MODELING THE YIELD CURVE DYNAMICS

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June 2015

CONTENTS:

1. Motivation

- Investors, traders, and everyone that manages risk on the bond market are depending on modelling the term structure of yields for **hedging**, **valuation** and **strategy building**.
- The **long-term interest rates** reflect the **expectations for the evolution of the future interest rates on the short-term**. This assumes that they include information about the future state of the economy.
- In developed states, central banks use the yield curve as a transmission mechanism for the monetary policy, trying to influence it, not only to utilize the information that it sends.
- The Romanian government bond market has made some progress in the last years: the depth of this market, its liquidity and the average maturity of the new issued bonds have all increased comparing with the status from ten years ago.

2. Obiectives

- To calibrate the yield curve from Romania and Germany, using the Nelson Siegel (NS) model with two approaches:

  - Two-Step method

  - Maximum Likelihood (using the Kalman Filter)

- Analyze the results of the two approaches;
- Evaluate the performance of an out-of-sample forecast using NS on multiple time horizons (1 month, 6 months, 1 year) by comparing the results obtained with those generated by a Random Walk model.

3. Literature review

- **Univariate models**: Cox, Ingersoll and Ross (1985); Vasicek (1997); Hull & White (1990). Generally, this factor is considered the short-term yield because despite the fact that in practice yields for different maturities are not perfectly correlated, the correlation coefficient is high.
- **Multivatiate models**: Longstaff & Schwarts (1992), Chen (1994), Balduzzi et al. (1996) și Dai & Singleton (2000). These models impose either the absence of arbitrage or specify a risk premium.
- **Nelson Siegel** is a model with three factors which has been proven it can calibrate the yield curve really well. Extensions with four and five factors have been proposed by Svensson (1994) and Bjork & Christensen (1999). They have demonstrated that these models forecast better compared with others (as the “Random Walk” model) on multiple time horizons.
- Christensen et al. (2011) proposed a **free arbitrage model** in extension of NS, that overpasses the difference between theory and practice.
- Models have evolved even more in the last years by determining the **existing correlations between the macroeconomic indicators and the yield curve**. A reference paper is that of And and Piazzesi (2003), in which they demonstrate that introducing the inflation and the real activity from the economy in forecasting the evolution of the yield curve model is useful, yet the effects are limited. These help explaining the short term and medium term movement of the yields (maturities smaller than 1 year), but in mostly the yield movement on the long term is determined by unobservable factors.

4. Methodology: Nelson Siegel and Diebold-Li extension

Nelson-Siegel (NS) equation:

$$f_{t}(\tau) = b_{1t} + b_{2t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right) - b_{3t}e^{-\lambda_t\tau} \qquad (1)$$

Diebold-Li equation:

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right) \qquad (2)$$

where: y_t = yield to maturity (YTM); τ = maturitaty; β_i = factors, i=1,2,3; λ = decay parameter

- **Level (β_{1t})** is a proxy for 10Y yields and a measure of parallel shifts of the yield curve on all maturities.
- The short-term factor **(β_{2t})** is closely related to the slope of the yield curve. An increase of β_{2t} generates an increase of short-term yields more than long-term yields, therefore the slope of the curve changes, it *flattens*. A decrease would lead to an increase of long end more than of short end of the curve; the yield curve therefore is *steepening*.
- **β_{3t}** measures the curvature of the term structure of interest rates and can be interpreted as a medium term factor.
- **λ_t** governs the exponential decay rate and where the loading on β_{3t} achieves its maximum.

4. Methodology: Fitting the yield curve

By estimating the optimal parameters of the three factors, actually it is necessary to determine the minimum of the following function:

$$\min_{\lambda_t, \beta_{1t}, \beta_{2t}, \beta_{3t}} \sum_{i=1}^{m} \left(\beta_{1t} + \beta_{2t} \left(\frac{1-e^{-\lambda_t \tau_i}}{\lambda_t \tau_i} \right) + \beta_{3t} \left(\frac{1-e^{-\lambda_t \tau_i}}{\lambda_t \tau_i} - e^{-\lambda_t \tau_i} \right) - y_t(\tau_i) \right)^2 = \qquad (3)$$

$$= \min_{\lambda_t, \beta_t} \left(X_{\lambda_t} \beta_t - y_t \right)^T \left(X_{\lambda_t} \beta_t - y_t \right) \qquad (4)$$

In order to estimate the three latent factors and λ, in the financial literature are used two approaches:

- **Two-Step Method** – Diebold-Li (2006);
- **Maximum Likelihood Method (using the Kalman filter)** – Diebold et al (2006).

4. Methodology: Two-Step Method

Nelson Siegel model is nonlinear and to make easier the estimations, the shape parameter λ_t is set a priori:

$$\max_{\lambda} \left(\frac{1 - e^{-\lambda_{\tau}}}{\lambda_{\tau}} - e^{-\lambda_{\tau}} \right) \quad (5)$$

Therefore, the problem becomes simple, linear, and factors can be estimated on each month, using Ordinary Least Square Method.

On the second step, the parameters' dynamic is estimated using an autoregressive vector VAR(1), resulting a forecasting of the curve.

$$\hat{y}_{t+h/t}(\tau) = \hat{\beta}_{1,t+h/t} + \hat{\beta}_{2,t+h/t} \left(\frac{1 - e^{-\lambda_{\tau}}}{\lambda_{\tau}} \right) + \hat{\beta}_{3,t+h/t} \left(\frac{1 - e^{-\lambda_{\tau}}}{\lambda_{\tau}} - e^{-\lambda_{\tau}} \right) \quad (6)$$

where $\hat{\beta}_{t+h/t} = \hat{c} + \hat{\Gamma} \hat{\beta}_t$ (7)

4. Methodology: Maximum Likelihood Method

Using the Kalman filter is constructed the likelihood function, which is maximized in order to obtain the estimated parameters.

