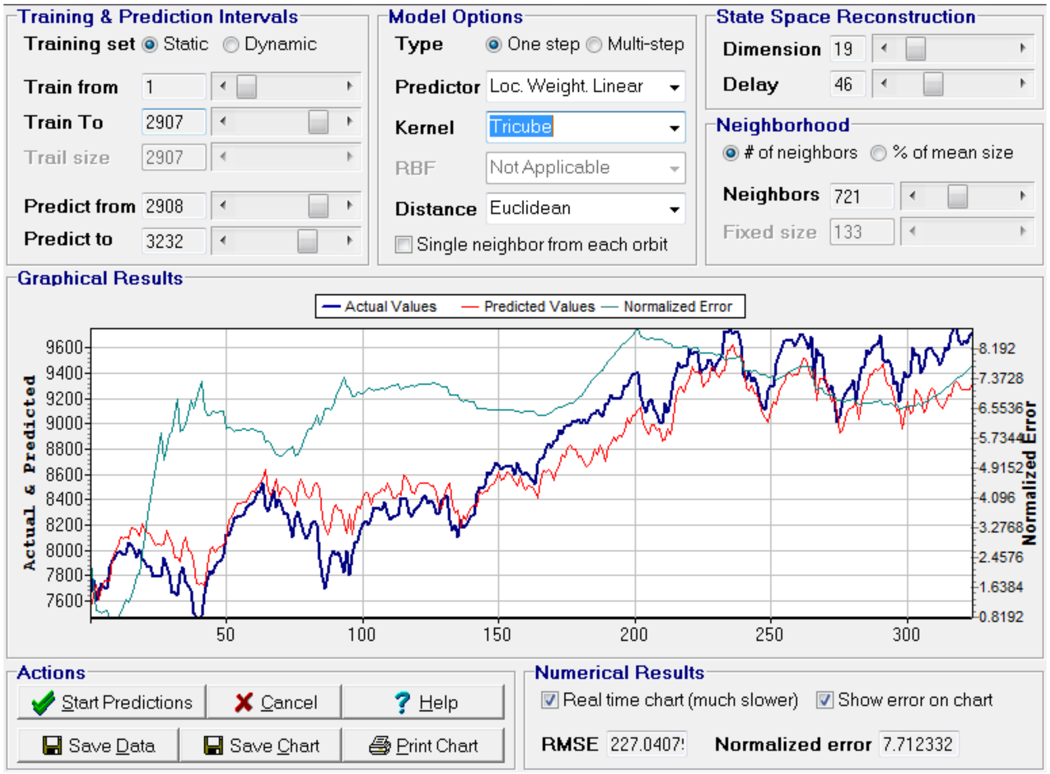


Figure 37.3. Prediction - DAX

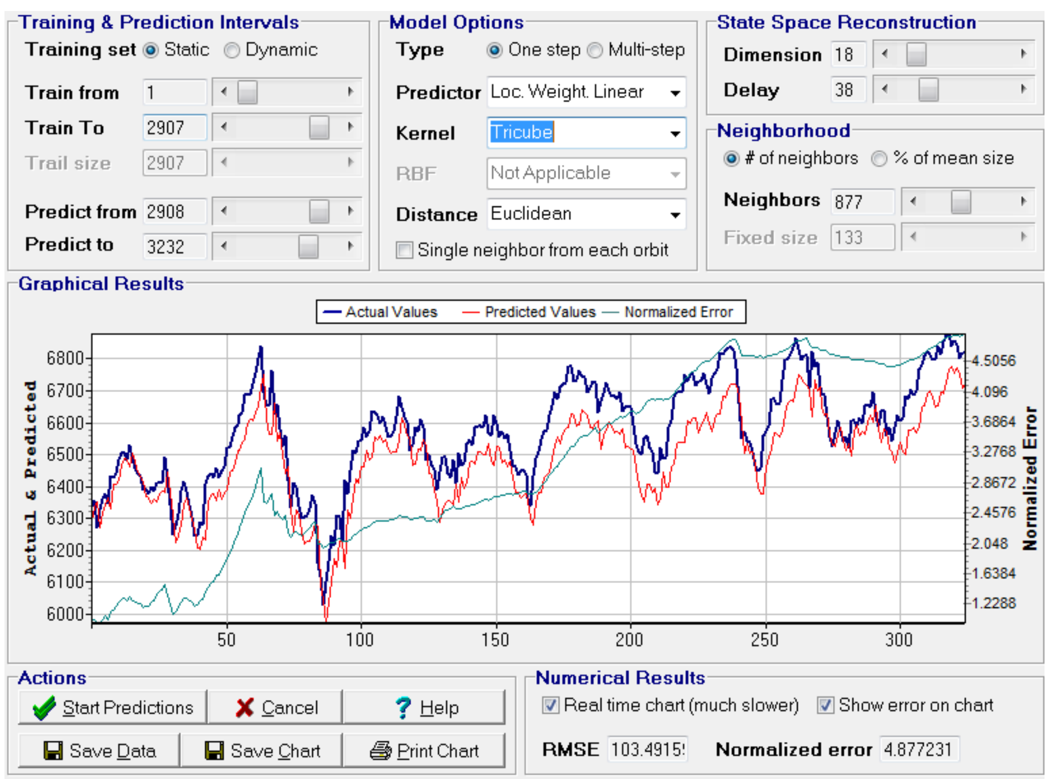