The **state space model** (ssm):

1. **Transition equation**: $$\begin{pmatrix} L_t - \mu_L \\ S_t - \mu_S \\ C_t - \mu_C \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} L_{t-1} - \mu_L \\ S_{t-1} - \mu_S \\ C_{t-1} - \mu_C \end{pmatrix} + \begin{pmatrix} \eta_t(L) \\ \eta_t(S) \\ \eta_t(C) \end{pmatrix} \quad (8)$$

t = 1,...,T; L_t, = level; S_t = slope; C_t = curvature

1. **Measurement equation:** $$\begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \\ y_t(\tau_m) \end{pmatrix} = \begin{pmatrix} 1 & \frac{1-e^{-\lambda_t \tau_1}}{\lambda_t \tau_1} & \frac{1-e^{-\lambda_t \tau_1}}{\lambda_t \tau_1} - e^{-\lambda_t \tau_1} \\ 1 & \frac{1-e^{-\lambda_t \tau_2}}{\lambda_t \tau_2} & \frac{1-e^{-\lambda_t \tau_2}}{\lambda_t \tau_2} - e^{-\lambda_t \tau_2} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\lambda_t \tau_N}}{\lambda_t \tau_N} & \frac{1-e^{-\lambda_t \tau_N}}{\lambda_t \tau_N} - e^{-\lambda_t \tau_N} \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} + \begin{pmatrix} e_t(\tau_1) \\ e_t(\tau_2) \\ \vdots \\ e_t(\tau_N) \end{pmatrix} \quad (9)$$

As matrix / vector notation, the state space system can be written as follows:

$$(f_t - \mu) = A\,(f_{t-1} - \mu) + \eta_t \quad (10)$$
$$y_t = \Lambda\, f_t + e_t \quad (11)$$

4. Methodology: MLE – Kalman filter hypothesis

We assume that white noise residuals of the transition equation (η_t) and the measurement disturbances (ε_t) are orthogonal to each other and to the initial state: $\begin{pmatrix}\eta_t \\ e_t\end{pmatrix} \sim WN \left[\begin{pmatrix}0 \\ 0\end{pmatrix} \quad \begin{pmatrix}Q & 0 \\ 0 & H\end{pmatrix}\right],\ E(f_0\, \eta_t') = 0,\ E(f_0\, e_t') = 0.$

In addition, the covariance matrix of vectors η_t and Q is non-diagonal (η_t errors – shocks apply to factors – can be correlated), and the covariance matrix of e_t, H, is diagonal (deviations of yields at different maturities are not correlated), in this way reducing the number of parameters of the model.

To solve the abose system I used Matlab 2015a software, with the toolbox Econometrics Toolbox™:

State equation: $x_t = A_t\, x_{t-1} + B_t\, u_t$

Measurement equation: $y_t = C_t\, x_t + D_t\, \varepsilon_t$

- cov (u_t, ε_t) = 0, u_t~ N(0,σ_u^2) , ε_t~ N(0,σ_ε^2);
- A_t = state matrix
- B_t = state disturbance loading
- C_t = measurement sensitivity
- D_t = observation inovation matrix

Creating SSM model requires to set the initial matrixes by mapping results from VAR(1) model with Two Steps Diebold-Li :

- **Matrix A** from SSM model is asumed to be the tridemensional coefficient matrix of the VAR(1) model;
- **Matrix B** is the lower Cholesky factor of Q of the residual covariance matrix from VAR(1) model: Q=BB';
- **Matrix D** is the root square of the diagonal elements of the covariance matrix of residuals of VAR(1) model: H=DD';
- **Matrix C** results after solving the model because is not set a prori;
- **λ** is set as the value chosen in the Two Steps Model.

5. Data analysis

- Monthly average bid-ask yields (average yield on each month) for Romanian Government bond, starting with 31st of January 2011 until 30th of April 2015.
- **5 maturities** (6M, 1Y, 3Y, 5Y and 10Y) si **52 monthly observations**;
- Increasing market liquidity and international visibility by including various bond series in global bond indexes (JP Morgan and Barclays), as well as upgrading the Romanian bonds rating to *investment grade* (BBB-) lead to an increase in the exposure of nonresidents by the end of 2012. As a result, the price of bonds increased and the yields lowered.

Based on results obtained above, I identified the following stylized facts that can be applied also on the Romanian market:

- The average curve is positive. Thus, the longer the maturity of yields, the bigger is the average yield because the investor receives a premium for the undertaken risk;
- The yield curve takes a variety of shapes along the period analyzed: positive, flatten (when the slope and curvature are close to 0) and humped;
- Yield's dynamics is persistent (the autocorrelation coefficient of lag 1M is 0.9439), while spread's dynamics is less persistent ($\hat{\rho}(1)$ of the slope is slightly smaller, 0.9078);
- Long-term yields are less volatile than the short ones (standard deviation from the average lower as the maturity increases – from 1.37% to 0.53%).

Maturity (Months)	Mean (bps)	Std.dev. (bps)	Minimum (bps)	Maximum (bps)	autocorr_1	autocorr_12	autocorr_30
6.0000	4.3140	1.8073	1.2478	6.6928	0.9445	0.5170	0.2044
12.0000	4.4970	1.8661	1.3938	6.8441	1.0113	0.5655	0.2484
36.0000	5.1302	1.7932	1.9233	7.4562	0.9315	0.5441	0.2545
60.0000	5.3644	1.6757	2.0975	7.5264	0.8115	0.4484	0.2381
120.0000	5.7714	1.3746	2.8400	7.5990	0.5460	0.2857	0.1714

Table 1. Descriptive statistics of the yield curve presented for monthly yields related to the available maturities.

Factor	Mean	Std_dev	Minimum	Maximum	autocorr_1	autocorr_12	autocorr_30
'level'	5.7714	1.3746	2.84	7.599	0.9439	0.4938	0.2964
'slope'	1.4574	0.6785	0.1681	3.0614	0.9078	-0.222	0.0545
'curvature'	0.1751	0.5384	-0.9417	1.2667	0.8345	0.3862	0.2368

Tabel 2. Descriptive statistics that present information regarding the level, slope and curvature, being defined as: level = 10y yield (120m); slope=10y-6m; curvature=2*2y-6m-10y. Indicators presented are: mean, standard deviation, minimum and maximum yields and autocorrelation coefficients of lag 1, 12 and 30. $\hat{\rho}(k) = \frac{\sum_{t=k+1}^{n}(y_t - \bar{y})(y_{t-k} - \bar{y})}{\sum_{1}^{n}(y_t - \bar{y})^2}$

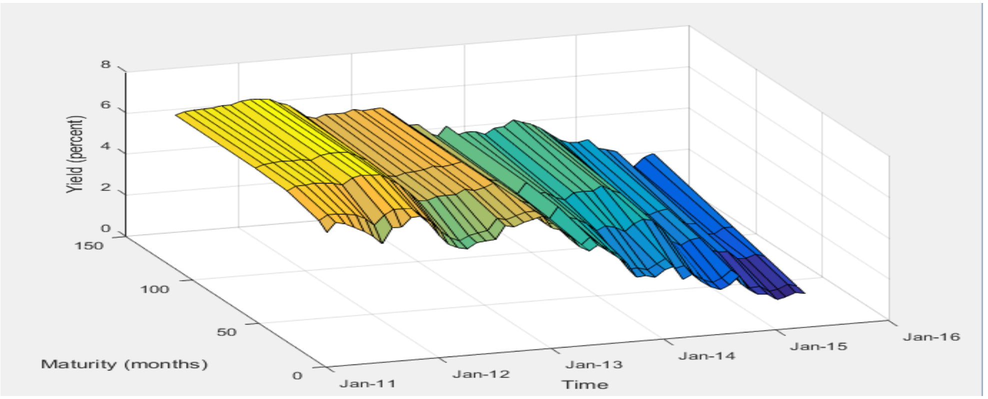

Fig 1. Romanian yield curve during 01.2011 – 04.2015, monthly data, with 6, 12, 36, 60 and 120 months maturities

6. Empirical results: Fitting the Romanian yield curve through the Two-Step Method

λ = 0.0597761 for τ = 2.5 years (30 months) maturity
λ is obtained by numerical optimization: http://www.wolframalpha.com/

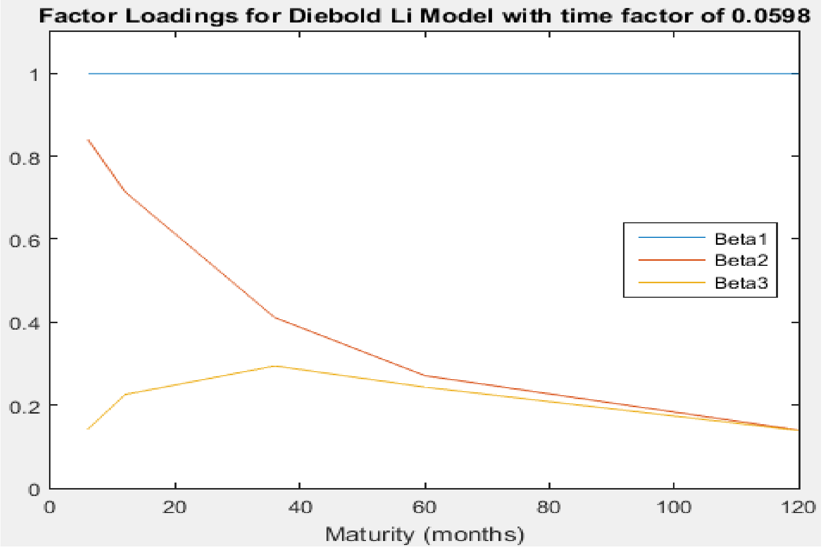

Fig 2. Decomposition of the yield curve, thus of the loadings on the three factors, β_{1t}, β_{2t} and β_{3t}: 1, $\left(\frac{1-e^{-\lambda_t \tau}}{\lambda_t \tau}\right)$ and $\left(\frac{1-e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau}\right)$.

The model becomes linear and by applying OLS are estimated $\{\hat{\beta}_{1t}, \hat{\beta}_{2t}, \hat{\beta}_{3t}\}$

- Level = 10y maturity yield;
- Slope = 10y yield – 6m yield;
- Curvature = 2 * 2y yield – 10 y yield – 6m yield;