Figure 37.4. Prediction – FTSE 100

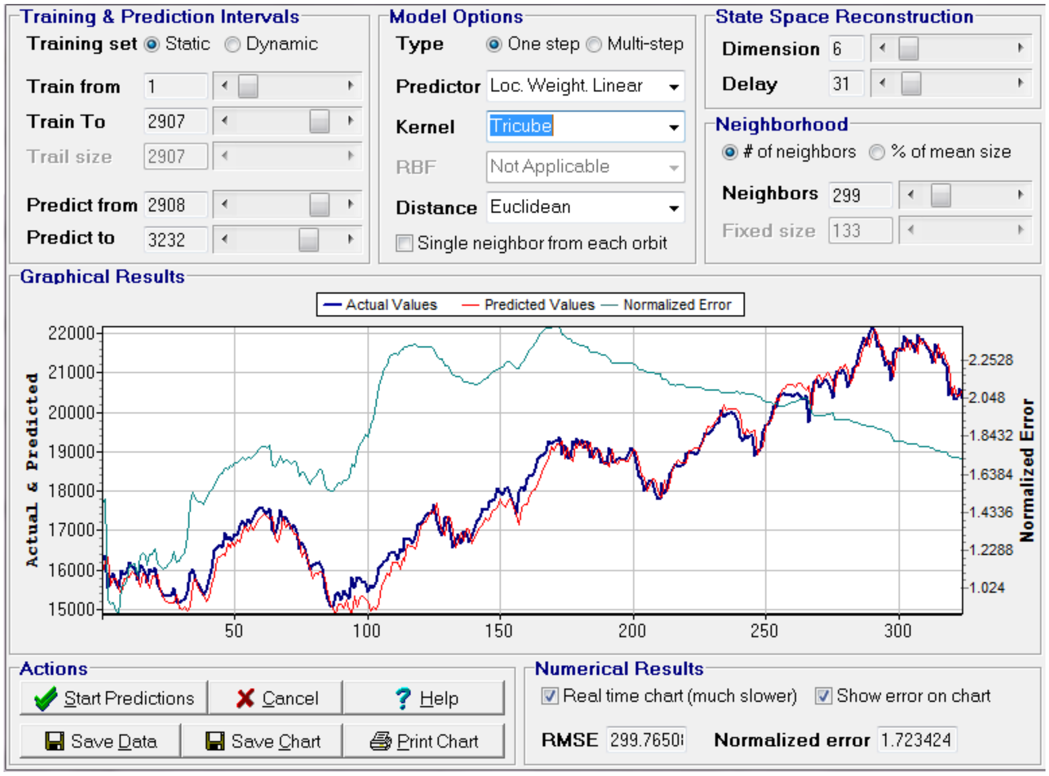


Figure 37.5. Prediction – FTSE MIB