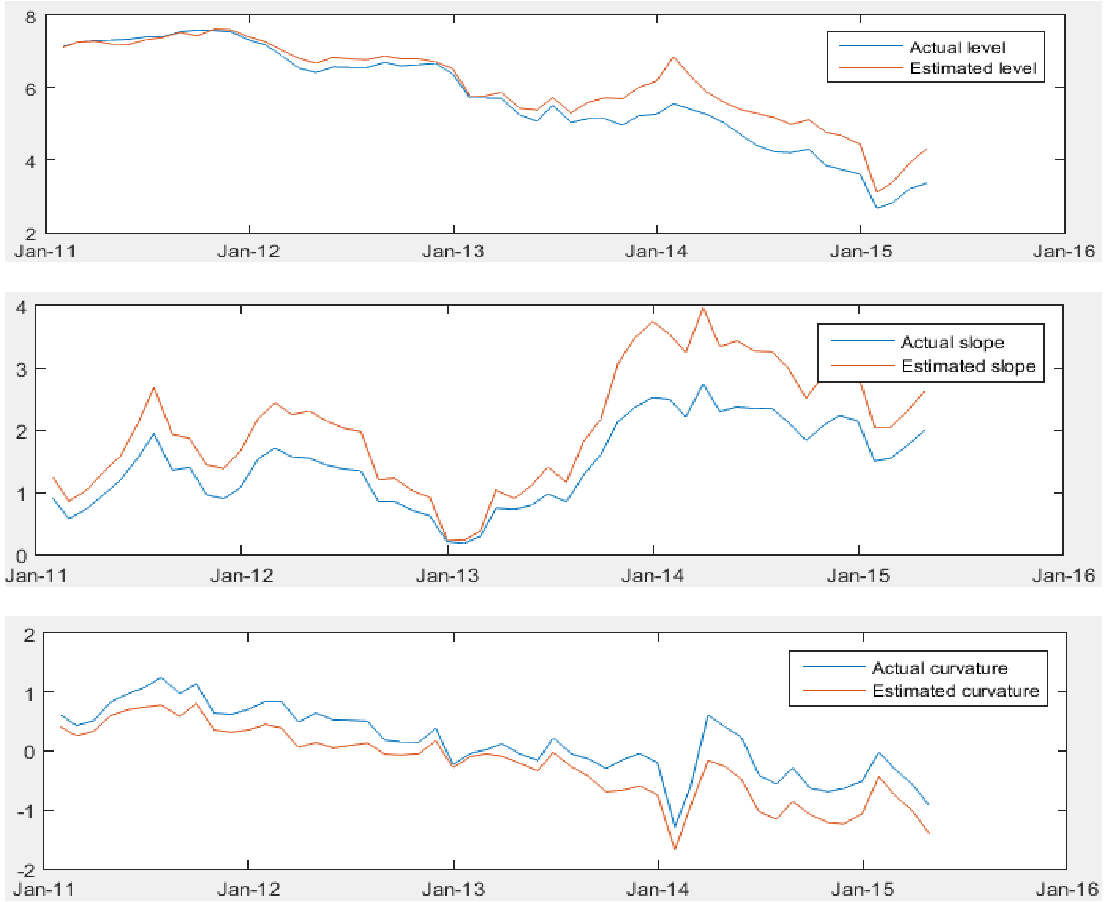

Fig 3. Estimated vs. actual level, slope and curvature.

Correlation between actual vs estimated: 0.9783, -0.9959, 0.9633

6. Empirical results: Fitting the Romanian yield curve through the Two-Step Method

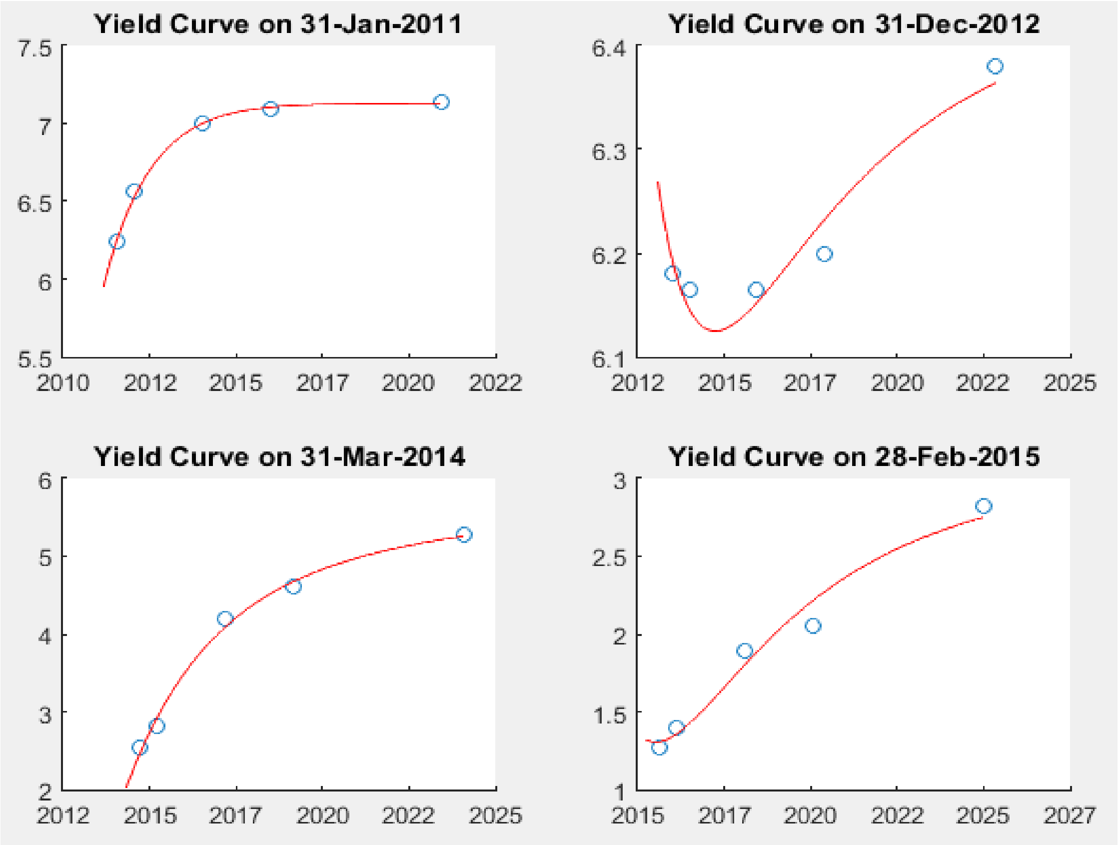

Fig 4. Fitted yield curves at various moments in time. Dots are the actual yields from the markets.

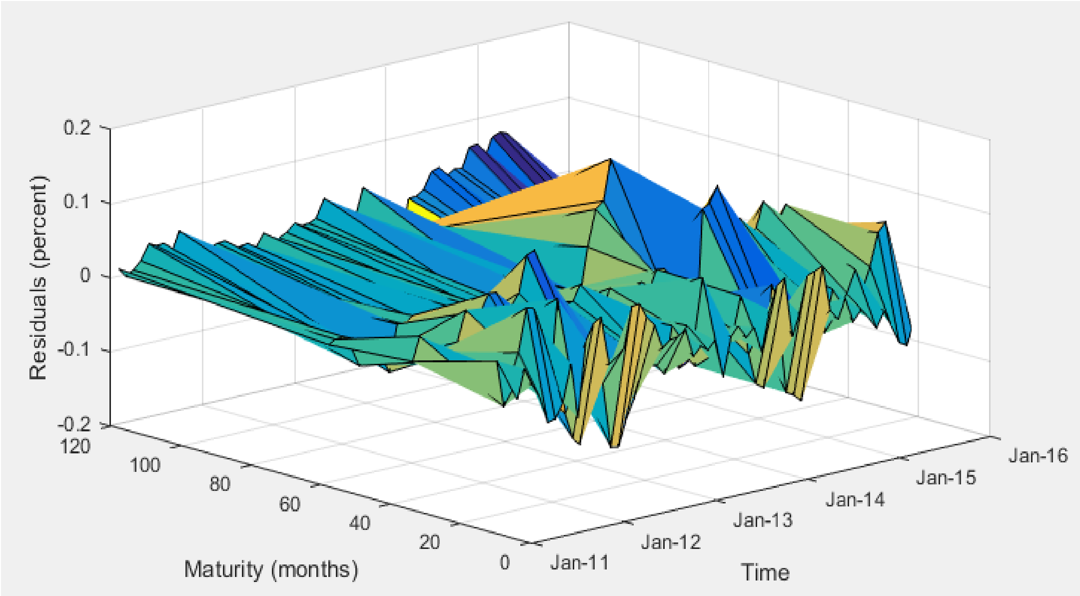

Fig 5. Residuals plot representation

6. Empirical results: Fitting the Romanian yield curve through the Two-Step Method

Before making the forecast, it is necessary to perform an additional analysis of parameters, to test the stationarity using Augmented Dickey-Fuller (ADF) – Eviews 7

By obtaining ADF statistics of -1.3051, -0.2027 and -0.9981 while the critical values related to the confidence levels of 1%, 5% and 10% are -2.6610, -1.9473, -1.6127, the null hypotesis cannot be rejected. Therefore, factors may have unit roots.

Generally, to stationarize time series is applied the first differentiation operator and the logaritm operator. In our case, parameters are associated with yields and is not indicated to apply any of them. As in Diebold and Li model (2006), two of three parameters are non-stationary. Therefore, I will continue to use the same time series.

On the second step of the model, factors are forecasted using a autoregressive vector VAR(1). In order to use the results at this step also in Kalman filter, where we work with mean adjusted factors, I will include also a constant, which is considered the mean of each parameter.

$$\hat{y}_{t+h/t}(\tau) = \hat{\beta}_{1,t+h/t} + \hat{\beta}_{2,t+h/t}\left(\frac{1-e^{-\lambda_\tau}}{\lambda_\tau}\right) + \hat{\beta}_{3,t+h/t}\left(\frac{1-e^{-\lambda_\tau}}{\lambda_\tau} - e^{-\lambda_\tau}\right) \quad (12)$$

$$\hat{\beta}_{t+h/t} = \hat{c} + \hat{\Gamma}\hat{\beta}_t \quad (13)$$

6. Empirical results: Fitting the Romanian yield curve using Kalman filter

The parameter vector is composed by: the transition matrix A that has 9 parameters; μ vector with the average of the three dynamic factors; Λ matrix – just λ; the residual covariance matrix Q of the transition equation (6 parameters); the residual covariance matrix H of yields on each maturity (5 parameters, being a diagonal matrix).

Therefore, the vector is composed by 24 parameters that will be numerical optimized through Kalman filter. The big number of parameters while having available just 5 maturities and 52 observations, most probably will make difficult the estimations.

Results obtained by using SSM vs. Two-Step method

SSM State Transition Matrix (A):

0.8357	-0.0338	0.0978
0.1214	0.9605	-0.0476
0.6267	0.0243	0.7204

Two-Step State Transition Matrix (A):

0.9221	-0.0320	0.0612
0.0099	0.8833	0.0247
0.4945	0.2493	0.6656

Table 3. Comparison of transition matrix A from SSM model with the coefficient matrix obtained from VAR(1) model.

SSM State Disturbance Covariance Matrix (Q = BB'):

0.0517	-0.0230	-0.0245
-0.0230	0.1279	-0.0100
-0.0245	-0.0100	0.3393

Two-Step State Disturbance Covariance Matrix (Q):

0.0588	-0.0251	-0.0465
-0.0251	0.1426	-0.0311
-0.0465	-0.0311	0.4357

Table 4. Comparison of innovation covariance estimate through SSM model with the covariance matrix obtained in the VAR(1)

6. Empirical results: Fitting the Romanian Yield Curve

Results obtained by using SSM vs. Two-Step method

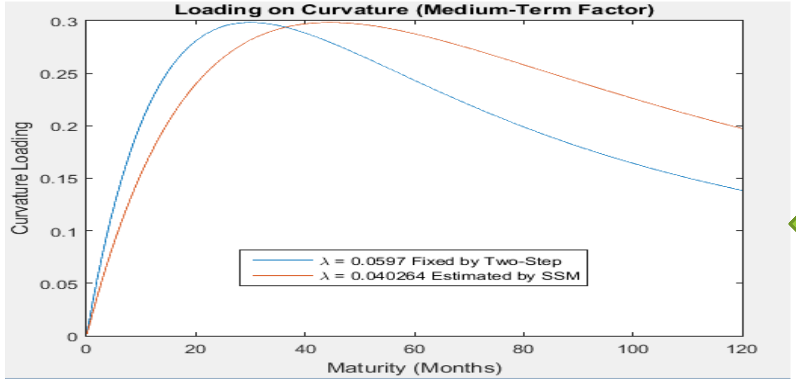

Fig. 6. Loading of curvature computed by setting λ at 0.597 in the Two-Step approach and by estimation with Kalman filter.