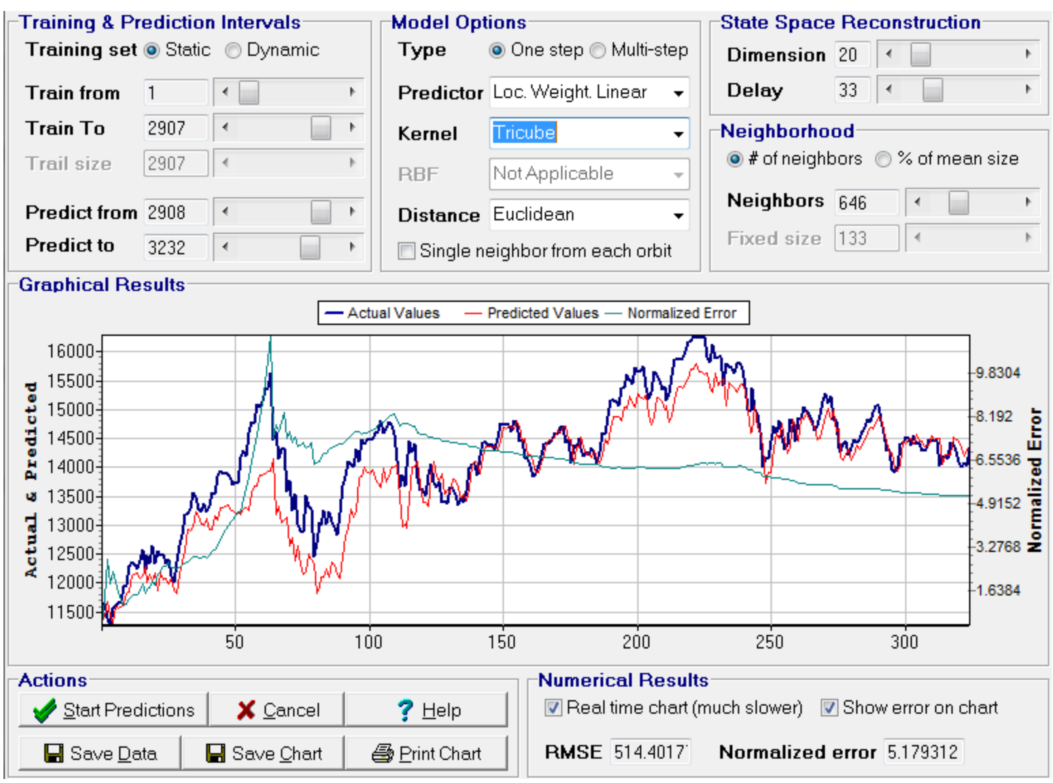


Figure 37.6. Prediction – NIKKEI 225

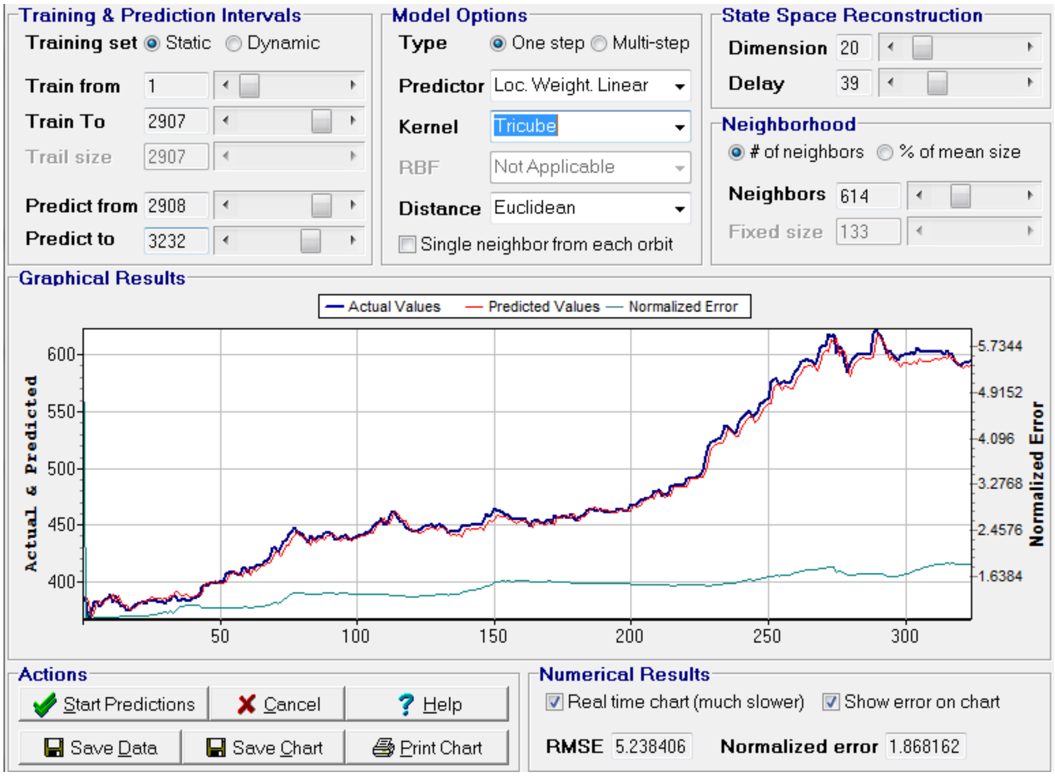


Figure 37.7. Prediction – SOFIX

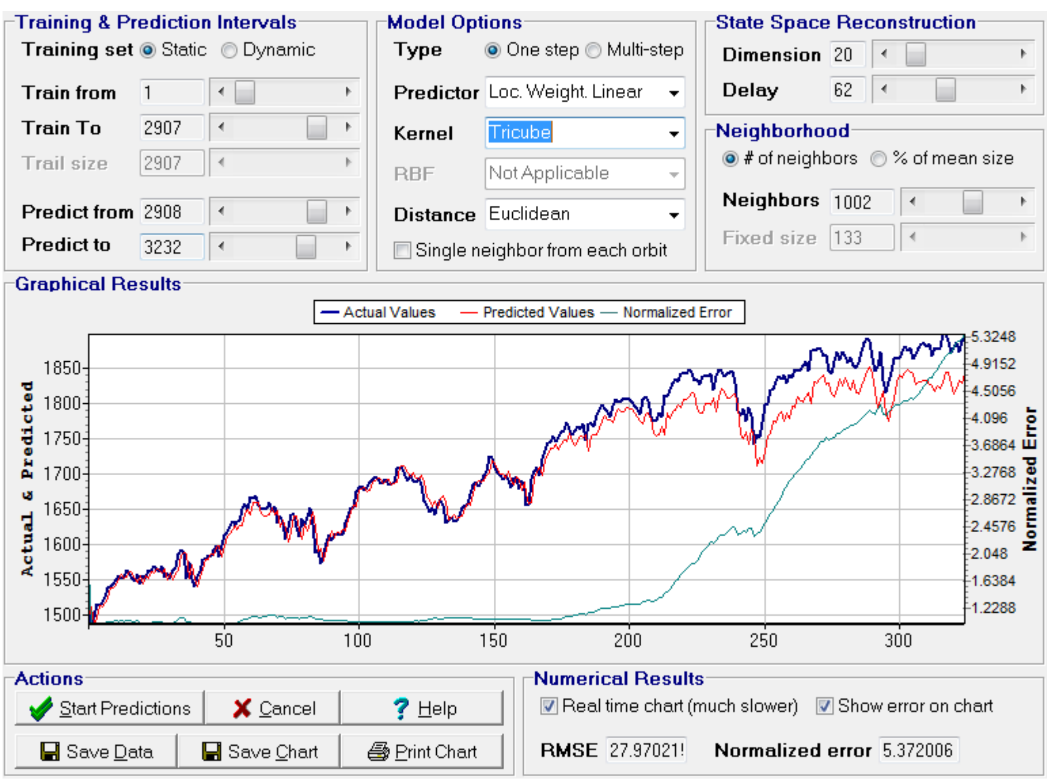


Figure 37.8. Prediction – S&P 500