Estimated λ is 0.0403, which corresponds to a maturity of τ = 44 months (3.6 years), where the loading of the curvature is maximum.

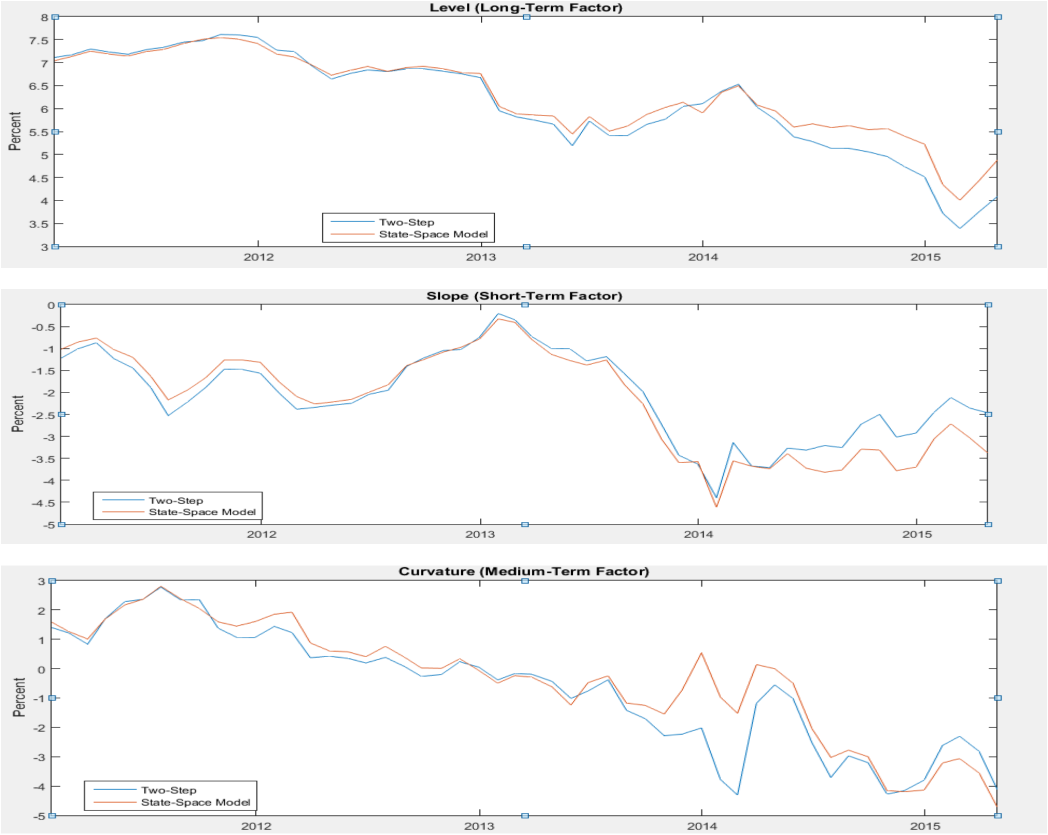

Fig. 7. Evolution of level, slope, and curvature using the Two Step method and the SSM

Maturity (Months)	State-Space Model Mean (bps)	State-Space Model Standard Deviation (bps)	Two-Step Mean (bps)	Two-Step Standard Deviation (bps)
6.0000	0.1250	10.3781	0.5321	3.8003
12.0000	-0.4168	3.8536	-1.4289	6.0318
36.0000	8.3350	10.4992	3.8880	4.5621
60.0000	-0.6466	1.9017	-4.6086	6.0478
120.0000	0.0003	0.0021	1.6174	2.7920

SSM model, even if it does not have always the best performance on each maturity, offers a better calibration on short maturities (6M) and long maturities (5Y and 10Y).

Table 5. Average errors and standard deviations of estimated yields resulted in Two Step approach and SSM

6. Empirical results: Fitting the German Yield Curve

- Monthly average bid-ask yields extracted from Thomson Reuters;
- Period analyzed is 31.05.2005 – 31.05.2015;
- 121 observations; 12 maturities: 3, 6, 12, 24, 36, 48, 60, 72, 84, 96, 108, 120 months;
- The bond market in GE is by far more developed that in RO and it is expected the NS to perform much better

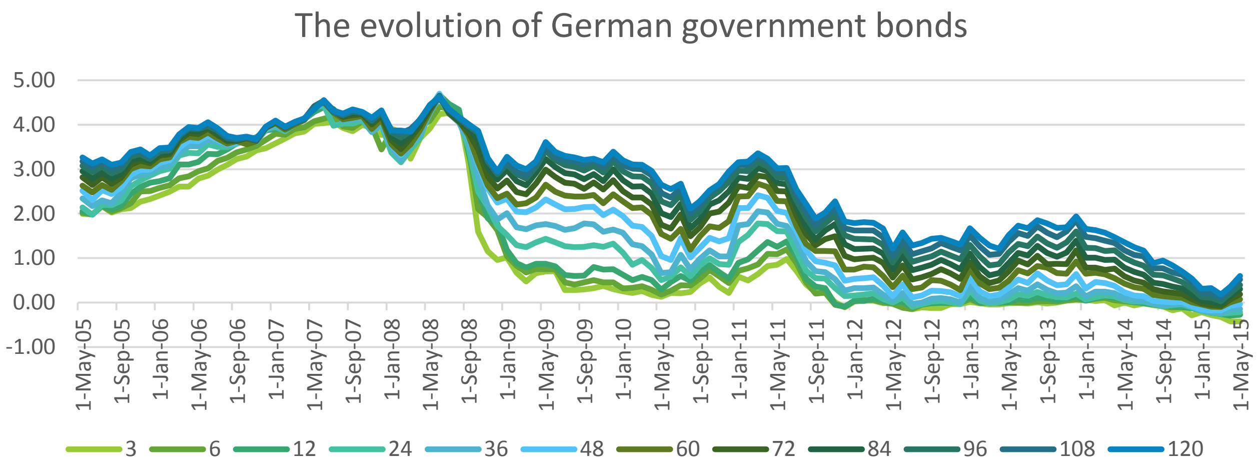

Fig 8. The evolution of German government bonds of various maturities on the analyzed period on the Nelson Siegel model

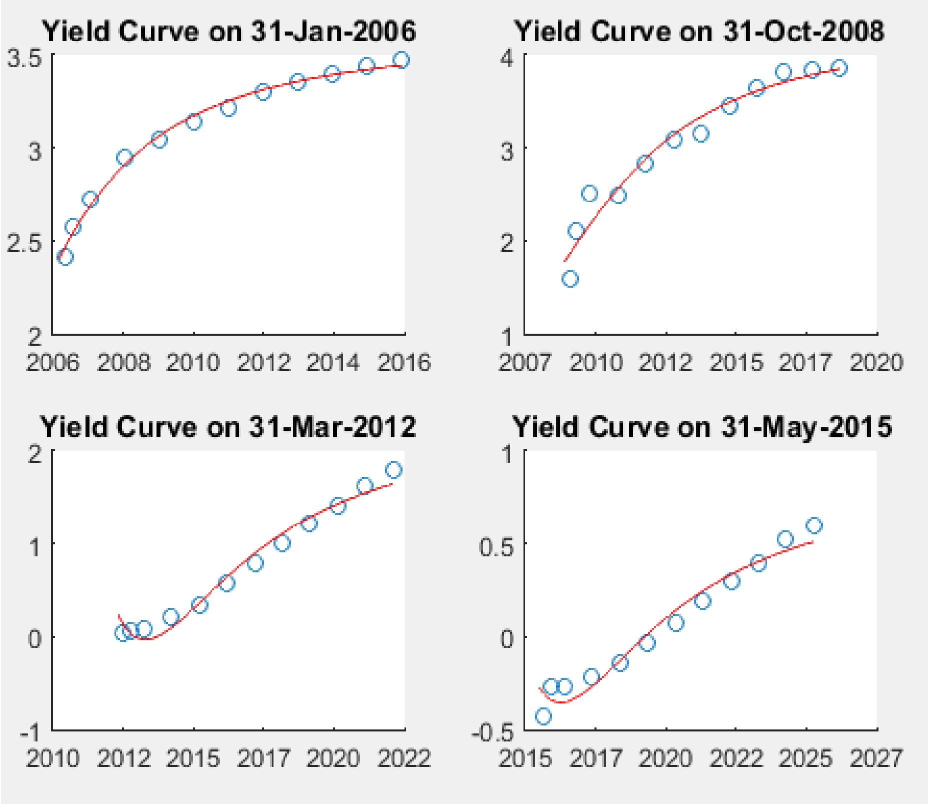

Fig 9. Yield curves fitted at various periods. Blue circles are actual yields from the market.

6. Empirical results: Fitting the German Yield Curve through Kalman filter

The vector of parameters has 31 components: the transition matrix A with 9 parameters; the vector μ with the averages of the three factors; matrix Λ that has only λ; matrix Q of residuals covariance of the transition equation (6 parameters); matrix H of residual covariance of estimated yields (12 parameters).

Results obtained by using SSM vs. Two-Step method

SSM State Transition Matrix (A):

0.9303	-0.0073	0.0074
0.0864	0.9850	-0.0015
0.2098	0.0706	0.8968

Two-Step State Transition Matrix (A):

0.9609	0.0135	-0.0041
0.0146	0.9261	0.0323
0.2288	0.2203	0.7905

Table 6. Comparison of transition matrix A from SSM model with the coefficient matrix obtained from VAR(1) model.

SSM State Disturbance Covariance Matrix (Q = BB'):

0.1395	-0.1490	-0.2224
-0.1490	0.2029	0.2094
-0.2224	0.2094	0.8235

Two-Step State Disturbance Covariance Matrix (Q):

0.0639	-0.0613	-0.0464
-0.0613	0.1117	0.0199
-0.0464	0.0199	0.3786

Table 7. Comparison of innovation covariance estimate through SSM model with the covariance matrix obtained in the VAR(1)

The dynamic of factors L_t, $S_t$and C_t is persistent and covariances between these factors and L_{t-1}, $S_{t-1}$and C_{t-1} in MLE is 0.93, 0.98 and 0.98.

The cross-factor dynamics is insignificant.

As in Romania's case, in the residuals covariance matrix obtained in both approaches the volatility of shocks increases from L_t to C_t.

6. Empirical results: Fitting the German Yield Curve

Results obtained by using SSM vs. Two-Step method

	State-Space Model		Two-Step	
Maturity (Months)	Mean (bps)	Standard Deviation (bps)	Mean (bps)	Standard Deviation (bps)
3.0000	-7.3576	13.3892	-9.0524	6.3615
6.0000	0.0725	7.0921	2.8509	5.8892
12.0000	3.7982	6.2715	10.6023	7.0227
24.0000	0.5956	6.0031	5.3941	6.8042
36.0000	-2.1995	6.1673	-2.8814	4.0368
48.0000	-0.1893	4.4436	-4.9484	4.7051
60.0000	-0.7964	4.3529	-7.0475	4.8502
72.0000	-0.1322	1.1168	-5.4821	5.1221
84.0000	0.0868	1.3138	-2.5560	4.3123
96.0000	0.0683	0.1850	1.3355	2.3015
108.0000	-1.3510	3.4947	4.5343	3.9295
120.0000	-3.5937	6.2316	7.2507	7.0685

Table 8. Errors average and standard deviations of estimated yields using Two-Step approach and SSM

For Germany, the high number of available observations offsets the big number of parameters estimated by Kalman filter. Therefore, except the short maturities (3 and 6 months), standard deviations of estimated errors are smaller than those resulted in the Two-Step approaches.

6. Empirical results: Nelson Siegel (NS) vs Random Walk (RW)

Random Walk:

Past changes of yields cannot be used to forecast future evolution because subsequent variations of yields are independent

$$y_t(\tau_i) = y_{t-1}(\tau_i) + \varepsilon_t(\tau_i)\,, \qquad \varepsilon_t(\tau_i) \sim \mathrm{N}(0,\sigma^2(\tau_i)) \tag{14}$$

AR(1):

$$\hat{y}_{t+h/t}(\tau) = \hat{\beta}_{1,t+h/t} + \hat{\beta}_{2,t+h/t}\left(\frac{1-e^{-\lambda_\tau}}{\lambda_\tau}\right) + \hat{\beta}_{3,t+h/t}\left(\frac{1-e^{-\lambda_\tau}}{\lambda_\tau} - e^{-\lambda_\tau}\right) \tag{15}$$

where: $\hat{\beta}_{i,t+h/t} = \hat{c}_i + \hat{\gamma}_i \hat{\beta}_t$, i=1, 2, 3

For both models, NS and RW, in the out−of−sample forecast I used as estimation subsample ¾ of the database, resulting ¼ of database used as forecasted subsample:

RO – estimation period 31.01.2011 – 30.04.2014, forecasted period 30.05.2014 – 30.04.2015

GE − estimation period 31.05.2005 – 31.01.2012, forecasted period 29.02.2012 – 31.05.2015

Forecasted and compared horizons are 1m, 6m and 1y. RMSE for a horizon of *h* months between periods t_1 and $t_2$is computed as follows:

$$RMSE_{t_1,t_2} = \sqrt{\frac{1}{t_2 - t_1 + 1}\sum_{t=t_1}^{t_2}\left(y_{t+h}^h + y_{t+h|t}^h\right)^2} \tag{16}$$

6. Empirical results: Nelson Siegel (NS) vs Random Walk (RW)

1M horizon	ROMANIA		GERMANIA	
	NS	RW	NS	RW
6 M	**0.0311**	0.0757	**0.0243**	0.0340
12 M	0.2952	**0.1464**	0.0928	**0.0455**
36 M	**0.2348**	0.3414	**0.0131**	0.3275
60 M	0.4560	**0.3619**	**0.0772**	0.7400
120 M	**0.3706**	0.3767	**0.1524**	1.7895

Table 9 Results of the out-of-sample forecasting using Nelson-Siegel (NS) and Random Walk (RW) for government bonds in Romania and Germany for 1 month horizon. Maturities analyzed are 6, 12, 36, 60 and 120 months.

6M horizon	ROMANIA		GERMANIA	
	NS	RW	NS	RW
6 M	0.5091	**0.4922**	**0.0150**	0.1004
12 M	0.8083	**0.6130**	0.1446	**0.0661**
36 M	**1.0829**	1.1421	**0.0202**	0.2005
60 M	1.1959	**1.0764**	0.1224	**0.2567**
120 M	0.8658	**0.8541**	**0.1544**	0.3308

Table 10 Results of the out-of-sample forecasting using Nelson-Siegel (NS) and Random Walk (RW) for government bonds in Romania and Germany for 6 months horizon. Maturities analyzed are 6, 12, 36, 60 and 120 months.

12M horizon	ROMANIA		GERMANIA	
	NS	RW	NS	RW
6 M	1.0405	**0.9529**	**0.0303**	0.0803
12 M	1.3085	**1.0557**	0.1539	**0.0649**
36 M	**1.6753**	1.6942	**0.0237**	0.2291
60 M	1.8585	**1.7122**	**0.1316**	0.2883
120 M	1.5144	**1.4855**	**0.1582**	0.3551

Table 11. Results of the out-of-sample forecasting using Nelson-Siegel (NS) and Random Walk (RW) for government bonds in Romania and Germany for 12 months horizon. Maturities analyzed are 6, 12, 36, 60 and 120 months.

For 1M horizon, NS model applied on the Romanian market offers a better performance on various maturities (6m, 3y, 10y) compared to RW model, while for Germany NS results are better on most of the selected maturities, excepting 1Y tenor.

Regarding forecasting the YC on a horizon of 6M and 12M, RMSE is smaller for Romania only on the 3y maturity.

7. Conclusions

- Results of the yield curve fitting on the Romanian market are encouraging, in spite of the small number of maturities and observations. On the other side, results on fitting the German yield curve are much better. These differences of performance were expected due to discrepancies between the economic development of the two countries, including of course their financial markets.

- The three estimated dynamic parameters can be associated on both states with the level, slope and curvature, according to Diebold and Li’s approach (2006), because they are highly corelated with the computed factors.

- Results obtained on the two aproaches of the NS model (the Two-Step and MLE) are similar, however more closely on Germany’s case. Using MLE method is preferable because in the Two-Step method parameters used in the 2nd step does not take into account the uncertainty of the estimations done in the first part, generating inefficient parameters estimations.

- On a 1M horizon, the NS model applied on the RO government bonds offers a better performance on various maturities (6m, 3y, 10y) comparing to RW, while for GE, results are better on most cases in the NS model, excepting 1y tenor. Regarding the forecast for a horizon of 6m and 12m, RMSE is smaller for RO just on the 3y maturity.

- For GE bond market, the out-of-sample forecasting of the yield curve realized using the NS model is much closer to the actual yield curve than using the RW model on almost all maturities, for all analyzed horizons. Obtaining a better performance on the German market was expected taking into account its high development.

- In spite of the progress done by the RO bond market by increasing the depth and the liquidity and by attracting foreign investors, RO has to make further and considerable progress in order to reach the level of GE.

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Thank you!